collision, we find

$$d\sigma^{NN}=(\mu_N/\mu_P)^2 d\sigma_m{}^{PP}. \tag{9}$$

It is interesting to compute the total cross section corresponding to a finite range of γ-ray energies. If we take $E_0=250$ Mev (the energy of the protons from the Rochester cyclotron), we obtain $\sigma^{NP}=0.28\cdot 10^{-29}$ cm^2, $\sigma^{PP}=0.23\cdot 10^{-29}$ cm^2 and $\sigma^{NN}=0.038\cdot 10^{-29}$ cm^2 for γ-rays with energies from $E_0/4$ to $E_0/2$ in the center of mass system. In the laboratory system this covers the energy region 85–170 Mev for γ-rays in the forward direction and 45–90 Mev for γ-rays in the backward direction. These cross sections predict therefore about one high energy γ-ray per 10,000 elastic collisions. If the nucleon collides with a nucleus, the nuclear cross section for bremsstrahlung should be, to a good approximation, the sum of the individual nucleon-nucleon cross sections.

It is also interesting to note that in our non-relativistic approximation a pure Majorana nuclear force would lead to exactly the same result as a pure ordinary force. Both would lead to a constant times the first term in $E(p)$ and $M(p)$; the interference between the Majorana and ordinary forces leads to the logarithmic term in $E(p)$ and $M(p)$. Examination of $E(p)$ and $M(p)$ also reveals that the result is fairly insensitive to the range of nuclear forces; as a matter of fact, if we set $\lambda=0$ (infinite range), the cross section is only increased by a factor 2.5. This shows us at the same time that the bremsstrahlung arising from the Coulomb force between two protons is down by a factor of $1/2.5(g_1/e^2)^2 \sim 600$ compared to our cross section.

After our calculation was completed, two papers[5] on the same subject came to our attention. Both papers study the continuous γ-emission accompanying low energy neutron-proton collisions (below 20 Mev) through the use of the low energy scattering phase shifts. This work was supported by the joint program of the Office of Naval Research and the Atomic Energy Commission.

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[5] Y. Nishina, S. Tomonaga, and H. Tamaki, Sci. Pap. Inst. Phys. Chem. Res. Tokyo **30**, 61 (1936); M. Krook, Proc. Phys. Soc. (London) **62**, 19 (1949).

PHYSICAL REVIEW VOLUME 76, NUMBER 1 JULY 1, 1949

On The Two-Meson Theory[1]

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(Received February 24, 1949)

Main consequences of the two-meson theory by Marshak and Bethe on the one hand and that by Sakata are compared with experiments concerning π- and μ-mesons. It is shown that the pair type interaction between π-mesons and nucleons in "M. B." theory contradicts with frequent occurrence of stars produced by π-mesons, whereas the assumption in Sakata theory that π-mesons with spin 0 or 1 are responsible for nuclear forces does not. Although the smaller range for the nuclear forces thus obtained from the latter theory is not at variance with the high energy neutron-proton scattering experiment, deuteron quadrupole moment cannot be accounted for by a π-meson field alone, so that the admixture of another meson field with larger range is necessitated. Both π-μ-decay and μ-nuclear capture can be consistently accounted for by assuming spin $\frac{1}{2}$ for μ-mesons as in Sakata theory. However, nuclear β-decay and μ-β-decay have to be considered as direct processes as in Fermi's theory of β-decay, instead of indirect processes through virtual emission and absorption of π-mesons as assumed usually in the meson theory.

THE recent investigations on cosmic rays by Powell and his co-workers have established the facts that there exist two groups of different mesons, i.e., heavy mesons (π-mesons) and light mesons (μ-mesons) and that the former transmute into the latter by emitting a neutral particle.

The theory which involves two mesons of this sort had already been proposed theoretically in 1942 by Sakata and Tanikawa, in cooperation with Inoue and Nakamura.[2] They introduced two kinds of mesons, light and heavy ones, in order to solve

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[1] Originally reported at the Annual Meeting of the Physical Society of Japan at Kyoto University, May 23, 1948. When we had sent the report dated August 1, 1948 to Professor J. R. Oppenheimer, he was kind enough to inform us of the new experimental results at the California Cyclotron and advised us to rewrite our report on his information. This is the revision based on the new experimental evidence. We should like to thank Professor Oppenheimer heartily for his kindness and valuable suggestions.

