The Optical Detection of Radiofrequency Resonance*

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Calculations are made of the frequency, intensity, and polarization of the light emitted by an atom in a $^2P-^2S$ transition in a weak magnetic field having an oscillating component. Near resonance, or when the frequency of the oscillating component approaches the Larmor frequency of one of the two states involved in the radiation process, all of the quantities calculated are modified. Qualitatively similar results are to be expected for other transitions. Observations in very weak fields would be of particular interest because they would give information on nuclear spins as well as atomic g-factors. Under such conditions, the Zeemann components will in general not be resolvable. There will, however, be a change in the polarization of the edges of a spectral line at resonance. It is shown that very small changes in this polarization structure are theoretically detectable.

A HOPE of drastically extending our knowledge of nuclear structure lies in further application of the resonance method, i.e., the direct measurement of the spacing between the hyperfine-structure components of an energy level. A new method is here proposed to detect the condition of resonance when it is established—namely a change in the radiation emitted by an atom when it is subjected to an oscillating field at the resonance frequency in the above sense.

For simplicity, the discussion is confined to the resonance line ($^2P-^2S$) of an atom having one optical electron, and it is further assumed that there is no hyperfine structure. The resonance to be investigated is that corresponding to the Larmor frequency of the system in very weak fields, or, in other words, corresponding to the Zeemann splitting of one of the levels concerned. Although this discussion specifically excludes nuclear effects, it is clear that any method of following the ordinary Zeemann effect into the region of very weak fields will produce information about nuclear moments when the Zeemann splitting becomes of the same order of magnitude as any hyperfine structures which may exist.

The wave functions describing an atom in a magnetic field having an oscillating component are known.[1] They are the solutions of the Schroedinger time-dependent equation, and contain one arbitrary constant which is usually so chosen as to define the initial state of the system. In an atomic-beam experiment, for instance, this constant specifies the state of the system when it enters the oscillating field, and the solution makes it possible to specify the probability of finding the system in any particular state at any subsequent time. The problem here proposed is physically somewhat different. Consider a gas of the atoms under consideration at extremely low pressures so that collision effects may be neglected. All the atoms in the container holding the gas are subjected to the same field. In this case, the arbitrary constant in the wave function of the atoms in the ground state must be so chosen that, if the oscillating field is slowly removed, the wave functions go over into the normal wave functions for the atom in a constant magnetic field.

For an atom having $J=\frac{1}{2}$, the desired solutions are

$$\frac{1}{\sqrt{2}}\left\{\left[\frac{-\delta}{(1+\delta^2)^{\frac{1}{2}}}+1\right]^{\frac{1}{2}}\psi_{\frac{1}{2}}+\left[\frac{\delta}{(1+\delta^2)^{\frac{1}{2}}}+1\right]^{\frac{1}{2}}\psi_{-\frac{1}{2}}\right\} \quad (1)$$

$$\frac{1}{\sqrt{2}}\left\{\left[\frac{\delta}{(1+\delta^2)^{\frac{1}{2}}}+1\right]^{\frac{1}{2}}\psi_{\frac{1}{2}}-\left[\frac{-\delta}{(1+\delta^2)^{\frac{1}{2}}}+1\right]^{\frac{1}{2}}\psi_{-\frac{1}{2}}\right\} \quad (2)$$

where δ is a number measuring the approach to resonance and defined below. At resonance $\delta=0$; on one side it is positive, and on the other negative. It is evident that these are the desired solutions in the sense that, far from resonance, they reduce to the standard forms describing the system for $m=\frac{1}{2}$ and $m=-\frac{1}{2}$. The mean

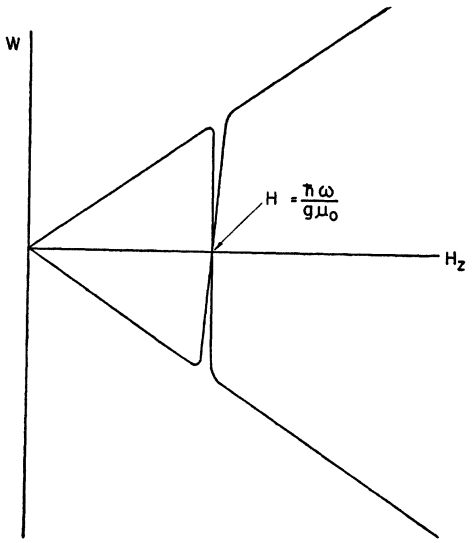

FIG. 1. Zeeman splitting of $J=\frac{1}{2}$ level in the presence of an oscillating field whose frequency is $\omega/2\pi$.

