

On the μ -Meson Capture

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SUPPLEMENT is made to the work, "On the two-meson theory" previously published.¹ It concerns the process of mu-meson capture: Firstly, Z -dependence of the capture probabilities; secondly, excitation energy of the nucleus after mu-meson capture; and thirdly, possibilities of interaction schemes between mesons, leptons, and nucleons.

The theories of mu-meson capture have already been given by various workers,²⁻⁹ and the capture probabilities are shown to depend on, at least, the fourth power of Z . In our paper,¹ using the formula for K -electron capture.

$$1/\tau_{\mu\text{-cap}} \propto (\alpha Z)^{2S+1}, \quad S = (1 - \alpha^2 Z^2)^{1/2}$$

which seems to be proportional to the third power of Z . However, if we take into account the Z -dependence of the unknown nuclear matrix elements, which may certainly be proportional to, at least, the first power of Z , the current result will be derived.

On the other hand, if we assume that the nuclear wave is described as a plane wave inside the nucleus and zero amplitude outside, the transition probability that the nucleon acquires the momentum (k , $k+d\mathbf{k}$) and the neutrino is emitted with the momentum (\mathbf{p} , $\mathbf{p}+d\mathbf{p}$) will be given as follows (in the case of scalar coupling):

$$\delta(1/\tau_{\mu\text{-cap}}) \sim Z^3 f(y) P^2 d\Omega_p \rho k, \quad y = |\mathbf{k} + \mathbf{p} - \mathbf{k}_0| R \quad (1)$$

where \mathbf{k}_0 is the average momentum of the initial protons and R is the nuclear radius. $f(y) = (y^{-1/2} J_{3/2}(y))^2$ has the value ~ 0.70 at $y=0$, and decreases rapidly as y increases, with $\exp(-y^2/4)$. Thus, the region where (1) has appreciable magnitude is confined in the small interval of $y \leq 4$. Further, the average initial momentum of protons and that of the neutrino do not depend on Z , since the latter carries away the greater part of the available energy, which enables one to put $|\mathbf{p}| \approx \kappa$, where $\kappa = m_\nu c^2/\hbar$. Accordingly, the region which contributes mainly to the probability may depend essentially on k and thus is proportional to R^{-3} . Since the final level density of k , ρ_k , is, however, proportional to R^3 , the integration with respect to the final level is almost independent on Z . The only factors which show the charge dependence are the volume concentration of mu-meson in the K -orbit, together with the number of protons Z . From these facts we can roughly estimate that the total probability would behave as the fourth power of Z .

Next, if the integration with respect to angle is done, the total probability will be given as a function of nucleon momentum k . This curve has a sharp maximum at x_0 , and the shape around the peak is given approximately by $\exp[-\frac{1}{2} R^2 \kappa^2 (x - x_0)^2]$, where $x = k/k'$, $k' = ((p + \kappa_0)^2/\hbar^2)^{1/2} \approx 2\kappa$. The value of Rk' is fairly large, excepting the light nuclei, and the maximum point, x_0 , is given,

$$x_0 = 1/\tanh \frac{1}{2} R^2 \kappa' x_0. \quad (2)$$

For large Z , the value of x_0 is nearly equal to unity and increases slowly as Z decreases. Since the peak is sharp, we may safely

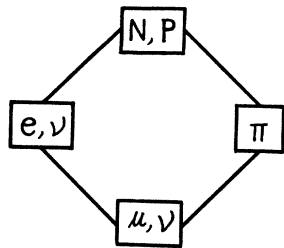


FIG. 1. Suggested interaction scheme between elementary particles.

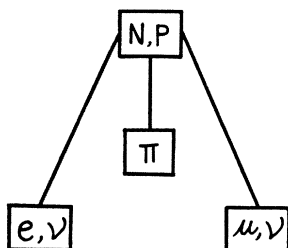


FIG. 2. Alternative interaction scheme proposed by others.

identify this x_0 with the average value of momentum of the excited nucleon, which is given as follows:

$$k^2 \hbar C / 2M\kappa - \bar{E} = x_0^2 / 2 - \bar{E}, \quad (3)$$

where \bar{E} is the average kinetic energy of the nucleon in the ground state and is 0.14 in the unit of $m_\nu c^2 \approx 100$ Mev. From (3) we can see that the excitation energy is 6 Mev for large Z , and ~ 14 Mev for small Z , and slowly varying between these two values for the intermediate Z .

