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On the Gyromagnetic Ratio and Spectroscopic Splitting Factor of Ferromagnetic Substances

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A discussion is given of the connection between the results of microwave resonance absorption experiments and gyromagnetic ratio experiments on ferromagnetic substances. A review of the experimental data indicates that the microwave experiments usually give $g>2$, while gyromagnetic measurements usually give values of the related quantity g' which are <2. The analogous situation obtaining in paramagnetic salts is explained by means of a simple example, and the application of the argument to ferromagnetic substances is indicated. The validity of the use of g in the macroscopic equations of motion in the resonance experiment is justified as a consequence of the approximate mutual cancellation of the orbital and lattice angular momenta. A critical discussion is given of other attempts to explain the $g\neq g'$ effect.

I. INTRODUCTION

THE purpose of this paper is to discuss the connection between the results of determinations of the gyromagnetic ratio of ferromagnetic substances and the results of microwave magnetic resonance absorption experiments on these substances. Further, a critical analysis will be given of the various attempts to "explain away" the apparently anomalous result of the resonance absorption experiments, namely the fact that the values of the g-factor deduced from the absorption experiments are usually >2, whereas the gyromagnetic experiments usually give values <2. We are concerned in this paper with an explanation of the differences between the two types of experiments. In the existing literature there is rather little discussion of the detailed theory of gyromagnetic effects: Gorter and Kahn[1] have treated the paramagnetic case, and Polder[2] has suggested the extension of their considerations to the ferromagnetic case. Broer[3] has discussed the diamagnetic case with particular reference to superconductors. Our arguments may be considered as an extension and justification of the suggestion made by Polder in the references cited.

In a microwave resonance absorption experiment one determines the separation $\hbar\omega$ between adjacent energy eigenvalues of the specimen in a constant magnetic field H. The classical theory[4] of the effect gives as the resonance condition

$$\hbar\omega=g\mu_B H_{\mathrm{eff}}, \tag{1}$$

where the effective magnetic field H_{eff} is given by

$$H_{\mathrm{eff}}=(BH)^{\frac{1}{2}} \tag{2}$$

for a plane specimen with the static magnetic field parallel to the plane of the specimen; and by

$$H_{\mathrm{eff}}=H-4\pi M_s, \tag{3}$$

for a plane specimen with the static magnetic field normal to the plane of the specimen; while in general

$$H_{\mathrm{eff}}=\{[H_z+(N_y-N_z)M_z][H_z+(N_x-N_z)M_z]\}^{\frac{1}{2}} \tag{4}$$

for an ellipsoid with demagnetizing factors N_x, N_y, N_z; here H_z is the static field. The effective field to be used in the resonance condition Eq. (1) differs from the applied field H because of the effect of the magnetic moment interactions within the specimen; these interactions give a dynamic demagnetization coupling which shifts the resonance condition in the manner prescribed by the effective field equations. The results of the classical theory of the effective field have been confirmed by the quantum-mechanical calculations of Van Vleck[5] and Polder[2] who worked with the complete microscopic Hamiltonian of the spin system; quantum-mechanical calculations by Luttinger and Kittel,[6] and Richardson,[7] give identical results starting from a macroscopic Hamiltonian. It is of course to be expected from the correspondence principle that the classical results will not be altered by quantization, since we are concerned here with large quantum numbers, of the order of 10^{15} or more.

We shall consider Eq. (1) as the definition of g, which will be called the *spectroscopic splitting factor*. That is, the g-value resulting from a microwave absorption experiment is to be calculated by substituting in the equation $\hbar\omega=g\mu_B H_{\mathrm{eff}}$ the experimental frequency ω and the experimental effective field H_{eff} as given by the appropriate special case of Eq. (4), taking H to be the static magnetic field strength at the resonance maximum.[8] This definition of g involves the implicit assumption that the effective field equation is valid

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[1] C. J. Gorter and B. Kahn, Physica **7**, 753–764 (1940).

