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Bremsstrahlung in High Energy Nucleon-Nucleon Collisions

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Formulas for the differential cross sections for the continuous γ-emission accompanying proton-neutron, proton-proton, neutron-neutron collisions have been derived. Numerical results are given for an incident nucleonic energy of 250 Mev.

THE operation of large synchro-cyclotrons and the improvements in the methods for detecting energetic γ-rays create an interest in the photons accompanying high energy nuclear collisions. When a nucleon of several hundred Mev hits a nucleus, one or more nuclear γ-rays (of several Mev) will, in general, be associated with the nuclear transmutation. If the nucleon produces a neutral meson which immediately disintegrates into two or more photons, a second source of γ-rays will be present. In this note we wish to consider a third source of γ-rays which must inevitably accompany individual nucleon-nucleon collisions (and therefore the collision of a nucleon with a nucleus).

The process in question is the bremsstrahlung arising from the coupling of the nucleons to the electromagnetic field (through the charge and magnetic moment of the proton and the magnetic moment of the neutron) and the nuclear interaction between the nucleons. The continuous spectrum of γ-rays which results has a maximum energy (in the center-of-mass system) of one-half the energy of the incident nucleon so that for incident nucleonic energies of several hundred Mev, an appreciable fraction of the γ-rays will be more energetic than nuclear γ-rays. The magnitude of the photon cross section should be of the order of the fine structure constant times the elastic scattering cross section for the nucleons. This rather small cross section is, however, comparable to the cross section for specific high energy processes (such as the production of neutral mesons at energies just above the threshold) and it seems worth while to perform an approximate calculation of the bremsstrahlung. This calculation will enable one to subtract out the continuous γ-ray background in looking for new effects and, in addition, the measurement of the continuous spectrum itself may throw some light on the nature of nuclear forces.

In order to obtain a first orientation and to eliminate unnecessary (and unknown) complications, we restrict ourselves to energies of the nucleon below the threshold for production of π-mesons (i.e. 290 Mev). We therefore do not concern ourselves with the continuous spectrum of γ-rays accompanying π-meson emission.[1] Moreover, we use the phenomenological theory of nuclear forces so that we need not consider virtual meson processes. In particular, the nuclear potential is chosen so as to give the best fit to the Berkeley neutron-proton scattering experiments at 90 Mev.[2] Since the energies we consider are large compared to the magnitude of the nuclear potential, we employ the Born approximation. At the same time, since our energies are small compared to the rest energy of the nucleon, we treat the nucleons non-relativistically.[3] For the coupling of the nucleons with the electromagnetic field, we therefore write

$$H=-(e/Mc)\mathbf{p}\cdot\mathbf{A}-(\mu_P\boldsymbol{\sigma}_\mathbf{P}+\mu_N\boldsymbol{\sigma}_\mathbf{N})\cdot\mathrm{curl}\mathbf{A} \qquad (1)$$

where p is the momentum of the proton, $\mathbf{A}$ is the vector potential, and μ_P, μ_N are the magnetic moments of the proton and neutron respectively; the other symbols have their usual significance. We expand $\mathbf{A}$ in the usual way (the normalization is for unit volume):

$$\mathbf{A}=(2\pi)^{\frac{1}{2}}\hbar c\sum_k[\boldsymbol{\varepsilon}/(\omega_k)^{\frac{1}{2}}]e^{i\mathbf{k}\cdot\mathbf{r}/\hbar}(a_k+a_{-k}{}^*)$$

where k is the momentum of the photon, $\omega_k=ck$, $\boldsymbol{\varepsilon}$ its polarization and a and a^* are creation and destruction operators respectively. For the nuclear potential[4] we write

$$V(r)=\frac{(1+P_M)}{2}g_i\frac{e^{-\lambda r}}{r} \quad \text{with} \quad \begin{array}{l}\lambda^{-1}=1.18\cdot 10^{-13}\ \mathrm{cm}\\ g_1=0.280\hbar c\\ g_3=0.404\hbar c\end{array} \qquad (2)$$

where P_M is the Majorana operator and g_1 and g_3 refer to the singlet and triplet states respectively.

