Polarization Effects in $n-p$ and $p-p$ Scattering

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Calculations have been made of the left-right asymmetry to be expected in neutron-proton scattering if polarized neutrons are used. The detection of such an asymmetry would be direct evidence of a tensor force interaction in the scattering. A 4 percent asymmetry might be expected at 100 Mev using a (p,n) exchange reaction as a source of 20 percent polarized neutrons. Effects are smaller in proton-proton scattering.

POSSIBLE methods for producing polarized beams of fast neutrons and protons have been discussed recently.[1–3] It has been hoped that such beams might provide a useful tool for studying the spin dependence of nuclear interactions. The present paper indicates the effect of using polarized incident beams in the experiments on the angular distribution of neutron-proton and proton-proton scattering. The same analysis, of course, also gives the polarization of the scattered particles when an unpolarized incident beam is used.

Three types of spin-dependent nucleon-nucleon interactions can be considered: (a) direct spin-spin coupling, which has the effect of differentiating the singlet and triplet interactions, (b) the tensor force interaction, and (c) the electromagnetic interaction of the magnetic moment of a nucleon with the Coulomb field through which it is moving. In order to detect the first of these by the use of polarized projectiles it is necessary either to use polarized target nuclei[4] or to detect the change in polarization[5] by an additional scattering. The tensor force, on the other hand, can produce a difference between the polarized and unpolarized angular distributions because it couples the spin with the orbital motion. It might be thought that this effect would also go to zero once one averages over the spin of the target nucleon; this is true, however, only to the Born approximation. An informal argument follows. The tensor interaction has a spin dependence of the form $(\boldsymbol{\sigma}\cdot\mathbf{r}\quad\boldsymbol{\sigma}_N\cdot\mathbf{r})$, where $\boldsymbol{\sigma}_N$ is the spin of target nucleon. In the Born approximation this gives a scattered wave with the spin dependence

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}\quad\boldsymbol{\sigma}_N\cdot\boldsymbol{\kappa})u_0, \tag{1}$$

where $\boldsymbol{\kappa}$ is parallel to the change in the momentum and u_0 is the initial spin state. Averaged over the directions of $\boldsymbol{\sigma}_N$, this equals zero. However, if (1) is considered as an intermediate state in a second-order approximation, final states are found with a spin dependence

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}'\quad\boldsymbol{\sigma}_N\cdot\boldsymbol{\kappa}')(\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}\quad\boldsymbol{\sigma}_N\cdot\boldsymbol{\kappa})u_0, \tag{2}$$

where $\boldsymbol{\kappa}+\boldsymbol{\kappa}'$ is parallel to the change in momentum. When averaged over the directions of $\boldsymbol{\sigma}_N$, (2) yields a non-zero term dependent on the spin $\boldsymbol{\sigma}$:

$$i\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}'[\boldsymbol{\sigma}\cdot(\boldsymbol{\kappa}\times\boldsymbol{\kappa}')]u_0.$$

The Born approximation is found to be so inaccurate for the case of the tensor force that such second order terms cannot be neglected.

The tensor force interaction for the case of $n-p$ scattering will be treated exactly using the general notation of Ashkin and Wu.[6] An incident triplet state is written

$$\psi_0 e^{ikz}=\sum_{m_s=-1}^{1} a_{m_s}\chi^{m_s}e^{ikz},$$

where χ^{m_s} are the triplet spin functions quantized along the z-axis (axis of incidence) and the a_{m_s} specify the polarization of the state. The scattered wave is

$$\psi_f(e^{ikr}/r)=\sum_{m_s'=-1}^{1}\chi^{m_s'}\sum_{m_s=-1}^{1}\mathcal{S}_{m_s'm_s}a_{m_s}(e^{ikr}/r),$$

with

$$\mathcal{S}_{m_s'm_s}=\frac{1}{2ik}\sum_{L,J}[4\pi(2L+1)]^{\frac{1}{2}}\{\exp(2i\delta_L^{Jm_s})-1\}\times(SLm_s-m_s'm_s'|SLJm_s)\times(SLJm_s|SL0m_s)Y_L^{m_s-m_s'}(\theta,\varphi),$$

where $\delta_L^{Jm_s}$ is the phase shift corresponding to an orbital angular momentum L and a total angular momentum J with z-component m_s, and $Y_L^m(\theta,\varphi)$ is a spherical harmonic function of the center-of-mass scattering angle θ and the azimuth angle φ. In matrix notation

$$\psi_0=a,\quad\psi_f=\mathcal{S}a$$

and the scattered intensity is given by

$$\psi_f^\dagger\psi_f=a^\dagger\mathcal{S}^\dagger\mathcal{S}a.$$

If the direction of initial polarization is taken as the y-axis,[7] this gives

$$\psi_f^\dagger\psi_f=\tfrac{1}{4}\operatorname{Tr}(\mathcal{S}^\dagger\mathcal{S})+\tfrac{1}{2}\bar{S}_y\operatorname{Tr}(S_y\mathcal{S}^\dagger\mathcal{S}),$$

where S_y is the indicated triplet spin operator, $\bar{S}_y$ is the

[1] J. Schwinger, Phys. Rev. **69**, 681 (1946).
[2] J. Schwinger, Phys. Rev. **73**, 407 (1948).
[3] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).
[4] M. E. Rose, Phys. Rev. **75**, 213 (1949).
[5] J. Schwinger and I. Rabi, Phys. Rev. **51**, 1003 (1937).
[6] J. Ashkin and T. Y. Wu, Phys. Rev. **73**, 973 (1948).
[7] Only polarization perpendicular to the axis of incidence can be detected (reference 3).

