Absolute Speed Gauge for High Voltage Particles*

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A precision method has been devised to measure nuclear reaction voltages, and equipment has been built capable of measuring reaction energies of charged heavy particles in the Mev range with accuracies of ~ 0.1 percent. Limitations of the operation are discussed and the possibility of a direct determination of the electronic charge-to-mass ratio and related fundamental constants is pointed out. To verify the conclusions reached, test runs have been carried out with electron beams, and performance was as expected.

INTRODUCTION

THE need for a direct and precise method for measuring speeds of fast charged particle beams arises in the study of nuclear reactions because the precise threshold and resonance energies at which these reactions occur form the basis of evaluation of nuclear constants besides being significant in their own right. Existing instruments such as the generating voltmeter are valuable for linear interpolation between reference voltages but are inherently unsuited for absolute calibration. Consequently, energy quotations from different observers are frequently found difficult to correlate even though different measurements by the same observer may be quite consistent relative to each other. It is fortunate that the threshold and resonance energies are extremely sharp and so will themselves provide the most convenient scale, once their values are established with commensurate precision.

For instance, the half-width of the 985 kv Al(p, γ) reaction is only 300 volts or less when a thin target[1] is used, and other reactions occur with comparable sharpness. Equipment described in the following had for its immediate purpose a determination of the Li(p, n) threshold at 1.89 Mev, described in the companion paper.[2] We were content with an accuracy of 0.1 percent, in view of more stringent limitations arising in the present application from the voltage constancy attainable with the Van de Graaff generator. But this is by no means the highest accuracy of the speed gauge. Given a constant voltage source, a precision-built r-f cavity and a carefully focused electron or ion beam, the accuracy can probably be increased by a factor of the order 10.

For the present purpose the charge-to-mass ratio, which enters into the evaluation of the measurements, is known for protons, deuterons and electrons with sufficient accuracy. It is possible to reverse the procedure, however, and thus to obtain a precise value for the electronic charge-to-mass ratio, of an accuracy at least comparable to the most accurate non-direct methods, and far exceeding the accuracy of direct e/m determinations heretofore used. Besides, in view of a slight discrepancy now existing between the two non-direct methods it is interesting and practicable to design a speed gauge precision-built for the purpose of obtaining e/m within 0.01 percent.

A typical application of the new device would be e.g., the determination of the proton-neutron mass difference by combining the results from (p, n) reactions with those of other suitable reactions.

Perhaps more interesting from a fundamental viewpoint, the device offers an absolute determination of speed and kinetic energy of the particle just before entering into a reaction. Unlike the electrostatic analyzer which compares voltages in the Mev region with much lower known voltages, the r-f speed gauge establishes an absolute voltage scale directly related to the fundamental atomic constants.

II. DESCRIPTION OF OPERATING PRINCIPLE

The principle on which the method is based, has been reported earlier.[3] Transit of the charged particle through two successive, accurately spaced r-f gaps is timed in terms of the period of an r-f oscillation of accurately known frequency. Thus one finds the particle speed as the ratio of the two most accessible and most direct data. Among several possible schemes for effecting the experiment[4] the one best suited provides for exceedingly tight coupling between gaps by means of a high frequency resonator cavity of high Q, of which the two gaps form a part. The high cavity-Q assures one of a high degree of phase synchronism at the two gaps, in addition to the obvious gain in power economy which results in higher signal voltage; thus a high Q is necessary for the required accuracy (0.1 percent in voltage) and also advantageous for sensitivity (beam currents of the order of 10^{-6} amp. or less).