We first calculate the cross section for continuous γ-emission accompanying a proton-neutron collision and then the cross sections for proton-proton and neutron-neutron collisions. The γ-emission takes place through one intermediate step: either the proton or neutron emits the γ-ray and is then scattered by the other nucleon or the scattering occurs first and the γ-ray is then emitted. The total matrix element can be written

$$H'=\sum[H_{AI}V_{IF}/(E_A-E_I)+V_{AII}H_{IIF}/(E_A-E_{II})], \qquad (3)$$

[1] See S. Hayakawa and S. Tomonaga, Prog. Theor. Phys. 2, 162 (1947); L. I. Schiff, Phys. Rev. 75, 1459 (1949).

[2] R. Christian and R. Serber, private communication.

[3] We use exactly the same approximations as were used by one of the authors in calculating the cross section for meson production just above threshold (see L. L. Foldy and R. E. Marshak, Phys. Rev. 75, 1493 (1949)).

[4] We calculate with the central force fit of the 90 Mev neutron-proton scattering experiments since only the angular distribution of the γ-rays, not the magnitude of the cross section, will be affected by tensor forces.

where A, F refer to the initial and final states respectively, and I, II refer to intermediate states corresponding to transitions of the first and second types respectively. If $\mathbf{p}_0$ and $-\mathbf{p}_0$ denote the initial momenta of the proton and neutron respectively (in the center of mass system) and $\mathbf{p}-\mathbf{k}/2$ and $(-\mathbf{p}-\mathbf{k}/2)$ their respective final momenta, the differential cross section becomes

$$d\sigma^{NP}=\frac{M^2}{128\pi^5\hbar^7}\frac{p}{p_0}\langle|H'|^2\rangle k^2dkd\Omega_k d\Omega_p. \tag{4}$$

In Eq. (4), the brackets around $|H'|^2$ denote the average over the initial spins of the nucleons and the summation over the polarization of the γ-ray and the final spins of the nucleons; the meaning of $d\Omega_k$ and $d\Omega_p$ is clear.

Since we are including the magnetic interaction, we must distinguish the transitions corresponding to the different spin states. If we choose the axis of spin quantization as $(\mathbf{k}\times\boldsymbol{\varepsilon})$, we get for the six possible transitions (we omit the common factor $(2\pi)^{\frac{1}{2}}\hbar c/\omega_k^{\frac{1}{2}}$):

$$\chi^{(1)}\rightarrow\chi^{(1)}:\quad H'=e/Mc(\mathbf{p}_0-\mathbf{p})\cdot\boldsymbol{\varepsilon}[V_{\mathbf{p}_0-\mathbf{p}}^{(1)}+V_{\mathbf{p}_0+\mathbf{p}}^{(1)}], \tag{5a}$$

$$\chi^{(1)}\rightarrow\chi_0^{(3)}:\quad H'=-ik/\hbar(\mu_P-\mu_N)\times[(V_{\mathbf{p}_0-\mathbf{p}}^{(1)}+V_{\mathbf{p}_0+\mathbf{p}}^{(1)})-(V_{\mathbf{p}_0-\mathbf{p}}^{(3)}+V_{\mathbf{p}_0+\mathbf{p}}^{(3)})], \tag{5b}$$

$$\chi_1^{(3)}\rightarrow\chi_1^{(3)}:\quad H'=e/Mc(\mathbf{p}_0-\mathbf{p})\cdot\boldsymbol{\varepsilon}[V_{\mathbf{p}_0-\mathbf{p}}^{(3)}+V_{\mathbf{p}_0+\mathbf{p}}^{(3)}], \tag{5c}$$

$$\chi_0^{(3)}\rightarrow\chi_0^{(3)}:\quad H'=\text{same as (5c)}, \tag{5d}$$

$$\chi_0^{(3)}\rightarrow\chi^{(1)}:\quad H'=\text{negative of (5b)}, \tag{5e}$$

$$\chi_{-1}^{(3)}\rightarrow\chi_{-1}^{(3)}:\quad H'=\text{same as (5c)}. \tag{5f}$$

In the above expressions, $\chi^{(1)}$ and $\chi^{(3)}$ are the singlet and triplet spin functions respectively, the subscript on $\chi^{(3)}$ denoting the magnetic quantum number and $V_{\mathbf{P}}=\int V(r)e^{i\mathbf{P}\cdot\mathbf{r}/\hbar}d\mathbf{r}$, the superscript on $V_{\mathbf{P}}$ specifying singlet or triplet scattering; we have neglected both the nucleonic recoil energies in the energy denominators and $\mathbf{k}/2$ compared to $\mathbf{p}_0\pm\mathbf{p}$ in the Fourier transforms of V, consistent with our non-relativistic approximation. Since the four spin functions are orthogonal, the electric and magnetic transitions do not interfere, and we find

$$\langle|H'|^2\rangle=2\pi/\omega_k U^2[(\mu_P-\mu_N)^2(1-\delta)^2+(e^2\hbar^2/4M^2\omega_k^2)(\delta^2+3)(p_0^2\sin^2\theta_0+p^2\sin^2\theta-2pp_0\sin\theta_0\sin\theta\cos(\phi_0-\phi))]. \tag{6}$$

