Two-Component Wave Equations

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In a recent letter Jehle[1] has considered a two-component wave equation; it is of interest to know the relationship of such equations to Dirac's equations. The object of this letter is to show that by modifying Eddington's[2] method of deriving Dirac's equations we can obtain an equation additional to Dirac's and simpler in form, which may, under certain circumstances, be put in two-component form. This differs somewhat from Jehle's equation, but its derivation suggests that it is the two-component equation most likely to have physical interest.

It is convenient to use quaternion notation; f_i, $(i=1, 2, 3)$, are the usual quaternion units, and $f_0=i$. For a quaternion q, where

$$q=q^k f_k,$$

we write

$$Sq=q^0 f_0, \quad Vq=q-Sq.$$

When the position quaternion,

$$q=x^k f_k \tag{1}$$

undergoes the orthogonal transformation

$$q'=aqb, \quad (|a|=|b|=1), \tag{2}$$

a "wave quaternion," ψ, is defined as one which undergoes the transformation,

$$\psi'=a\psi.$$

If φ is any other quaternion and

$$p=V\cdot\psi\varphi, \quad q=V\cdot\varphi\psi, \tag{3}$$

the obvious identities

$$p\psi=\psi q, \quad \varphi p=q\varphi, \tag{4}$$

are equivalent to Eddington's[3] simple wave equation; and their covariance under (2) is equivalent to

$$P'=apa^{-1}, \quad q'=q, \quad \bar{\varphi}'=a\bar{\varphi}, \tag{5}$$

where $\bar{\varphi}$ is the conjugate quaternion to φ.

The divergence of a mixed (see Silberstein[4]) quaternion, j, is the invariant

$$\mathrm{div} j=S(\partial j/\partial x^0-\nabla j); \tag{6}$$

since the four quaternions,

$$j(\alpha)=S\cdot(\psi f_\alpha \varphi)+V\cdot\psi\varphi, \quad (\alpha=0\cdots 3), \tag{7}$$

are mixed, the identification of one of them with the four-dimensional probability vector is suggested. Using (6) leads to the equations

$$\nabla\psi-(\partial\psi/\partial x^0)f_\alpha=M\psi, \quad \varphi\nabla'-f_\alpha(\partial\varphi/\partial x^0)=-\bar{M}\varphi, \tag{8}$$

where M is a linear quaternion function of a quaternion, $\bar{M}$ its conjugate (Hamilton[5]) and under field-free conditions M, $\bar{M}$ have special forms so that iteration produces the second-order wave equation. *If* $\alpha\neq 0$, *these are identical with Dirac's equations* as written by Conway,[6] and are well known to admit a 4×4 matrix representation. *If* $\alpha=0$, *however*, the ψ-equation, for instance, becomes

$$\nabla\psi-i(\partial\psi/\partial x^0)=M\psi, \tag{9}$$

or, it may be written

$$f^k(\partial\psi/\partial x^k)=-M\psi, \tag{10}$$

where

$$f^k=g^{k\lambda}f_\lambda=\hat{f}_k, \quad \tfrac{1}{2}(f^k f^\lambda+f^\lambda f^k)=S(f^k f^\lambda)=-g^{k\lambda}. \tag{11}$$

The special form of the left-hand side of (10) allows of a 2×2 matrix representation of the f^k, and ψ is then represented as a two-component column matrix. Such a representation of M is not, in general, possible, but when it is, a two-component equation is true. One such possibility, suggested by considering the case when ψ has real quaternion components (i.e., $S\cdot\psi f_i$), $(i=1\cdots 3)$, $S\cdot\psi$, real), is the equation, where now f^k and λ, μ, ψ represent matrices,

$$f^k(\partial\psi/\partial x^k)=\lambda\psi+\mu\psi^*. \tag{12}$$

Under field-free conditions iteration must give the usual wave equation; this requires $\lambda=0$, and if we choose the representation

$$f_1=\begin{pmatrix} i & \cdot \\ \cdot & -i \end{pmatrix}, \quad f_2=\begin{pmatrix} \cdot & -1 \\ 1 & \cdot \end{pmatrix}, \quad f_3=\begin{pmatrix} \cdot & -i \\ -i & \cdot \end{pmatrix}, \tag{13}$$

then

$$\mu=\nu f_2,$$

where ν is a scalar. Writing now

$$f^2 f^k=\gamma^k, \quad \lambda=\lambda_k\gamma^k, \tag{14}$$

we find that

$$\bar{\gamma}^k=-\gamma^k \text{ for } k\neq 2, \quad \bar{\gamma}^2=\gamma^2,$$

and the γ^k obey the same rules as the f^k. The equation becomes

$$\gamma^k[(\partial/\partial x^k)-\gamma_k]\psi=\nu\psi^*,$$

where the λ_k describe the applied field. Jehle's choice of a special form for s^k, involving complex conjugates, restricts the λ_k to the form, $i\varphi_k$, which he considers; however, Eddington[7] has remarked that, in Dirac's theory, this is an unnecessary survival of Hermitic conditions from the Schrödinger theory, and it seems as if the same should apply here.