[2] D. Polder, Phys. Rev. **73**, 1116 (1948); Phil. Mag. **40**, 99–115 (1949); see also the comments by J. H. Van Vleck, Physica **15**, 197–206 (1949).

[3] L. J. F. Broer, Physica **13**, 473–478 (1947).

[4] C. Kittel, Phys. Rev. **71**, 270–271 (1947); **73**, 155–161 (1948).

[5] J. H. Van Vleck, unpublished.

[6] J. M. Luttinger and C. Kittel, Helv. Phys. Acta **21**, 480–482 (1948).

[7] J. M. Richardson, Phys. Rev. **75**, 1630 (1949).

[8] It is supposed merely for ease in discussion that the crystalline anisotropy energy of the substance is zero; the effect of finite anisotropy is discussed in the author's 1948 paper. Professor J. H. Van Vleck has kindly pointed out that a term was neglected in Eq. (26) of the latter paper, so that Eq. (25b) should read

$$N_y^e=(\tfrac{3}{2}+\tfrac{1}{2}\cos 4\theta)(K_1/M_s^2),$$

and the K_1 term in Eq. (27b) should be corrected similarly.

TABLE I. Comparison of representative experimental values of spectroscopic splitting factor (g) and magnetomechanical ratio. (g') for ferromagnetic substances.[*]

	Microwave resonance g	Gyromagnetic experiments g'
Iron	2.12–2.17	1.93
Cobalt	2.22	1.87
Nickel	2.19–2.42	1.92
Magnetite	2.20	1.93
Heusler alloy	2.01	2.00
Permalloy	2.07–2.14	1.91
Supermalloy	2.12–2.23	—

[*] The values of g' are taken from the summary by S. J. Barnett, Proc. Am. Acad. **75**, 109 (1944). The values of g are based largely on J. H. E. Griffiths, reference 13, and unpublished data kindly communicated by Dr. Griffiths; also on various papers and unpublished measurements by W. A. Yager and his collaborators, to which detailed reference is made in the text. The value of g for magnetite is from L. R. Bickford, Jr., Phys. Rev. **76**, 137 (1949). For Permalloy, g is from Kip and Arnold, reference 14.

under more general conditions that envisaged by any of the previous derivations of the effective field equation.[2, 4–7] A more general derivation is given in Section IV.

In a gyromagnetic experiment one determines the change of magnetic moment resulting from a change of angular momentum, as in the Barnett experiment; or else one determines the change of angular momentum resulting from a change of magnetic moment, as in the Einstein-de Haas experiment. In either case we define the *magnetomechanical factor* g' by the equation

$$g'(e/2mc)=\Delta M/\Delta J, \tag{5}$$

where M is the magnetization and J is the angular momentum per unit volume. We shall require later the following more explicit statement of this relation. One actually observes the angular velocity (and, hence, the angular momentum) of the lattice of the specimen; because of conservation of angular momentum the change of angular momentum of the lattice ΔJ_{lat} is equal but opposite to the sum of the changes in the angular momenta[*] J_{spin} and J_{orb} associated with the spin and orbital motion of the atomic electrons:

$$\Delta J_{\mathrm{lat}}=-\Delta(J_{\mathrm{spin}}+J_{\mathrm{orb}}). \tag{6}$$

We have supposed here that, unlike the microwave case, the changes of J_{spin} and J_{orb} are non-radiative, but proceed by relaxation effects entirely internal to the specimen. We now make the additional assumption that the magnetic moment associated with the rotation of the lattice may be neglected; this assumption may introduce fractional errors of the order of less than $m/M_H\approx 1/2000$; an error of this magnitude is not detectable with present experimental technique. Thus we may set

$$\Delta M=\Delta(M_{\mathrm{spin}}+M_{\mathrm{orb}}), \tag{7}$$

so that Eq. (5) becomes

$$g'\frac{e}{2mc}=\frac{\Delta(M_{\mathrm{spin}}+M_{\mathrm{orb}})}{\Delta(J_{\mathrm{spin}}+J_{\mathrm{orb}})}, \tag{8}$$

apart from questions of algebraic sign. We are omitting diamagnetic effects throughout.