The beam current is modulated at the source with 70 megacycles/second (Fig. 1a), and after acceleration passes in succession through the two gaps which are separated by a field-free drift space. The cavity is capable of electromagnetic resonance at the modulation frequency, in a mode which energizes both gaps in the same field directions at any instant. A particle will

* Assisted by the joint program of the ONR and the AEC.
[1] Bender, Shoemaker, and Powell, Phys. Rev. **71**, 905 (1947).
[2] Shoupp, Jennings, and Jones, Phys. Rev., this issue.
[3] Altar, Garbuny, and Coltman, Phys. Rev. **72**, 528 (1947).
[4] Coltman, Shoupp, and Altar, HV Ion Speed Gauge and Voltmeter. U. S. Patent appl. In particular, the operating scheme finally adopted was suggested to us by Dr. Coltman.

generally contribute net energy at the two gaps toward excitation of the mode, and the resulting r-f power is fed to an indicator. Only when transit time between gaps is exactly an odd multiple (preferably 11, 13) of half-cycles, will the two contributions from a given particle cancel each other exactly. The resulting null at the indicator serves as the timing criterion. To make impulses, contributed by all particles toward mode excitation, most effectively additive, it would be best to modulate the particle beam in very short pulses which would enter the cavity an integral number of cycles apart. It suffices, however, to modulate the beam periodically at the operating frequency, in which case the amplitude of the fundamental Fourier component is indicative of the excitation effect.

If the two gap fields are symmetric and have identical geometries, particles at the second gap will encounter an exact repetition of the field interacted with at the first gap; it will then clearly be permissible to identify effective gap spacing L with the relative displacement between two identical field configurations. Drift distance can then be ascertained with a very small error, considerably less than the longitudinal extension of the fields encountered and, indeed, much smaller than the gap width.

The beam voltage V corresponding to the n-th indicator minimum is related to drift length L, frequency f and speed $s=2fL/n$ as follows:

$$V=\frac{m_0c^2}{e}\left[\frac{1}{\left(1-\frac{s^2}{c^2}\right)^{\frac{1}{2}}}-1\right]$$

$$\cong\frac{2m_0}{e}\frac{f^2L^2}{n^2}\left(1+\frac{3f^2L^2}{n^2c^2}+\cdots\right) \quad (1)$$

where kinetic energy is represented by its relativistically corrected value. This correction, amounting to 0.3 percent for 2 Mev-protons, cannot be ignored.

Least accurate factor in (1) is the length L. If the drift distance is about 1 meter, L has to be measured with an accuracy of 0.05 cm or better to be satisfactory.

Frequency measurements and fundamental constants involved offer no problem at the currently attempted accuracy level.

III. SIGNAL AND NOISE CONSIDERATIONS

The probable error incurred in the presence of noise may be computed as the smallest deviation from the timing criterion (1) which results in a detectable signal-to-noise ratio. Signal power for given transit angles (differing from π by small amounts δ) is fixed by the condition of energy balance; the gap voltage will build up to a steady-state amplitude v such that power transfer P_{in} from the beam to the gap fields equals P_{out} lost by dissipation and other causes. Thus:

$$P_{\text{in}}=\int dt[i_0+i\cos\omega t][v\sin(\omega t-\varphi)+v\sin(\omega\{t+L/s\}-\varphi)]$$

$$=iv\sin\delta/2\cos(\delta/2+\varphi)$$

in view of

$$\omega L/s=n\pi-\delta.$$

If both gaps contribute equally to the build-up, the phase of the excited electromagnetic oscillation is halfway between the two phases of the (sinusoidal) beam current when entering and, respectively, when leaving the cavity. This is true because interaction with the two fields may be represented as the vector sum of two a.c. vectors of equal magnitude, representing the respec-