[2] S. Sakata and T. Inoue, Prog. Theo. Phys. **1**, 143 (1946); S. Sakata, Symposium on the Meson Theory at Tokyo, 1943.

TABLE I

Ordinary force $a+b\equiv g$	Tensor force f	Quadrupole moment $Q(10^{-27}\text{ cm})$	Cut-off radius $x_0(10^{-13}\text{ cm})$
−0.50	0.80	0.696	0.32
0.50	1.05	1.344	0.37
1.81	1.60	1.64	0.69
2.00	2.40	1.87	0.82
3.00	3.20	2.14	1.10
4.00	4.00	2.53	1.28

the difficulties of the meson theory which make the quantitative connection between the nuclear forces, beta decay of nuclei, and the scattering and the decay of cosmic mesons unsuccessful.

In 1947 in the light of the new knowledge obtained concerning the absorption and the decay of cosmic mesons stopped in matter, preliminary reports by Powell and his co-workers, and the various cosmic ray evidences observed, Marshak and Bethe[3] proposed a two-meson theory. Their schemes, however, coincide in most parts with those of Sakata's, and the only difference between them seems to lie in that "M.B."[3] assumed a pair theory for nuclear forces, while in Sakata's theory nuclear potentials were supposed to be Yukawa type. In the present stage, we can decide which theory is preferable in view of the experimental evidences found by Powell and his co-workers.[4]

As mentioned above, the interaction between nucleons ($n=p$) and *pi*-mesons (π) in M.B. theory is of a pair theory type. Thus, if we denote the neutral particle, or M.B.'s neutrino, by ν, this interaction may be written as follows:

$$\text{(I)}\quad N\rightarrow P+\pi+\nu \qquad \text{(M.B.)}$$

while in Sakata's theory, it is

$$\text{(I}'\text{)}\quad N\rightarrow P+\pi. \qquad \text{(S.)}$$

Secondly, the interaction between π-mesons and μ-mesons is just the same in both theories,

$$\text{(II)}\quad \pi\rightarrow\mu+\nu. \quad \text{(M.B.) and (S.)}$$

Now the interactions between μ-mesons and nucleons is given in M.B. theory

$$\text{(III)}\quad P+\mu\rightarrow\pi+\nu+P\rightarrow N \qquad \text{(M.B.)}$$

whereas in Sakata's theory

$$\text{(III}'\text{)}\quad P+\mu\rightarrow\pi+\nu+P\rightarrow N+\nu. \qquad \text{(S.)}$$

From these illustrations one can find that although the process (II) coincides in both theories, (I) and (III) will lead to different results. Here the most recent reliable values for the masses of new found particles are given by Berkeley experiments as follows:[5]

$$m_\pi=286m,\quad m_\mu=216m,\quad m_\nu=0$$

where m_π, m_μ, and m_ν are the masses of π-meson, μ-meson and the neutral particle, respectively, m being the mass of an electron.

Now we first discuss the absorption of π-mesons by the nucleus. In Sakata's theory, the whole energy of π can be transferred to the nucleus which amounts to $286m\cong 140$ Mev, enough to admit a nuclear star formation subsequently. Moreover, the disintegration energy thus estimated agrees satisfactorily with the conclusion that Lattes and others have obtained by comparing their results with the disintegration experiments of nuclei bombarded by 200 Mev deuterons. M.B. theory, on the other hand, predicts that a π-meson may be absorbed by the nucleus with simultaneous emission of a neutrino. If we confine ourselves to the simple case where a free proton absorbs a π-meson and emits a neutrino, it will be easily seen by the conservation law for the energy and momentum that the mass energy of π-meson, being equal to 140 Mev, is carried away by the neutrino and there remains little energy to be shared by the proton. Actually the binding energy of a proton in the nucleus is known to be small compared to the mass energy of a π-meson; hence we may roughly approximate it as a free proton. Therefore, if we apply a consideration similar to the photo-effect concerning the center of gravity of the proton and the neutrino, the residual nucleus will obtain little energy in this process and the excess of the π-meson will be carried away by the neutrino, which precludes definitely the possibilities of subsequent nuclear star formation. For this reason we may conclude that M.B. theory could not accord with Bristol's experiments.