Because the following sections of this paper are concerned largely with the differences which exist between g and g' it may be well here to prove that $g=g'$ for a solid in which the individual atoms are not subjected to any *non-central* electric fields from neighboring atoms in the solid: this amounts in effect to the assumption that the electrostatic field intensity caused by neighbors is zero over the volume of the atom under consideration. A consequence of this assumption is that the orbital and spin angular momenta are not coupled to the lattice, although the orbit and spin may be coupled to each other. We permit only a fictitious small coupling to the lattice in order to make possible the exchange of angular momentum between the electronic and lattice systems which occurs in the gyromagnetic experiments, but we neglect the momentum exchange in the microwave experiments. On the above model the resonance absorption effect will be described by the equation of motion

$$d(\mathbf{J}_{\mathrm{spin}}+\mathbf{J}_{\mathrm{orb}})/dt=(\mathbf{M}_{\mathrm{spin}}+\mathbf{M}_{\mathrm{orb}})\times\mathbf{H}; \tag{9}$$

on substituting Eq. (8), we have

$$d(\mathbf{M}_{\mathrm{spin}}+\mathbf{M}_{\mathrm{orb}})/dt=g'(e/2mc)(\mathbf{M}_{\mathrm{spin}}+\mathbf{M}_{\mathrm{orb}})\times\mathbf{H}, \tag{10}$$

which leads in the usual way to the resonance condition

$$\omega=g'(e/2mc)H_{\mathrm{eff}}, \tag{11}$$

or

$$\hbar\omega=g'\mu_B H_{\mathrm{eff}}; \tag{12}$$

this is identical with Eq. (1) if

$$g'=g. \tag{13}$$

We have thus shown that the spectroscopic splitting factor is equal to the magnetomechanical factor when the electronic angular momentum is not coupled to the crystal lattice. We shall show below that for numerous solid substances the electronic angular momentum is coupled to the lattice, and that to a good approximation one should in calculating the resonance condition omit J_{orb} from the left side of Eq. (9), while Eq. (8) is unaltered. In this situation g will no longer be equal to g', but we have (Eq. 37))

$$\frac{g'}{g}=\frac{J_{\mathrm{spin}}}{J_{\mathrm{spin}}+J_{\mathrm{orb}}}, \tag{14}$$

which may be greater or less than unity depending on the sign of the spin-orbit coupling constant.

In some ways g is similar to the effective mass in the theory of metals; in both cases hidden momentum transfer processes are present and are important.

II. EXPERIMENTAL VALUES OF g AND g'

A summary of the results of experimental measurements of g and g' is given in Table I. It may be noted that $g>2$ and $g'<2$ in every instance except for Heusler

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[*] For convenience in writing we shall imagine that we are always dealing with unit volume of material.

alloy, where $g \cong g' \cong 2.00$. A possible explanation of the unique position of Heusler alloy will be suggested in Section III. We now proceed to give a detailed discussion of certain fundamental aspects of the experimental results.

Shape Dependence of Effective Field Equation

Perhaps the most convincing test of Eq. (4) for the effective field is provided by a comparison of g-values from the parallel [Eq. (2)] and normal [Eq. (3)] field experiments, as first suggested to the author by Professor C. J. Gorter. The experiments were performed on Supermalloy at 1.25 cm by Kittel, Yager, and Merritt,[9] and resonance was obtained at the widely different field strengths of 4880 oersteds with the field parallel to the plane sample and 15,500 oersteds with the field normal to the sample; the corresponding g values as calculated using Eqs. (2) and (3) are

