On the μ-Meson Capture

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SUPPLEMENT is made to the work, "On the two-meson theory" previously published.[1] It concerns the process of mu-meson capture: Firstly, Z-dependence of the capture probabilities; secondly, excitation energy of the nucleus after mu-meson capture; and thirdly, possibilities of interaction schemes between mesons, leptons, and nucleons.

The theories of mu-meson capture have already been given by various workers,[2-9] and the capture probabilities are shown to depend on, at least, the fourth power of Z. In our paper,[1] using the formula for K-electron capture.

$$1/\tau_{\mu}\cdot\text{cap.}\alpha(\alpha Z)^{2S+1},\quad S=(1-\alpha^2Z^2)^{\frac{1}{2}}$$

which seems to be proportional to the third power of Z. However, if we take into account the Z-dependence of the unknown nuclear matrix elements, which may certainly be proportional to, at least, the first power of Z, the current result will be derived.

On the other hand, if we assume that the nuclear wave is described as a plane wave inside the nucleus and zero amplitude outside, the transition probability that the nucleon acquires the momentum $(k, k+dk)$ and the neutrino is emitted with the momentum $(\mathbf{p}, \mathbf{p}+d\mathbf{p})$ will be given as follows (in the case of scalar coupling):

$$\delta(1/\tau_{\mu}\cdot\text{cap.})\sim Z^3f(y)P^2d\Omega_p\rho_k,\quad y=|\mathbf{k}+\mathbf{p}-\mathbf{k}_0|R \tag{1}$$

where $\mathbf{k}_0$ is the average momentum of the initial protons and R is the nuclear radius. $f(y)=(y^{-\frac{1}{2}}J_{\frac{3}{2}}(y))^2$ has the value ~ 0.70 at $y=0$, and decreases rapidly as y increases, with $\exp(-y^2/4)$. Thus, the region where (1) has appreciable magnitude is confined in the small interval of $y\leq 4$. Further, the average initial momentum of protons and that of the neutrino do not depend on Z, since the latter carries away the greater part of the available energy, which enables one to put $|\mathbf{p}|\approx\kappa$, where $\kappa=m_\mu c^2/\hbar$. Accordingly, the region which contributes mainly to the probability may depend essentially on k and thus is proportional to R^{-3}. Since the final level density of k, ρ_k, is, however, proportional to R^3, the integration with respect to the final level is almost independent on Z. The only factors which show the charge dependence are the volume concentration of mu-meson in the K-orbit, together with the number of protons Z. From these facts we can roughly estimate that the total probability would behave as the fourth power of Z.

Next, if the integration with respect to angle is done, the total probability will be given as a function of nucleon momentum k. This curve has a sharp maximum at x_0, and the shape around the peak is given approximately by $\exp[-\frac{1}{4}R^2\kappa'^2(x-x_0)^2]$, where $x=k/\kappa'$, $\kappa'=\langle(p+\kappa_0)^2\rangle_{\text{Av}}^{\frac{1}{2}}\approx 2\kappa$. The value of $R\kappa'$ is fairly large, excepting the light nuclei, and the maximum point, x_0, is given,

$$x_0=1/\tanh\tfrac{1}{2}R^2\kappa' x_0. \tag{2}$$

For large Z, the value of x_0 is nearly equal to unity and increases slowly as Z decreases. Since the peak is sharp, we may safely identify this x_0 with the average value of momentum of the excited nucleon, which is given as follows:

$$k^2\hbar C/2M\kappa-\bar{E}=x_0^2/2^0-\bar{E}, \tag{3}$$

where $\bar{E}$ is the average kinetic energy of the nucleon in the ground state and is 0.14 in the unit of $m_\mu c^2\cong 100$ Mev. From (3) we can see that the excitation energy is 6 Mev for large Z, and ~ 14 Mev for small Z, and slowly varying between these two values for the intermediate Z.

Finally, possible interaction schemes between elementary particles will be discussed. We have suggested a model given in Fig. 1. Alternative models, such as those shown in Fig. 2, are put forward by various workers. In the latter, a pi-meson may decay, through virtual nucleon pairs, either into leptons or into a mu-meson and a neutrino. Whereas the direct coupling between nucleons-leptons or nucleons-mu-mesons are shown to be the same,[2] the final momentum density is larger for leptons than for mu-mesons, which will make the lifetime of pi-mesons for pi-electron decay essentially faster than that for pi-mu-decay.[10] This seems to be at variances with the recent experiments of Berkeley Therefore the model given in Fig. 2 should be ruled out.

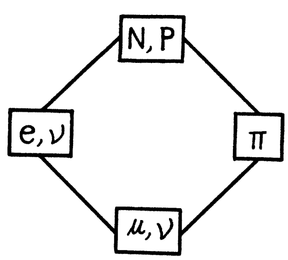

FIG. 1. Suggested interaction scheme between elementary particles.

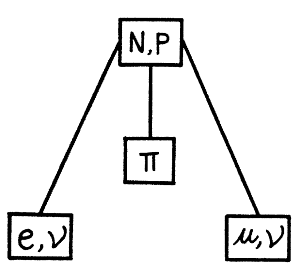

FIG. 2. Alternative interaction scheme proposed by others.

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[9] Y. Tanikawa, Prog. Theor. Phys. 3, 314, 315 (1948).
[10] Of course, this conclusion may suffer some ambiguities due to divergent difficulties of present field theory and varieties of interaction types between elementary particles. Private discussions by H. Fukuda and Y. Miyamoto.

Statistical Mechanics of Mixtures of He^3 and He^4

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THE measurements[1] of the vapor-liquid concentration ratio of dilute solutions of He^3 and He^4 show that for temperatures *above the* λ-*point* these solutions behave as "perfect solutions" leading to the validity of Raoults law:

$$C^V/C^L=P_3^0/P_4^0, \tag{1}$$

where P_3^0 and P_4^0 are the vapor pressures[2] of the pure substances He^3 and He^4.

If the mixture of He^3 and He^4 were a perfect mixture at all temperatures and concentrations, the expression for the free energy for a perfect mixture

$$F=(1-x)F_4^0+xF_3^0+RT\{x\ln x+(1-x)\ln(1-x)\}. \tag{2}$$

F_3^0 and F_4^0 being the free energies of the pure substances and x being the relative concentration $N_3/(N_3+N_4)$ of He^3, leads to the conclusion that the temperature of the transition of phase I into II is independent of concentration ($dT/dx=0$) and that the transition remains second order ($x^{\text{I}}=x^{\text{II}}$) at all concentrations.

According to Taconis and Beenakker[3] the vapor-liquid concentration ratio deviates from Rauoult's law at temperatures *below the* λ-*point*, being given by

$$C^V/C^L=(1/y)(P_3^0/P_4^0), \tag{3}$$

where $y=N_4^n/N_4$ is the ratio of the number of molecules in the "normal" state to the total number of molecules of He^4, a result which is in agreement with experiments of Daunt *et al.*[4,5] showing that the molecules of He^3 do not take part in the superflow of liquid helium II. This was explained by assuming that the He^3 molecules only mix with the normal part of He^4. If this assumption