Next, we shall discuss the case of (III) and (III′). The disintegration of a nucleus by μ-mesons has been scarcely observed so far. It is easily seen that M.B. theory encounters difficulties in explaining this fact if we note that in M.B. theory the nucleus would absorb the whole energy of a μ-meson, including its rest mass energy, $m_\mu=216m$ $\cong 108$ Mev which can only be released by a nuclear star. On the other hand, Sakata's theory predicts that one neutrino should be emitted by the absorption of a μ-meson and if, in this case we take into account the above consideration in the case (I′), it would be clear that the neutrino would carry away the greater part of the incident energy. We can thus conclude that star type disintegration of

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[3] R. E. Marshak, and H. A. Bethe, Phys. Rev. **72**, 506 (1947). We denote Marshak and Bethe by M.B. throughout this paper.

[4] C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Nature **160**, 453, 486 (1947).

[5] E. Gardner and C. M. G. Lattes, Science **109**, 270 (1948). We owe the knowledge of the precise value of the neutral particle to Professor Oppenheimer.

nucleus by the absorption of μ-mesons is forbidden in Sakata's theory.

The above discussions indicate that Sakata's theory must be preferably chosen for the two-meson theory.

As for the nuclear problems, some questions have arisen in the two-meson theory. It is easily seen that only π-mesons play a role in the nuclear forces since the coupling between nucleons and μ-mesons turned out to be negligibly small as estimated by the capture lifetime of μ-mesons by the various nuclei. The range of nuclear forces, however, carried by π-mesons stretches only 1.35×10^{-13} cm which is shorter than the usually adopted value, $2.5\sim 2.8\times 10^{-13}$ cm. Some of the recent experiments on $n-p$ scatterings[6] seem not to be at variance with this reduced force range.

We investigated the deuteron problem by the two meson theory indicated. For the nuclear potentials we assumed a most general type (symmetrical):

$$V(x)=\tfrac{1}{3}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{a+b(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+fS_{12}(1/x^2+1/x+3)\}(e^{-x}/x), \quad (1)$$

where S_{12} is a tensor operator and $\hbar$, c, and m_π are put to unity. To avoid $1/r^3$ divergence we resorted to the zero cut-off procedure. The result is shown in Table I. The above figures are so determined as to give the binding energy of the ground state to be 2.17 Mev and D-state probability 4 percent for the deuteron. The results from the vector or pseudoscalar fields correspond to values somewhere between the first and the second column. It is seen that if the correct value, 2.75×10^{-27} cm^2 is to be assigned for the quadrupole moment (Q), the cut-off radius must be taken longer than the force range itself, i.e.,

$g=4$, $f=4$ (which corresponds to 0.6 for the dimensionless coupling constants in the usual notation)

$$x_0=1.28.$$

In the reasonable cut-off distances, Q is much smaller than the correct one. The undesirable reduction of Q seems to be caused by the deficiency of the tensor part at the outer region of the force range. To ascertain that there are no ways of escape, we tried the limiting case. Following Foldy's method,[7] where the effects of the tensor force reach to the longest distance, we get the value 1.50×10^{-27} cm^2 for the quadrupole moment, still too small to accept. Consequently, one may conclude that π-mesons only cannot reasonably explain the deuteron problems. In this situation it seems to us inevitable that a mixture of fields with different force range should be introduced. Possibilities of such forces are now being studied. One suggestion may be a nuclear field carried by π-mesons mixed with a third meson which is heavier than π-mesons, while another is a charged meson field with range 1.35×10^{-13} cm mixed with a neutral vector field with range $2.5\sim 2.8\times 10^{-13}$ cm.