$$g_{\parallel}=2.17; \quad g_{\perp}=2.23.$$

These values are quite sensitive to the value of the saturation magnetization, and the uncertainty in this quantity may perhaps account for the small differences between $g_{\perp}$ and $g_{\parallel}$. Unpublished results of Yager on nickel at 1.25 cm give

$$g_{\parallel}=2.19\text{–}2.22; \quad g_{\perp}=2.26.$$

An additional but less critical test of the theory is furnished by the measurements of Hewitt[10] on two specimens of zinc-manganese ferrite, one a plane and the other a small sphere. Application of the appropriate effective field equations leads to

$$g_{\text{plane}}=2.12; \quad g_{\text{sphere}}=2.16.$$

Frequency Dependence

The clearest test of the frequency dependence of Eq. (1) is provided by Supermalloy. At 1.25 cm Yager and Bozorth,[11] and Yager,[12] find $g=2.17$. At 3 cm Yager finds, in unpublished work, $g=2.17$. The resonance in Supermalloy is exceptionally sharp; so that the agreement of the g-values at the two wave-lengths is quite a critical confirmation of the theory. For iron, the measurements of Griffiths[13] give $g=2.12$ at 1.25 cm and 2.03 at 3 cm, while for nickel the measurements give $g=2.27$ at 1.25 cm and 2.40 at 3 cm; here, however, the g-values are subject to considerable uncertainty because the resonance peaks are relatively wide. For Permalloy, Kip and Arnold[14] find $g=2.08 \pm 0.03$ at 3 cm and $g=2.12 \pm 0.03$ at 1.25 cm.

TABLE II. Comparison of g-values of paramagnetic salts from susceptibility measurements and from microwave resonance absorption measurements.

Substance	g (Susceptibility)	g (Resonance absorption)
$NiSiF_6 \cdot 6H_2O$	2.25[a]	2.29–2.36[b]
$NH_4Cr(SO_4)_2 \cdot 12H_2O$	1.98–2.00[c, d]	1.99[e, f]
$MnSO_4 \cdot 4H_2O$	1.99[g]	1.98[h]
$K_2Cu(SO_4)_2 \cdot 6H_2O$		
$\parallel$ axis	2.42[i]	2.42[j]
$\perp$ axis	2.05[i]	2.09[j]

[a] J. Becquerel and W. Opechowski, Physica **6**, 1039–1056 (1939).
[b] Holden, Kittel, and Yager, Phys. Rev. **75**, 1443 (1949).
[c] A. Serres, Ann. de physique **17**, 5 (1932).
[d] W. J. de Haas and C. J. Gorter, Proc. Amsterdam Acad. **33**, 676 (1930).
[e] Whitmer, Weidner, Hsiang, and Weiss, Phys. Rev. **74**, 1478–1484 (1948).
[f] Yager, Merritt, Holden, and Kittel, Phy. Rev. **75**, 1630 (1949).
[g] Kamerlingh Onnes and Oosterhuis, Comm., Leiden 129b.
[h] Cummerow, Halliday, and Moore, Phys. Rev. **72**, 1233–1240 (1947).
[i] D. Polder, Physica **9**, 709 (1942).
[j] R. D. Arnold and A. F. Kip, Phys. Rev. **75**, 1199–1205 (1949).

Temperature Dependence

The data on the temperature dependence of g are scanty. We have unpublished results of Yager on Supermalloy in the temperature range between liquid nitrogen ($-196°C$) and 300°C. The Curie point is about 400°C. The data on one sample suggests a very gradual increase in g from about 2.1 at $-190°C$ to 2.17 at 300°C, but this conclusion is rather uncertain because of possible errors in the values of the saturation magnetization. For Permalloy, Kip and Arnold[14] find $g=2.14 \pm 0.075$ at room temperature, and $g=2.07 \pm 0.08$ at liquid nitrogen temperature.