[1] H. Jehle, Phys. Rev. **75**, 1609 (1949).
[2] Sir A. S. Eddington, *Relativity Theory of Protons and Electrons* (Cambridge University Press, London, 1936), §8.2.
[3] Reference 2, §5.4.
[4] L. Silberstein, Phil. Mag. (6) **23**, 798 (1912), Eq. (13).
[5] Sir W. R. Hamilton, *Elements of Quaternions* (Longmans Green and Company, New York, 1866).
[6] A. W. Conway, Proc. Roy. Soc. (A)**162**, 147) 1937).
[7] Reference 2, p. 131.

On the Temperature Dependence of Counter Characteristics in Self-Quenching G-M Counters*

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It has been reported[1] that the self-quenching counters, usually employing argon or some other gas mixed with some organic vapor, show a temperature dependence of their counting rate which can be sometimes troublesome. Korff, Spatz, and Hilberry[2] during their experiments on argon-alcohol counters found a marked dependence of the counting rate on temperature. They also found that the plateau disappeared at lower temperatures. Putman,[3] in trying to eliminate variations of counting rate with temperature, found that there is a bodily shifting of the plateau towards high operating potentials as the temperature was increased. In his case, however, when the plateau was flattened with an external quenching circuit, the counting rate became largely independent of temperature.

The temperature dependence of the counting rate has been explained in terms of the action of the quenching vapors in a counter. Most of the organic compounds are vapors at room temperature. At lower temperature, however, some of these will condense out with the following result: (1) There may be insufficient quenching vapor left. Under such conditions, the number of vapor molecules and consequently the effect of quenching will depend on temperature. (2) The liquid might condense near the electrodes in such a way as to cause semi-conducting paths across the insulating material between the wire and the cylinder. Leakage across such paths can manifest itself as spurious counts. It is therefore very important to know the range of temperatures within which we can safely use the G-M counters, and the present investigation was undertaken to this effect.

Two counters were chosen, one with internal and the other with external cathode. The counter with internal cathode, prepared in this laboratory, is a usual type of glass enveloped counter with a thin oxidized copper cylinder cathode, filled with commercial argon at 8.5 cm and absolute alcohol (100 percent pure) at 1.5 cm, to a total pressure of 10 cm of mercury, the temperature at the time of filling being 33–34°C. The second is a thin walled glass counter[4] of the self-quenching type with external cathode consisting of a thin layer of graphite. (This counter was brought by Dr. P. S. Gill from Paris and was given to Dr. H. R. Sarna of the East Punajb University. The author is grateful to Dr. Sarna for lending this counter for the present investigation.) The counters could be heated or cooled in a suitable enclosure to different temperatures and maintained constant within ±0.2°C by means of a simple thermostat constructed in this laboratory. The total range of variation extended from 8°C to 60°C. A number of plateau curves were obtained at different temperatures for the two tubes (Fig. 1).

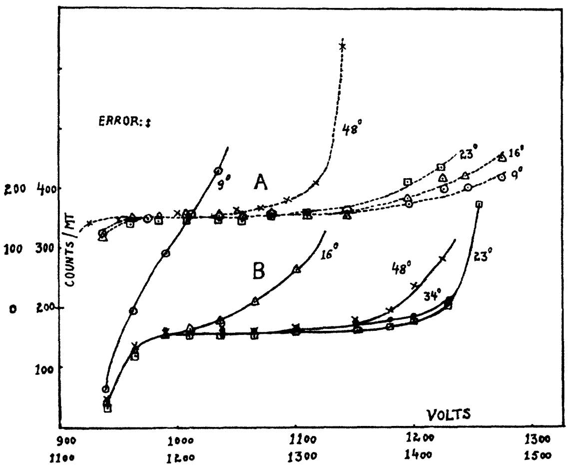

FIG. 1. The temperature dependence of the plateau curves for (A) external cathode and (B) internal cathode G-M counters.