TABLE I. Maximum value of polarization effect I_P for $n-p$ scattering divided by unpolarized intensity I_0. θ is the scattering angle in the center-of-mass system for which I_P is a maximum. (I_P'/I_0) is given for the same angle.

Theory	Energy (Mev)	θ	I_P/I_0	I_P'/I_0
Neutral	15	120	−0.23	−0.0005
Charged	15	90	−0.19	−0.001
Symmetrical	15	120	−0.026	−0.0007
Half-exchange	15	45	0.011	−0.003
	15	135	−0.011	−0.0004
Symmetrical	100	35	0.24	−0.006
Half-exchange	100	35	0.19	−0.003
	100	145	−0.19	−0.0003

mean value of the y-component of the total spin in the initial state, and the triplet state is assumed to be normalized to $\frac{3}{4}$. Assuming a completely polarized neutron incident on an unpolarized proton, one finds that the polarization adds to the differential scattering cross section the term

$$I_P=\tfrac{1}{4}\,\mathrm{Tr}(S_y\mathcal{S}^\dagger\mathcal{S})$$
$$=\frac{1}{2\sqrt{2}}\,\mathrm{Im}\sum_{m_s=-1}^{1}\mathcal{S}_{m_s0}{}^*(\mathcal{S}_{m_s1}-\mathcal{S}_{m_s,-1}).$$

Since this is proportional to $\cos\varphi$, the effect of the polarization is to produce a left-right asymmetry in the scattered intensity relative to the initial direction of polarization.[3]

Values of I_P at 15 Mev have been calculated using the phase shifts computed by Rarita and Schwinger[8] under three assumed forms for the exchange interaction, designated "neutral," "charged," and "symmetrical." Calculations were also made for the "half-exchange" interaction which has been suggested to fit the high-energy scattering data.[9] The maximum values of I_P are given in Table I in the form of I_P/I_0 (for $\varphi=0°$), where I_0 is the differential cross section for unpolarized neutrons. The large effect for the "neutral" and "charged" theories is associated with the large p-wave scattering, which is predicted by these theories but is contrary to experiment. The other theories, which fit present data much better, give a much smaller effect.

The values of I_P at 100 Mev have been calculated using the phase shifts given by Ashkin and Wu[6] for a square-well potential. For the "symmetric" interaction

$$I_P=(1/k^2)\cos\varphi\sin\theta\{0.03+0.18\cos\theta+0.05\cos^2\theta+0.69\cos^3\theta+0.07\cos^5\theta\}$$

while for the "half-exchange" interaction

$$I_P=(1/k^2)\cos\varphi\sin\theta\cos\theta\times\{0.20+0.71\cos^2\theta+0.07\cos^4\theta\}.$$

[8] W. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941).

[9] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949). This interaction is written $(1+P)/2$, where P is the Majorana exchange operator. For odd values of L it gives zero scattering.

In both cases (see Table I) the maximum value of I_P is about 20 percent of the theoretical unpolarized scattering (about 35 percent of the experimental unpolarized scattering). It follows that a 20 percent polarization of unpolarized neutrons can be accomplished by scattering from protons,[10] and thus an asymmetry of about 4 percent can be obtained in a double scattering experiment. A more practical possibility may be the scattering of high-energy neutrons produced in a (p,n) reaction. If this reaction can be viewed, in accordance with the general viewpoint of Serber,[11] as a single $p-n$ exchange collision it should exhibit the same polarization effects as $n-p$ scattering. In particular, assuming a "half-exchange" interaction the neutrons coming out at an angle of 35° should have a polarization of about 20 percent if 100 Mev unpolarized protons are incident. If these neutrons are then scattered from protons a left-right asymmetry of about 4 percent relative to the normal of the plane of the (p,n) reaction should be observed at a scattering angle of 35°.

The magnetic moment-Coulomb field interaction, which may be treated by the Born approximation,[2] adds another term to the angular distribution,[12]

$$I_P'=2(e/\hbar c)\mu\cot(\theta/2)\cos\varphi\,\mathrm{Im}g(\theta),$$

$$g(\theta)=\frac{1}{4k}\{\textstyle\sum_L(2L+1)\sin K_Le^{iK_L}P_L{}^0(\cos\theta)+\mathrm{Tr}\mathcal{S}\},$$

where μ is the magnetic moment of the neutron and K_L is a singlet phase shift. This term is found to be much less than 1 percent (see Table I) except at very small angles. Consequently any polarization effect that may be detected must be attributed to the tensor force interaction.

For the case of proton-proton scattering the analysis must be modified to account for the identity of the particles and the charge of the proton. At low energies the polarization effect is much less than for $n-p$ scattering because there is no s-wave triplet scattering and consequently the highest order terms entering I_P are interference terms between p-wave scattering and Coulomb scattering. At 100 Mev it is also considerably smaller; for the symmetric interaction neglecting Coulomb scattering completely

$$I_P=-(1/k^2)\cos\varphi\sin\theta\cos\theta\{0.10+0.08\cos^2\theta\}.$$

In the case of the "half-exchange" interaction the effect vanishes since there is no triplet $p-p$ scattering for this interaction.

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[10] The recoil protons, of course, are also polarized to the same extent.

[11] R. Serber, Phys. Rev. **72**, 1114 (1947).

[12] This interaction also contributes terms (not included in I_P') in the case of an unpolarized incident beam. These terms are also very small for the examples considered here.