III. ATOMIC MECHANISM RESULTING IN INEQUALITY OF g AND g'

We shall first give a calculation for a particular case in order to exhibit in as explicit a form as possible the mechanism by which one obtains $g \neq g'$. The calculation is given primarily with an educational motive: calculations based on similar principles but in a rather formal setting occur in the theory of paramagnetism and appear to have been given first by Schlapp and Penney.[15] In paramagnetic salts of the iron group it is quite often found that the observed susceptibilities lead to values of the Curie constant somewhat different from the Bose-Stoner "spin-only" values. The Curie constant is proportional to g^2, so that one may obtain values of g from susceptibility measurements, as well as from resonance absorption measurements. Table II gives a comparison of g-values obtained by these two methods, and it is seen that harmonious agreement prevails. Schlapp and Penney were concerned with the explanation of the departure of the susceptibility from the values predicted by the spin-only theory.

The mechanism by which one accounts for $g \neq g'$ in paramagnetic salts is rather special and unusual, and perhaps this aspect is not generally appreciated. It is a

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[9] Kittel, Yager, and Merritt, Amsterdam Conference on the Physics of Metals, 1948; Physica **15**, 256–257 (1949).

[10] W. H. Hewitt, Jr., Phys. Rev. **73**, 1118–1119 (1948).

[11] W. A. Yager and R. M. Bozorth, Phys. Rev. **72**, 80–81 (1947).

[12] W. A. Yager, Phys. Rev. **75**, 316–317 (1949).

[13] J. H. E. Griffiths, Nature **158**, 670 (1946).

[14] A. F. Kip and R. D. Arnold, R. L. E. Progress Report, M.I.T., October 15, 1948, pp. 31–32.

[15] R. Schlapp and W. G. Penney, Phys. Rev. **42**, 666 (1932).

result of the combined effects of spin-orbit interaction and the crystalline electric field, but it does not occur in an important way for a crystalline field of purely cylindrical symmetry. Let us consider the behavior of an atom in a ^2F state in an electric field of cubic symmetry. It is supposed that the strength of the electric field is sufficient to destroy the coupling of the **L** vector to the **S** vector, but is not sufficient to destroy the Russell-Saunders coupling in which the individual **l** vectors form a resultant **L**. In a cubic field the seven orbital states split into groups of 1, 3, and 3; we suppose that the non-degenerate orbital state is lowest.** There is also the twofold spin degeneracy. We may take the zero-order ground state wave functions as

$$\Psi=(1/\sqrt{2})(\psi_2-\psi_{-2})\begin{pmatrix}\alpha\\ \beta\end{pmatrix}. \tag{15}$$

Here ψ_2 has $L=3$; $M_L=2$; α and β are the spin functions for $m_s=\pm\frac{1}{2}$. This combination may readily be seen to be satisfactory, since ψ_2 depends on angle as $z(x+iy)^2$ and ψ_{-2} as $z(x-iy)^2$; the combination $\psi_2-\psi_{-2}$ depends on angle as xyz, which is a one-dimensional representation of the cubic group and therefore meets our need for a non-degenerate orbital function with $L=3$. The combination,

$$(1/\sqrt{2})(\psi_2+\psi_{-2}),$$

belongs to one of the threefold degenerate orbital levels separated by an energy difference Δ from the ground state. The spin-orbit interaction operator

$$\lambda\mathbf{L}\cdot\mathbf{S}=\lambda\{L_zS_z+\tfrac{1}{2}(A_LB_S+B_LA_S)\}, \tag{16}$$

where A and B are the usual raising and lowering operators, mixes various upper states in with the ground state. It turns out that for our purposes only the λL_zS_z part of the operator is important. This mixes states of the same spin and produces first order effects on g and g'; the remaining terms in Eq. (16) give only second order effects, which may usually be neglected. A simple perturbation calculation gives

$$\Psi^\alpha=[(1/\sqrt{2})(\psi_2-\psi_{-2})+(\epsilon/\sqrt{2})(\psi_2+\psi_{-2})]\alpha, \tag{17}$$

and

$$\Psi^\beta=[(1/\sqrt{2})(\psi_2-\psi_{-2})-(\epsilon/\sqrt{2})(\psi_2+\psi_{-2})]\beta, \tag{18}$$

where $\epsilon=-\lambda/\Delta$, and Δ is the level separation in the crystal field. We have neglected terms which arise from the $A_LB_S+B_LA_S$ part of the spin-orbit operator, and also the ϵ^2 correction to the normalization factor.