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[1] I. I. Rabi, Phys. Rev. **51**, 652 (1937); F. Bloch and I. I. Rabi, Rev. Mod. Phys. **17**, 237 (1945).

value of the energy of the system in these states is

$$W=\pm\frac{g\mu_0 H_z}{2}\left[\frac{\delta+H_0/H_z}{(1+\delta^2)^{\frac{1}{2}}}\right]; \tag{3}$$

where μ_0 is one Bohr magneton; H_z is the constant magnetic field; $\delta=H_z/H_0\times(\omega-\omega_0)/\omega_0$; H_0 is the amplitude of the rotating magnetic field. This field is at right angles to the constant field; ω is the angular velocity of the rotating field; ω_0 is the Larmor frequency $g\mu_0 H_z/\hbar$; and $ge/2mc$ is the gyromagnetic ratio. The derivation of Eqs. (1)–(3) is given in Appendix A.

We are now in a position to consider the effect of the rotating magnetic field H_0 on the Zeemann effect. The resonance line width in the above discussion is determined entirely by the amplitude of the rotating field. In actual gases, the levels will be broadened due to other causes. Investigations of resonance in the ground state will be of particular interest because of the sharpness of these levels.

From Eq. (3) it is clear that the energy levels have their normal positions except near resonance, where the level corresponding to $m=\frac{1}{2}$ changes places with the level corresponding to $m=-\frac{1}{2}$. A schematic plot of the effect on the energy levels is shown in Fig. 1, and the effect on the position of the Zeemann components is shown in Fig. 2.

The intensity and state of polarization of the lines can be readily computed. For the sake of brevity, we write a and b for the coefficients of the spin-wave functions in Eqs. (1) and (2), and x^2, y^2, and z^2 for numbers proportional to the squares of the components of the electric dipole moments along the x-, y-, and z-axes. The results are

$$\left.\begin{array}{llll} {}^2P_{3/2}-{}^2S_{\frac{1}{2}} & m=\frac{3}{2}\text{ to }\frac{1}{2} & x^2=y^2=3a^2 & z^2=0 \\ & \frac{3}{2}\text{ to }-\frac{1}{2} & x^2=y^2=3b^2 & z^2=0 \\ & \frac{1}{2}\text{ to }\frac{1}{2} & x^2=y^2=\ b^2 & z^2=4a^2 \\ & \frac{1}{2}\text{ to }-\frac{1}{2} & x^2=y^2=\ a^2 & z^2=4b^2 \end{array}\right\}. \tag{4}$$

$$\left.\begin{array}{llll} {}^2P_{\frac{1}{2}}-{}^2S_{\frac{1}{2}} & \frac{1}{2}\text{ to }\frac{1}{2} & x^2=y^2=2b^2 & z^2=2a^2 \\ & \frac{1}{2}\text{ to }-\frac{1}{2} & x^2=y^2=2a^2 & z^2=2b^2 \end{array}\right\}.$$

These results state that the resultant intensity and the polarization of all the light given off in any direction is independent of δ, and is the same in all directions. Further, since on one side of resonance $a=0$ and $b=1$, while on the other side of resonance $a=1$ and $b=0$, it would seem from Eq. (4) that there are certain differences in the Zeemann effect above and below resonance. When the interchange of the energy levels is taken into account, it appears that the Zeemann effect presents exactly the same appearance on either side of resonance.

Equation (4) does, however, predict certain changes in the position, intensity, and degree of polarization of spectral lines in the vicinity of resonance. The following consideration indicates that these changes will not be easy to detect experimentally. With specially constructed light sources having a very small Doppler effect, and with the best available instruments, usable resolving powers of 10^6 can be achieved. Since visible light has a frequency of the order of 0.5×10^{-15}, we have for the smallest resolvable frequency difference between two spectral lines

$$\frac{\Delta\nu}{\nu}=\frac{\Delta\nu}{0.5\times10^{-15}}=10^{-6}, \tag{5}$$

$$\Delta\nu=0.5\times10^{9}.$$

This is of the order of magnitude of the frequencies to be applied. It is therefore clear that the detail of the predicted phenomena will not be easy to observe. The presence of resonance can, however, be detected without resolving the Zeemann components, as may be seen from the following argument. Consider, for example, the polarization structure of the line resulting from the transition ${}^2P_{\frac{1}{2}}$ to ${}^2S_{\frac{1}{2}}$ in a field which is too weak to produce an observable resolution of the components. Although the line as a whole would be unpolarized, the edges would show a slight excess of σ-polarization, and the center of the line a slight excess of π-polarization. At resonance, this polarization structure disappears. Resonance might be detected by noting a change in this polarization structure. Extremely small effects of this kind can be observed by means of an optical system which transmits only the excess polarized light at the edge of a line, and by modulating the amplitude of this transmitted component by going in and out of resonance at some selectable frequency. The limitations of such a scheme are further discussed in Appendix B.