In Eq. (6), we have set $U=(V_{\mathbf{p}_0-\mathbf{p}}^{(3)}+V_{\mathbf{p}_0+\mathbf{p}}^{(3)})$ and $\delta=g_1/g_3$; the angles (θ_0,ϕ_0) and (θ,ϕ) are the polar and azimuthal angles of $\mathbf{p}_0$ and $\mathbf{p}$ with respect to $\mathbf{k}$ as z-axis.

If we now insert the expression for $\langle|H'|^2\rangle$ into Eq. (4) and perform the integrations over $d\Omega_p$ and $d\Omega_k$, we obtain the differential cross section for γ-emission irrespective of the angle of emission of the γ-ray and the final directions of the two nucleons, namely (p is measured in units of p_0):

$$d\sigma^{NP}=d\sigma_e^{NP}+d\sigma_m^{NP}, \tag{7}$$

where

$$d\sigma_e^{NP}=\frac{1}{3}\left(\frac{e^2}{\hbar c}\right)\left(\frac{g_3}{cp_0}\right)^2(\delta^2+3)E(p)dp \tag{7a}$$

$$d\sigma_m^{NP}=2(\mu_P-\mu_N)^2\frac{g_3^2}{(\hbar c)^3}(1-\delta)^2M(p)dp \tag{7b}$$

with

$$E(p)=\frac{p^2(1+p^2)}{(1-p^2)}\left\{\frac{2}{[\lambda^2+(1-p)^2][\lambda^2+(1+p)^2]}+\frac{1}{2p(\lambda^2+1+p^2)}\log\left[\frac{\lambda^2+(1+p)^2}{\lambda^2+(1-p)^2}\right]\right\}, \tag{7c}$$

$$M(p)=\frac{(1-p^2)^2}{(1+p^2)}E(p). \tag{7d}$$

The dimensionless quantity called λ in these formulas is the λ of Eq. (2) measured in units of $p_0/\hbar$; λ^2 is therefore inversely proportional to the energy of the incident proton.

For the electric emission Eq. (7) leads to a $(E_0/2-ck)^{\frac{1}{2}}dk/k$ spectrum for the γ-rays near the upper limit $E_0/2$($E_0/2=p_0^2/M$ is the energy of the incident proton in the center of mass system). For the magnetic emission the spectrum is $(E_0/2-ck)^{\frac{1}{2}}kdk$ near the upper end. With the constants chosen for the nuclear potential, however, the magnetic contribution is only a few percent of the electric for the NP collision. From Eq. (7c) it appears that near the upper end of the spectrum ($p\lesssim 1$), $E(p)$ is relatively insensitive to the energy of the incident proton; this will be true provided that the deBroglie wave-length of the incident proton is appreciably smaller than the range of the nuclear forces. In this case the cross section for emission of a γ-ray in any given *fractional* energy range will vary inversely with the energy of the incident proton.

In order to obtain the cross sections for γ-emission associated with proton-proton or neutron-neutron collisions, we observe that the potential represented by Eq. (2) is zero for states of odd angular momentum so that only singlet even states contribute to the scattering. Consequently, the only terms which contribute in Eqs. (5a)–(5f) are the ones which involve the $V^{(1)}$'s. It can easily be shown that if we take each non-vanishing term in Eqs. (5a)–(5f) and multiply by 2, we get the same result as we would get by using properly antisymmetrized wave functions for the two like nucleons. For the γ-emission accompanying a proton-proton collision, we find for the differential cross section

$$d\sigma^{PP}=d\sigma_e^{PP}+d\sigma_m^{PP} \tag{8}$$

where

$$d\sigma_e^{PP}=[4\delta^2/(\delta^2+3)]d\sigma_e^{NP},$$
$$d\sigma_m^{PP}=[4\delta^2\mu_P^2/(1-\delta)^2(\mu_P-\mu_N)^2]d\sigma_m^{NP}.$$