It may be noted that

$$(\psi_2-\psi_{-2}|L_z|\psi_2-\psi_{-2})=0 \tag{19}$$

and

$$(\psi_2+\psi_{-2}|L_z|\psi_2+\psi_{-2})=0; \tag{20}$$

however,

$$(\Psi^\alpha|L_z|\Psi^\alpha)=4\epsilon, \tag{21}$$

$$(\Psi^\beta|L_z|\Psi^\beta)=-4\epsilon. \tag{22}$$

It is seen that the spin-orbit interaction is effective in endowing the ground states with an orbital angular momentum component because of interference effects between the $(\psi_2-\psi_{-2})$ and $(\psi_2+\psi_{-2})$ terms. An instructive way to understand the behavior of the wave functions is to write the orbital part of Eq. (17) as $2^{-\frac{1}{2}}[(1+\epsilon)\psi_2-(1-\epsilon)\psi_{-2}]$, which shows that the term ψ_2 with positive momentum outweighs the term ψ_{-2} which has negative momentum. The expectation values of the energy in a magnetic field are

$$(\Psi^\alpha|L+2S_z|\Psi^\alpha)\mu_BH=(1+4\epsilon)\mu_BH; \tag{23}$$

$$(\Psi^\beta|L_z+2S_z|\Psi^\beta)\mu_BH=-(1+4\epsilon)\mu_BH; \tag{24}$$

so that the energy difference is

$$\Delta E=\hbar\omega=2(1+4\epsilon)\mu_BH, \tag{25}$$

or

$$g=2(1+4\epsilon). \tag{26}$$

The magnetomechanical factor is

$$g'=\frac{(\Psi^\alpha|L_z+2S_z|\Psi^\alpha)}{(\Psi^\alpha|L_z+S_z|\Psi^\alpha)}=\frac{(1+4\epsilon)}{(\frac{1}{2}+4\epsilon)} \tag{27}$$

or, for $\epsilon\ll1$,

$$g'\cong2(1-4\epsilon). \tag{28}$$

Thus we have demonstrated for a particular case the mechanism by which g may be expected to be larger than two if g' is less than two. The relationship stated here was first pointed out in a general way by Gorter and Kahn, who give a comparison with experiment for the paramagnetic salts on which gyromagnetic measurements were made by Sucksmith.

The algebraic sign of $\epsilon(=-\lambda/\Delta)$ is opposite to the algebraic sign of the spin-orbit constant λ, since the splitting Δ is defined so as to be positive. Now λ is positive[16] for an electronic shell which is less than half full, and negative for a shell more than half-full. We then expect ϵ to be positive for paramagnetic salts containing Fe^{++}, Co^{++}, Ni^{++}, and Cu^{++} ions; and this conclusion is generally confirmed by experiment; ϵ also appears to be positive for the ferromagnetic metals Fe, Co and Ni, and their alloys. At Mn^{++} the $3d$ shell is approximately half-filled so that ϵ should be quite small, again in agreement with experiments on paramagnetic manganous salts. We have perhaps in this circumstance the explanation of the equality of g and g' for Heusler alloy (Cu_2AlMn), if we wish to consider manganous ions as the magnetic moment carriers in the alloy. It may be of interest to measure g-values in the Fe-Mn alloy system, and particularly in the ferromagnetic compounds of Mn and Cr, such as MnBi, MnP, CrTe, etc. In the mixed ferrites one might expect

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** This assumption is made so that we may study the situation which occurs when the orbital momentum is quenched, in the zeroth-order approximation. Presumably in ferromagnetic materials the orbital momentum is quenched by exchange interactions if it is not quenched by the ordinary crystalline field.