It is clear from the figure that in the case of a counter with external cathode, there has been practically no change in the counting rate within this temperature range, while with the other counter with internal cathode there has been a marked effect near the lower temperatures. In the latter case as well, the counting rate remains independent within about 18° to 60°C; however, below 18°C the plateau has decreased and disappeared at about 9°C. Further it is also clear that the plateau curve becomes flatter at lower temperatures, while there is a definite increase in slope of the plateau at higher temperatures together with a decrease in over-all width of the plateau, possibly due to increase in the number of spurious counts at higher temperatures. (This point is being investigated.) The decrease in plateau width and its disappearance at lower temperatures in the case of a counter with internal cathode might be explained on the supposition that some of the quenching vapor condenses forming semi-conducting paths between the cathode and the central wire, thus giving rise to spurious counts. In the case of the counter with external cathode there is no possibility of such conducting paths and hence no such effect. The extent to which the decrease in the partial pressure of the quenching vapor is responsible for this change will be discussed and a full account of the present investigation published later.

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* The work was completed in the Delhi University Physics Laboratories, Delhi, India.

[1] S. A. Korff, *Electron and Nuclear Counters* (D. Van Nostrand Company, Inc., New York, 1948).

[2] Korff, Spatz, and Hilberry, Rev. Sci. Inst. **13**, 127 (1948).

[3] J. L. Putman, Proc. Phys. Soc. London **61**, 312 (1948).

[4] Roland Maze, J. de phys. et rad. **7**, 164 (1946).

On the Latitude Dependence of Nuclear Disintegrations and Neutrons at 30,000 Feet*

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PRELIMINARY measurements of the change in the rate of production of low energy nuclear disintegrations between geomagnetic latitudes 40°N and 55°N have been made at 30,000 feet pressure altitude in a B-29 aircraft using electron collection ionization pulse chambers. Since both fast and thermal neutrons in the atmosphere at this altitude display a latitude dependence much larger (Fig. 1) than for other components observed in the

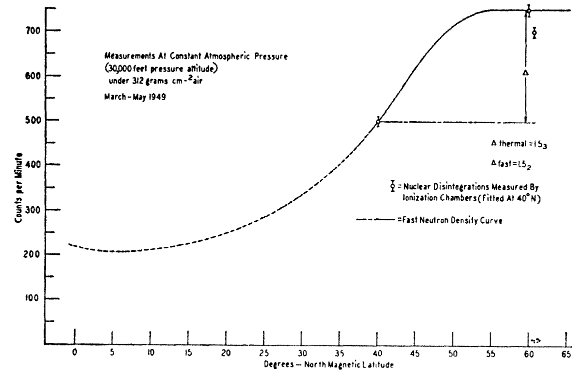

FIG. 1. Comparison of the latitude dependence of nuclear disintegrations and neutrons at 30,000 feet.

cosmic radiation,[1,2] a measurement of nuclear disintegration rates may indicate the extent to which the observed neutrons are associated with these nuclear disintegrations or "star" events.

Two identical thin wall ionization chambers filled with 6 atmospheres of highly purified argon (similar to those used at Los Alamos and by Bridge, Rossi, and co-workers[3]) were operated using fast, linear amplifiers with their output pulses mixed in a non-loading circuit before sorting pulse sizes in a multi-channel pulse height analyzer. The ionization chambers were separated so that the probability of a nuclear event being simultaneously detected in both chambers was negligible.[3] Three G-M counter trays were located to cover a shower area of 1.5 m^2 including the area occupied by the ionization chambers. They were connected to a three-fold shower recording circuit, and this circuit was, in turn, connected to one of the ion chamber pulse height discriminator channels to form four-fold coincidences.

To determine the absolute energy loss in a chamber due to a nuclear event, the maximum pulse size from a retractable polonium alpha-particle source in the chamber was used as a standard to calibrate the pulse discriminator circuits. A linear oscilloscope deflection system and electronic pulsing circuit provided bias calibrations from 0.8 to 2.0× Po alpha-pulse maximum. The system was recalibrated in intervals of not more than two hours with consequent discriminator bias drifts less than ±5 percent. All data were recorded both manually and by photographic film recording. Test intervals to search for spurious pulses due to vibration or electrical pick-up showed that no spurious pulses were observed on any of the flights.

The results at 40°N geomagnetic latitude in Table I show the reproducibility of results over the period of several weeks in which the flights occurred. The same kind of measurements are given in Table II for latitudes greater than 55° (the aircraft was flown to 65°N). It will be noted that Flight No. 3 gives consistently lower values than Flight No. 4. This might be explained as the kind of variation in time previously found for the fast neutrons.[2]

The latitude factor of increase between 40° and 55° has been calculated separately for Flight No. 3 and Flight No. 4 in Table III.