As for the new-found particles, the π-meson and the μ-meson, it is rather essential to assign them the field type more precisely. For one thing the most recent experimental values for the lifetime of π-mesons for $\pi-\mu$ decay[8]

$$\tau_{\pi\mu}=0.85\times 10^{-8}\text{ sec.} \quad (2)$$

and the lifetime of μ-mesons in the K shell for the nuclear capture by various nuclei,[9] for instance, in the case of $Z=10$

$$\tau_{\mu\text{cap.}}=3.1\times 10^{-6}\text{ sec.} \quad (3)$$

may certainly serve to solve this problem. On account of the above discussions, it may be natural to assume π-mesons as Bosons. In view of the results of Christy and Kusaka[10] concerning the cosmic-ray analysis, the spin of μ-mesons should be $(1/2)\hbar$ or 0. We first investigated tentatively the case, π-mesons: spin $1\hbar$ (vector symmetrical), and μ-mesons: spin $(1/2)\hbar$. Calculated lifetime of π-mesons for $\pi-\mu$ decay is given as follows:

$$1/\tau_{\pi\mu}=(m_\pi c^2/3\hbar)(2(\gamma_1{}^2/\hbar c)+\gamma_2{}^2/\hbar c)\times(1-\theta^2)^2(1+\theta^2), \quad \theta=m_\mu/m_\pi \quad (4)$$

where $\gamma_1{}^2/\hbar c$ and $\gamma_2{}^2/\hbar c$ are the dimensionless coupling constants between π and μ. Inserting the experimental values of (2) in it, one may estimate the $\pi-\mu$ coupling,

$$\begin{aligned}&(a)\ \gamma_1\neq 0,\quad \gamma_2=0,\quad \gamma_1{}^2/\hbar c=2.9\times 10^{-15},\\&(b)\ \gamma_1=0,\quad \gamma_2\neq 0,\quad \gamma_2{}^2/\hbar c=5.8\times 10^{-15}.\end{aligned} \quad (5)$$

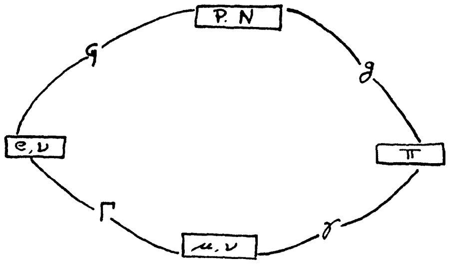

FIG. 1.

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[6] Lavatelli, Long, Snyder, and Williams, Phys. Rev. **72**, 1147 (1947). C. G. Shull, E. O. Wollan, G. A. Morton, and W. L. Davidson, Phys. Rev. **73**, 262, 842 (1948).

[7] L. I. Foldy, Phys. Rev. **72**, 125 (1947).

[8] Professor E. Gardner has kindly informed us, in response to our request, of the experimental results which were recently investigated at Berkeley. We are benefited so much by his kind information, and we should like to express our many thanks to Professor E. Gardner.

[9] M. Conversi, E. Pancini, and O. Piccioni, Phys. Rev. **71**, 209 (1947).

[10] R. F. Christy and S. Kusaka, Phys. Rev. **59**, 414 (1941).

The absorption of the μ-meson in the K shell can be calculated in just the same way as in the K electron capture. Inserting the corresponding values in Yukawa-Sakata's formula[11] for the K electron capture, we get

$$\frac{1}{\tau_{\mu\,\mathrm{cap.}}}=\left(\frac{M_1^2}{T_{01}}+\frac{M_2^2}{T_{02}}\right)\frac{4\pi(\alpha Z)^{2s+1}}{\Gamma(2s+1)}\times\frac{(2m_\mu c^2 r_0)^{2s-2}}{\hbar}(\Delta\overline{W}+s)^2,$$

$$\Delta\overline{W}=\frac{20}{216}\qquad \text{assumed } s=(1-\alpha^2Z^2)^{\frac{1}{2}}, \tag{6}$$

$$\frac{1}{T_{01}}=\frac{8\theta^2}{\pi}\frac{m_\pi c^2}{\hbar}\left(\frac{g_1\gamma_1}{\hbar c}\right)^2,$$

$$\frac{1}{T_{02}}=\frac{32\theta^2}{3\pi}\frac{m_\pi c^2}{\hbar}\left(\frac{m_\mu}{m_\pi}\right)^2\left(\frac{g_2\gamma_2}{\hbar c}\right)^2,$$