[16] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1935), p. 210.

g to vary with composition, perhaps in a predictable way.

The translation of the example worked out in this section to an actual ferromagnetic metal offers very considerable difficulties because of the inadequacy of present treatments of the wave functions of $3d$ shell electrons in metals. The quenching of the orbital angular momentum may occur largely through exchange effects, as Van Vleck has suggested, rather than through crystal field effects. Brooks[17] has carried out a calculation of g on an itinerant electron model, according to which the quenching occurs by exchange effects, but his results are somewhat inconclusive. It does not appear to be practical to make more detailed calculations at present. It is well to emphasize that the $g\neq g'$ effect can be understood for paramagnetic salts, and in principle nothing fundamentally new appears to be added (apart from exchange quenching) on going to the metal; one adds only the grave computational difficulties associated with the breakdown of simple atomic coupling schemes.** There appears further to be a residual discrepancy, as for small departures from 2.00 one expects g to be as much larger than 2.00 as g' is smaller than 2.00. This relation is definitely not satisfied by the available data. Further theoretical consideration needs to be given to this discrepancy.

The experimental values of g may be used in breaking down the number of effective Bohr magnetons per atom into the separate contributions from spin and orbit.

It should be pointed out that there is no contradiction involved in having $g>2$ in a material for which the crystalline anisotropy energy may be zero, as in Supermalloy. This may be appreciated on the theory of anisotropy given by Brooks,[17] for example.

IV. MACROSCOPIC CONSEQUENCES OF ATOMIC MECHANISM

In the preceding section we have shown how an individual atomic system may have $g\neq g'$ as a consequence of incomplete quenching of the orbital angular momentum. We shall now discuss the effect on the macroscopic equations of motion, with the idea of justifying the use of the effective field equations which were originally derived for the central field case. In a magnetic dipole transition the z component of the total angular momentum of the system changes by $+\hbar$ in absorption:

$$\Delta J^z=+\hbar. \tag{29}$$

For wave functions of the form of Eqs. (17) and (18) which describe individual atomic systems,

$$\Delta J_{\text{spin}}^z=+\hbar \tag{30}$$

in absorption, and

$$\Delta J_{\text{orb}}^z=4\epsilon\hbar. \tag{31}$$

Now by definition

$$\Delta J=\Delta J_{\text{spin}}+\Delta J_{\text{orb}}+\Delta J_{\text{lat}} \tag{32}$$

so that we must have

$$\Delta J_{\text{lat}}^z=-\Delta J_{\text{orb}}^z, \tag{33}$$

and, by symmetry, similar relations apply to the x and y components. That is, the model adopted for describing an individual atomic system leads to the conclusion that in a resonance absorption experiment the orbital angular momentum changes are neutralized by compensating changes in the lattice angular momentum. The latter could in principle be observed on a freely-suspended specimen, but is probably undetectable in practice. The accuracy of Eq. (33) is limited by only the neglect of terms in ϵ^2 as a consequency of the neglect of terms in β in the wave function Ψ^α.

The macroscopic equation of motion

$$\frac{d}{dt}(\mathbf{J}_{\text{spin}}+\mathbf{J}_{\text{orb}}+\mathbf{J}_{\text{lat}})=(\mathbf{M}_{\text{spin}}+\mathbf{M}_{\text{orb}}+\mathbf{M}_{\text{lat}})\times\mathbf{H} \tag{34}$$

reduces to

$$\frac{d}{dt}\mathbf{J}_{\text{spin}}=(\mathbf{M}_{\text{spin}}+\mathbf{M}_{\text{orb}})\times\mathbf{H} \tag{35}$$

on application of Eq. (33) and neglecting M_{lat} as before. Eq. (35) leads to the resonance condition $\hbar\omega=g\mu_BH_{\text{eff}}$ if we set

$$g\frac{e}{2mc}=\frac{M_{\text{spin}}+M_{\text{orb}}}{J_{\text{spin}}}, \tag{36}$$

which is equivalent to the expression for g resulting from the microscopic calculation. The expression Eq. (4) for the effective field holds also for the present calculations. We have further

$$\frac{g'}{g}=\frac{J_{\text{spin}}}{J_{\text{spin}}+J_{\text{orb}}}. \tag{37}$$