For the γ-emission accompanying a neutron-neutron collision, we find

$$d\sigma^{NN}=(\mu_N/\mu_P)^2 d\sigma_m{}^{PP}. \quad (9)$$

It is interesting to compute the total cross section corresponding to a finite range of γ-ray energies. If we take $E_0=250$ Mev (the energy of the protons from the Rochester cyclotron), we obtain $\sigma^{NP}=0.28\cdot 10^{-29}$ cm^2, $\sigma^{PP}=0.23\cdot 10^{-29}$ cm^2 and $\sigma^{NN}=0.038\cdot 10^{-29}$ cm^2 for γ-rays with energies from $E_0/4$ to $E_0/2$ in the center of mass system. In the laboratory system this covers the energy region 85–170 Mev for γ-rays in the forward direction and 45–90 Mev for γ-rays in the backward direction. These cross sections predict therefore about one high energy γ-ray per 10,000 elastic collisions. If the nucleon collides with a nucleus, the nuclear cross section for bremsstrahlung should be, to a good approximation, the sum of the individual nucleon-nucleon cross sections.

It is also interesting to note that in our non-relativistic approximation a pure Majorana nuclear force would lead to exactly the same result as a pure ordinary force. Both would lead to a constant times the first term in $E(p)$ and $M(p)$; the interference between the Majorana and ordinary forces leads to the logarithmic term in $E(p)$ and $M(p)$. Examination of $E(p)$ and $M(p)$ also reveals that the result is fairly insensitive to the range of nuclear forces; as a matter of fact, if we set $\lambda=0$ (infinite range), the cross section is only increased by a factor 2.5. This shows us at the same time that the bremsstrahlung arising from the Coulomb force between two protons is down by a factor of $1/2.5(g_1/e^2)^2 \sim 600$ compared to our cross section.

After our calculation was completed, two papers[5] on the same subject came to our attention. Both papers study the continuous γ-emission accompanying low energy neutron-proton collisions (below 20 Mev) through the use of the low energy scattering phase shifts. This work was supported by the joint program of the Office of Naval Research and the Atomic Energy Commission.

[5] Y. Nishina, S. Tomonaga, and H. Tamaki, Sci. Pap. Inst. Phys. Chem. Res. Tokyo **30**, 61 (1936); M. Krook, Proc. Phys. Soc. (London) **62**, 19 (1949).

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On The Two-Meson Theory[1]

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Main consequences of the two-meson theory by Marshak and Bethe on the one hand and that by Sakata are compared with experiments concerning π- and μ-mesons. It is shown that the pair type interaction between π-mesons and nucleons in "M. B." theory contradicts with frequent occurrence of stars produced by π-mesons, whereas the assumption in Sakata theory that π-mesons with spin 0 or 1 are responsible for nuclear forces does not. Although the smaller range for the nuclear forces thus obtained from the latter theory is not at variance with the high energy neutron-proton scattering experiment, deuteron quadrupole moment cannot be accounted for by a π-meson field alone, so that the admixture of another meson field with larger range is necessitated. Both π-μ-decay and μ-nuclear capture can be consistently accounted for by assuming spin $\frac{1}{2}$ for μ-mesons as in Sakata theory. However, nuclear β-decay and μ-β-decay have to be considered as direct processes as in Fermi's theory of β-decay, instead of indirect processes through virtual emission and absorption of π-mesons as assumed usually in the meson theory.

THE recent investigations on cosmic rays by Powell and his co-workers have established the facts that there exist two groups of different mesons, i.e., heavy mesons (π-mesons) and light mesons (μ-mesons) and that the former transmute into the latter by emitting a neutral particle.

The theory which involves two mesons of this sort had already been proposed theoretically in 1942 by Sakata and Tanikawa, in cooperation with Inoue and Nakamura.[2] They introduced two kinds of mesons, light and heavy ones, in order to solve

[1] Originally reported at the Annual Meeting of the Physical Society of Japan at Kyoto University, May 23, 1948. When we had sent the report dated August 1, 1948 to Professor J. R. Oppenheimer, he was kind enough to inform us of the new experimental results at the California Cyclotron and advised us to rewrite our report on his information. This is the revision based on the new experimental evidence. We should like to thank Professor Oppenheimer heartily for his kindness and valuable suggestions.

[2] S. Sakata and T. Inoue, Prog. Theo. Phys. **1**, 143 (1946); S. Sakata, Symposium on the Meson Theory at Tokyo, 1943.