Finally, possible interaction schemes between elementary particles will be discussed. We have suggested a model given in Fig. 1. Alternative models, such as those shown in Fig. 2, are put forward by various workers. In the latter, a pi-meson may decay, through virtual nucleon pairs, either into leptons or into a mu-meson and a neutrino. Whereas the direct coupling between nucleons-leptons or nucleons-mu-mesons are shown to be the same,² the final momentum density is larger for leptons than for mu-mesons, which will make the lifetime of pi-mesons for pi-electron decay essentially faster than that for pi-mu-decay.¹⁰ This seems to be at variance with the recent experiments of Berkeley. Therefore the model given in Fig. 2 should be ruled out.

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- ³ J. L. Lopes, *Phys. Rev.* **74**, 1722 (1948).
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- ⁷ T. Inoue, and S. Ogawa, *Prog. Theor. Phys.* **3**, 319 (1948).
- ⁸ S. Hayakawa, *Prog. Theor. Phys.* **3**, 200 (1948).
- ⁹ Y. Tanikawa, *Prog. Theor. Phys.* **3**, 314, 315 (1948).
- ¹⁰ Of course, this conclusion may suffer some ambiguities due to divergent difficulties of present field theory and varieties of interaction types between elementary particles. Private discussions by H. Fukuda and Y. Miyamoto.

Statistical Mechanics of Mixtures of He³ and He⁴

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THE measurements¹ of the vapor-liquid concentration ratio of dilute solutions of He³ and He⁴ show that for temperatures above the λ -point these solutions behave as "perfect solutions" leading to the validity of Raoult's law:

$$C^V/C^L = P_3^0/P_4^0, \quad (1)$$

where P_3^0 and P_4^0 are the vapor pressures² of the pure substances He³ and He⁴.

If the mixture of He³ and He⁴ were a perfect mixture at all temperatures and concentrations, the expression for the free energy for a perfect mixture

$$F = (1-x)F_4^0 + xF_3^0 + RT \{x \ln x + (1-x) \ln(1-x)\}, \quad (2)$$

F_3^0 and F_4^0 being the free energies of the pure substances and x being the relative concentration $N_3/(N_3+N_4)$ of He³, leads to the conclusion that the temperature of the transition of phase I into II is independent of concentration ($dT/dx=0$) and that the transition remains second order ($x^I=x^{II}$) at all concentrations.

According to Taconis and Beenakker³ the vapor-liquid concentration ratio deviates from Raoult's law at temperatures below the λ -point, being given by

$$C^V/C^L = (1/y)(P_3^0/P_4^0), \quad (3)$$

where $y = N_4^v/N_4^l$ is the ratio of the number of molecules in the "normal" state to the total number of molecules of He⁴, a result which is in agreement with experiments of Daunt *et al.*^{4,5} showing that the molecules of He³ do not take part in the superflow of liquid helium II. This was explained by assuming that the He³ molecules only mix with the normal part of He⁴. If this assumption

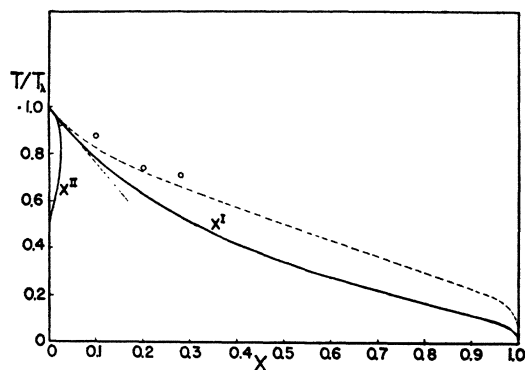


FIG. 1. Variation of concentrations x^I and x^{II} with temperature. The experimental points were measured by Abraham, Weinstock, and Osborne.

could be generalized to the hypothesis, that He^3 and He^4 behave like a perfect mixture of He^3 in the normal component of He^4 , then the free energy (2) must be replaced by

$$F = (1-x)F_4^0 + xF_3^0 + RT(x/x_a) \{x_a \ln x_a + (1-x_a) \ln(1-x_a)\}, \quad (4)$$

where x_a is the "apparent" concentration; $N_3/(N_3+N_4^n)$, $y = N_4^n/N_4$ as before and where the connection between x_a , x and y is given by $x_a = x/(y+x(1-y))$.