We thus see that different combinations of momenta are involved in the Einstein-de Haas experiment, for which $\Delta J=0$, and in the resonance absorption experiments, for which $\Delta J=\hbar$. The discussion of this section justifies the use of g, instead of g', in discussing resonance absorption on either classical or quantum equations of motion. In the gyromagnetic experiments direct spin lattice relaxation effects are important; we neglect relaxation effects in the discussion of microwave resonance, to the present approximation.

V. DISCUSSION OF OTHER EXPLANATIONS

In this paper we have supposed that the $g\neq g'$ effect is a real effect. Other authors have suggested that the effect is not real, but that the difference between g and

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[17] H. Brooks, Phys. Rev. **58**, 909–918 (1940). In this reference g and g' are somewhat confused; g is actually calculated, although it is spoken of as if it were g'.

** It may be that the coupling schemes are simpler in the ferromagnetic oxides than in the metals; Néel has for example given an explanation of many properties of the ferrites on the basis of R-S coupling schemes.

g' is only apparent, being caused by experimental disturbances of various kinds.

Birks[18] has suggested that the resonance maximum is shifted by random anisotropy line shape distortion effects associated with the polycrystalline nature of the material. This hypothesis fails to account for the best-established high g-value in the field, that of Supermalloy, where the line width is very narrow. Birks supposed that rolling anisotropy was responsible for the high g-value of Supermalloy; unpublished measurements by Yager give too small a value of the rolling anisotropy for this supposition to be adequate. Deliberately cold-rolling Supermalloy broadened the resonance, but did not shift the position of the maximum,[12] contrary to the behavior predicted by Birks.

Rado[19] has suggested that a certain expression which occurs in the theory of the approach to magnetic saturation[20] in polycrystalline materials may be interpreted as an effective field for the purpose of ferromagnetic resonance. This identification is not justified, and leads to incorrect results, as may be confirmed by examining the simple limiting case of zero anisotropy; and also in the case of large H, where Rado proposes that $H=4\pi M_s/3$ be used in place of H in the effective field equations. The Lorentz term $(4\pi/3)M_s$ should not, however, be included in the effective field equation, as proved earlier.[4] On experimental grounds the fact that g appears to be independent of frequency to a fair accuracy in the case of Supermalloy may be cited against the Rado hypothesis. Further, the behavior of Heusler alloy would be quite anomalous on this hypothesis. W. F. Brown[21] has raised essentially the same objectives as we have here.

Griffiths[22] has obtained results on thin ferromagnetic films which indicate that g decreases as the films become thinner and approaches 2.00 for films below a critical thickness of the order of 0.5 micron. These results have not yet been satisfactorily explained, and it may be that the explanation is to be found in differences between the physical properties of actual thin films and the mathematical model of a thin film. The results at first sight may suggest that exchange force effects caused by non-uniform magnetization accompanying eddy-currents may play a significant part in the resonance process. Theoretical calculations by Polder[2] and by the author indicate that it is not to be expected that significant exchange effects enter for the eddy-current skin depths characteristic of existing measurements, although they may enter at very low temperatures in pure metals. Certainly the high g-values in the semiconducting ferrites cannot be ascribed to exchange-eddy current effects of this type. Further, the exchange effect would be accompanied by a line-broadening: the sharp line in Supermalloy therefore rules out a shift in the resonance maximum due to non-uniform magnetization.

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