$$M_1=\int\psi^*\varphi du,\quad M_2=\int\psi^*\alpha\varphi du.$$

where ψ^* and φ are the nuclear wave functions after and before μ-meson capture, respectively. The Z-dependence of $1/\tau_{\mu\mathrm{cap.}}$ given in (6) is not at variances with the experiments.[9] Using the value for $\tau_{\mu\mathrm{cap.}}$ given in (3), with the help of (5), we find the following values for the nucleon-π coupling multiplied by the nuclear matrix elements for the μ-meson capture,

$$\kappa_1=g_1^2/\hbar c\left|\int\psi^*\varphi du\right|^2=0.094,$$
$$\kappa_2=g_2^2/\hbar c\left|\int\psi^*\alpha\varphi du\right|^2=0.035. \tag{7}$$

In the case of the pseudoscalar π-meson, we have estimated them in the analogous way, using Nelson's formula[12]

$$\gamma_3^2/\hbar c=1.8\times10^{-15},$$
$$\kappa_3=g_3^2/\hbar c\left|\int\psi^*\sigma\varphi du\right|^2=0.5, \tag{8}$$

where $\gamma_3^2/\hbar c$ and $g_3^2/\hbar c$ are dimentionless coupling constants between pseudoscalar π-mesons and μ-mesons, or nucleons, respectively. In (8), it is seen that the values of κ's in the pseudoscalar theory of π-mesons is about ten times greater than those in the vector theory. Therefore, we could determine which of the field types may be preferable for π-mesons, if we know the nuclear matrix elements for μ-meson capture, the estimation of which will be published later. In view of the relation $|\int\psi^*\sigma\varphi du|^2<1$, the value followed from the pseudoscalar theory would be $g_3^2/\hbar c>0.5$, rather too large to accept, which may exclude the pseudoscalar theory of π-mesons in our preliminary estimations.

For the interaction of π-mesons and μ-mesons with light particles, there appear other questions for the two-meson theory. If we assume that the beta-decay of nucleus takes place through π-mesons, computed lifetime of the beta-decay of π-mesons becomes $\tau_{\pi\beta}=6\sim2\times10^{-9}$ sec. which is faster than the lifetime of π-mesons for $\pi-\mu$ decay given in (2). Consequently, we should be forced to forbid π-mesons to contribute nuclear beta-decay except for the forbidden transitions, and essentially may rely on Fermi theory.

Next, if the beta-decay of μ-mesons through π-mesons is assumed, the estimated lifetime of π-mesons for their beta-decay would be too fast to accept. For this reason, we have assumed that the beta-decay of μ-mesons should occur directly. Since we have taken μ-mesons as Fermi particles, the decay process may be naturally either of the following two:

(IV) $\mu\rightarrow$electron+a Bose neutrino

or

(V) $\mu\rightarrow$electron+2 Fermi neutrinos.

Which of the two processes is valid will depend on the further experiments such as the decay electron spectrums or the neutrino loss in the cosmic rays although, in the latter case, the existence of neutral mesons may make the problems more complicated. Using the experimental lifetime of μ-mesons for their beta-decay

$$\tau_{\mu\beta}=2.2\times10^{-6}\text{ sec.}, \tag{9}$$

we find the values for the coupling between μ-mesons and the light particles,

$$\text{(IV)}\qquad \Gamma^2/\hbar c\cong3\times10^{-18} \tag{10}$$

or

$$\text{(V)}\qquad \Gamma=6\times10^{-14}, \tag{11}$$

where $\Gamma^2/\hbar c$, Γ is the dimensionless coupling constants between μ-mesons and the light particles. A schematic diagram illustrating the conceivable couplings is shown in Fig. 1. It may be a noticeable fact that, in our scheme, the decay processes which are followed by the emission of neutrino could only occur with the extraordinary (and about the same) small couplings, as indicated in (5), (8), (10), (11), γ's, Γ's and the dimensionless Fermi constant G.

We owe some of the numerical calculations to Miss Masako Hanai.

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[11] H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. Japan **17**, 467 (1935); **18**, 128 (1936).

[12] E. C. Nelson, Phys. Rev. **60**, 830 (1941).