The equations of equilibrium: $\mu_3^I = \mu_3^{II}$ and $\mu_4^I = \mu_4^{II}$ for the thermodynamic potentials of the components 3 and 4 in both phases I and II, derived from (4), lead to the result:

$$x_a^I = x_a^{II}, \quad (5a)$$

$$\ln(1-x_a^I) = -(F_4^0 I - F_4^0 II)/RT(y^I - y^{II}). \quad (5b)$$

It is assumed that y is a function of temperature *only* and independent of concentration as it should be for a perfect solution. For the high temperature phase I, $y^I = 1$; whereas in the low temperature phase II, $y^{II} = N_4^n/N_4 = (T/T_\lambda)^\nu$ with $\nu = 5.6$. Equation (5a) shows that the concentrations of He_3 in phases I and II are no longer equal. Therefore, the transition is no longer of the second order but becomes of the first order, the relation between the concentrations being $x^I = x^{II}/(y^{II} + x^{II}(1-y^{II}))$. The actual calculation of the concentrations x^I and x^{II} as a function of the transition temperature depends on the extrapolation of the free energy curve of the high temperature phase I below $T = T_\lambda$. In Fig. 1, the results have been plotted extrapolating the entropy curve of the phase II, linearly to $\tau = 0$, as has been suggested by Daunt and Mendelssohn.⁶ The broken curve corresponds to a linear extrapolation of the free energy curve, as suggested in the liquid He^4 model of Gorter. In both cases the concentration of He_3 in the liquid phase at lower temperatures is extremely small, its value being approximately $x^{II} = x^I y^{II}/(1-x^I)$. The experimental points in Fig. 1 were measured by Abraham, Weinstock, and Osborne.⁷

The general formula for the vapor-liquid concentration ratio is given by

$$C^V/C^L = (1/y)(1-x_a)^{(1-y)}(P_3^0/P_4^0) \quad (6)$$

approaching to Taconis and Beenakker's formula at low concentrations.

The first-order character of the transition of phase I into phase II, which is demonstrated in Fig. 1, leads to interesting conclusions. The fact that below temperatures of about $0.5T_\lambda$ the value of x^{II} is approximately zero makes it possible to speak in this temperature region of a pure phase II of He^4 which is in equilibrium with an ideal mixture of He^4 and He^3 in phase I. If a mixture of He^3 and He^4 of concentration x is cooled down, pure liquid He^4 starts to be formed in a separate phase II at the transition temperature determined by $x = x^I(T)$. When the temperature is decreased still more, the amount of pure liquid He^4 in phase II increases, the total fraction of substance, which at a

temperature T is left as a perfect mixture in phase I, being $x/x^I(T)$.

Of course, the present remarks must only be considered as a possibility, which is based entirely on the hypothesis that He^3 only mixes as a perfect mixture with the normal part of He^4 and that the interpretation in terms of the free energy expression (4) is true. A more detailed account will appear in *Physica*. The author is indebted to Dr. Daunt for many stimulating discussions on this subject.

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** The author is very much indebted to Dr. Osborne for putting these data at his disposal before publication.

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The Beta-Spectrum of Tritium

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IN continuation of the investigation of the β -spectrum of H^3 reported elsewhere,¹ a more detailed study has been made of the shape of the spectrum near the end point. The proportional counter technique previously described² has again been used with a photographic method of recording and analyzing the output pulses.

This study was designed to increase the accuracy of the determination of the shape of the spectrum close to the end-point energy E_0 , and also to obtain a more accurate estimate of the correction which must be made to the spectrum in view of the experimental limitations resulting from the finite resolution of the instrument.

Energy calibration was provided by a beam of K_α x-rays of molybdenum (17.4 kev). The half-width for half-intensity for this line was 900 ev (~ 5 percent), and the parts of the theoretical Fermi curves shown in Fig. 1 have been modified to allow for the spread arising from a resolution corresponding to this line width. The curves have been calculated for true upper energy limits (E_0) of 18.0 and 18.6 kev with a neutrino mass $\mu = 0$ kev, and for $E_0 = 18.0$ kev and $\mu = 1$ kev. They may therefore be compared directly with the observed experimental points. For $\mu = 0$ the points lie between the two curves for $E_0 = 18.0$ and $E_0 = 18.6$ kev, which brings our results into rather close agreement with those of Hanna and Pontecorvo;³ the higher energy value would also give slightly better agreement with the theoretical Fermi curve over the whole energy range previously investigated.¹ Alternatively, for $E_0 = 18.0$ kev, it may be assumed that μ lies between 0 and 1

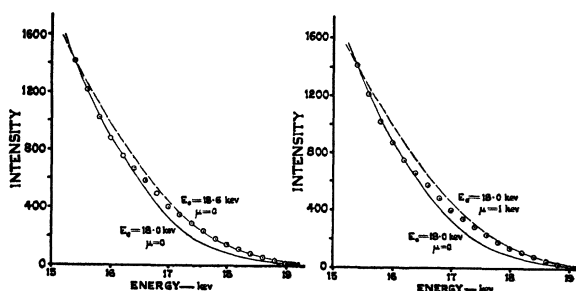


FIG. 1. Comparison of experimental and theoretical Fermi curves for tritium near the end point.