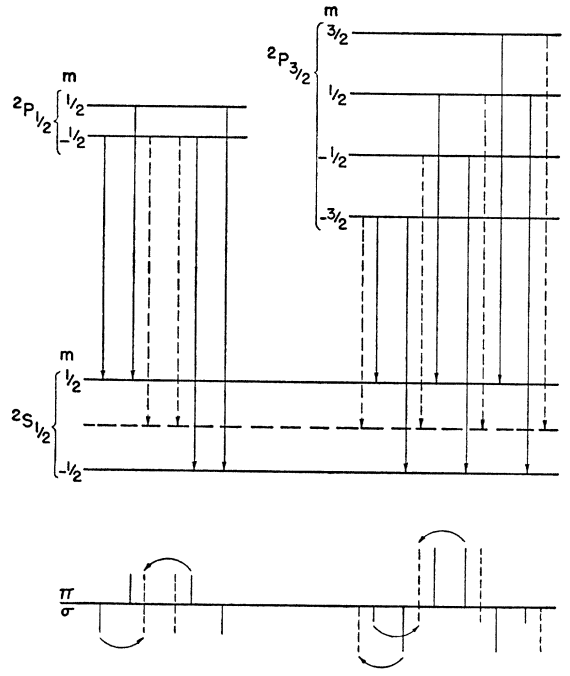

FIG. 2. ——— Normal Zeemann lines; — — — Zeemann lines at resonance in the ${}^2S_{\frac{1}{2}}$ state.

APPENDIX A

The equation to be solved is

$$i\hbar\dot{\psi}=g\frac{\mu_0}{2}(\sigma_1H_1+\sigma_2H_2+\sigma_3H_3),$$

where

σ are the spin matrices;
$H_1=H_0\cos\omega t$;
$H_2=H_0\sin\omega t$;
$H_3=H_z$.

The normalized solutions are

$$\psi=C_{\frac{1}{2}}\psi_{\frac{1}{2}}+C_{-\frac{1}{2}}\psi_{-\frac{1}{2}},$$
$$C_{\frac{1}{2}}=A_1\sin\alpha e^{ip_1t}+A_2\cos\alpha e^{ip_2t},$$
$$C_{-\frac{1}{2}}=-A_2\sin\alpha e^{-ip_2t}+A_1\cos\alpha e^{-ip_1t},$$
$\alpha=$integration constant,
$$p_1=-\frac{\omega}{2}+\frac{H_0}{H_z}\frac{\omega_0}{2}(\delta^2+1)^{\frac{1}{2}},$$
$$p_2=-\frac{\omega}{2}-\frac{H_0}{H_z}\frac{\omega_0}{2}(\delta^2+1)^{\frac{1}{2}},$$
$$A_1=\left[\frac{\delta+(\delta^2+1)^{\frac{1}{2}}}{2(\delta^2+1)^{\frac{1}{2}}}\right]^{\frac{1}{2}},$$
$$A_2=\left[\frac{-\delta+(\delta^2+1)^{\frac{1}{2}}}{2(\delta^2+1)^{\frac{1}{2}}}\right]^{\frac{1}{2}}.$$

The expression for the energy is found by evaluating

$$W=-(\hbar/i)\bar{\psi}\dot{\psi}.$$

In general, the energy is a function of the time, and this is interpreted as periodic emission and absorption from the rotating field. This is not true for the particular solutions in which we are interested, corresponding to $\alpha=0$ or $\pi/2$.

As we have seen, above and below resonance the magnetic moment is parallel or antiparallel to the constant field H_z. At resonance $\delta=0$, and the expression for the energy becomes, according to Eq. (3),

$$\pm g(\mu_0/2)H_0,$$

that is, the magnetic moment is parallel or antiparallel to the oscillating field H_0. No work is done on the atom by the rotating field, and therefore the energy is constant.

APPENDIX B

The problem is to detect a very small amount of polarized light in the presence of unpolarized light. The intensity of the polarized component alone may be modulated, for instance, by passing the light first through a rotating polaroid and then through a fixed polaroid.** If such light is used to activate an electron-multiplier tube whose output contains a tuned amplifier, very small amounts of polarization can be detected. Let the current through the output of the tube be

$$Gi_0(1+f\cos\omega t),$$

where $f=$fraction of total light falling on the tube which is originally polarized, and therefore modulated, and $G=$gain of tube.

The signal-to-noise ratio across a tuned circuit through which this current flows may be estimated as follows:

signal voltage$=L(di/dt)=GLfi_0\omega\sin\omega t$;
noise voltage$=GRi_0(2eb/i_0)^{\frac{1}{2}}$;
$b=$band width of amplifier;
signal/noise$=Qf(i_0/2eb)^{\frac{1}{2}}$.

Intense light sources are advantageous. Assuming that the light intensity can be increased to the point for which the limiting factor is the tube current $i_{\max}=i_{0\max}G$, we see that the smallest detectable fraction of polarized light is

$$f=1/Q(G2eb/i_{\max})^{\frac{1}{2}}$$

for $Q=100$, $G=10^6$, $i_{\max}=1$ ma, $b=1$ cycle/sec., $f\sim10^{-7}$.

** More elegant ways of doing this by means of a vibrating glass block have been developed by Prof. H. Mueller of the Massachusetts Institute of Technology.