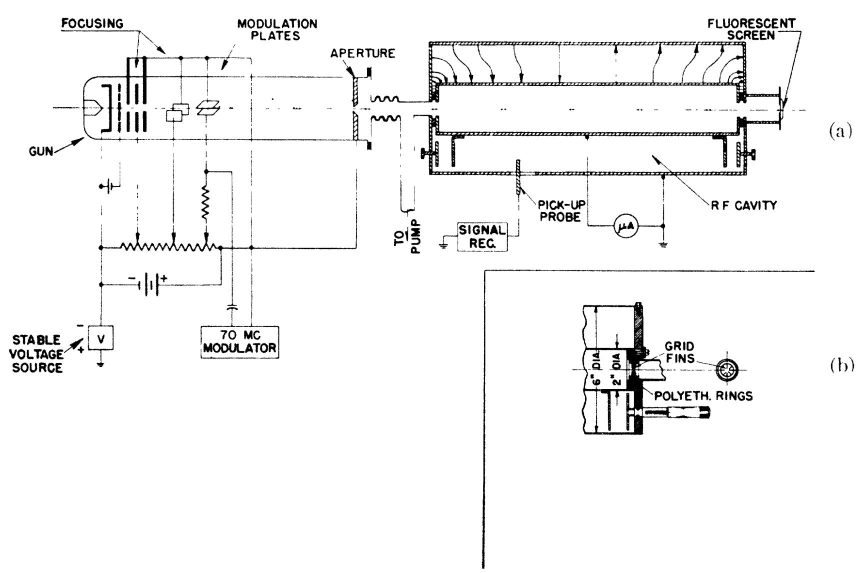

FIG. 1. (a) Schematic diagram of speed gauge. Experimental arrangement to measure particle speeds shown for the case of electron beams using a preferred method of deflection modulation. Spacings between bunches drawn for $n=9$ near minimum cavity response. Arrowed lines in cavity indicate electric field of the excited mode. (b) Design of cavity gap.

tive contribution at each gap. Hence:

$$\varphi = -\delta/2, \tag{2}$$

so

$$P_{\text{in}} = iv\cos(\omega L/2s) \cong (n\pi iv/2)(\Delta s/s), \tag{3}$$

where Δs is the deviation from the due value s_0 satisfying the timing criterion. If we define shunt resistance r_{sh} as that resistance which, when shunted across one of the two gaps will simulate the entire power loss in the cavity, we have for the steady-state condition

$$P_{\text{out}} = v^2/2r_{sh} = P_{\text{in}}; \tag{4}$$

hence from (3) and (4)

$$v = n\pi i r_{sh}(\Delta s/s). \tag{5}$$

Of course, under loaded conditions r_{sh} is considerably less than the purely dissipative value R_{sh} obtained when there is no power transfer to an indicator. If power is fed through a transformer of suitable "turns" ratio k, to an amplifier offering an effective resistance r across the first amplifier grid, it will be seen that the grid voltage is given by

$$V^2_{\text{grid}} = \frac{v^2}{k^2} = \frac{2r_{sh}}{k^2}P_{\text{out}} = \frac{2R_{sh}r}{R_{sh}+k^2r}P_{\text{out}}, \tag{6}$$

which can be maximized by making $k^2 = R_{sh}/r$. Comparing this with Eq. (3) one has

$$V_{\text{grid, opt}} = \frac{n\pi i}{2}r\left(\frac{v}{V_{\text{grid, opt}}}\frac{\Delta s}{s}\right) = \frac{n\pi i}{2}(R_{sh}r)^{\frac{1}{2}}\frac{\Delta s}{s}. \tag{7}$$

For instance, if the cavity has a shunt resistance $r_{sh} = 150{,}000$ ohms (300,000 ohms/gap) and operates into an amplifier showing an input resistance $r = 20{,}000$ ohms at the first grid, a signal of about 50 microvolts can be derived from a beam with a modulated component of 10^{-7} A operating at $n = 11$, for a deviation of 0.05 percent in speed (0.1 percent in particle voltage). Total r-f power generated would be 65×10^{-13} watt, half of which is dissipated in the cavity. The gap voltage is 130 microvolts for optimal loading, just half the value obtained if the cavity were unloaded.

Since the cavity represents in effect two tuned circuits, thermal noise originates to the extent

$$P_{\text{noise}} = 8ktf/Q \sim 4\times10^{-16}\text{ watt},$$

clearly far below the signal level. Noise originating in the first amplifier tube is larger but not objectionable. Since there is no limitation on the time available for completion of a measurement, receiver bandwidth can be reduced almost at will. If a 6AK5 tube were used for the first amplifier stage, the noise component in the plate current would be $2.5\times10^{-14}\Delta f$ amp. The receiver bandwidth Δf actually used was 10 kc and since the signal at this level of amplification is 2×10^{-7} amp., noise from extraneous sources is not a limitation.

Another—intrinsic—limitation is more interesting because it arises directly from the statistical nature of the beam current itself. As a consequence of beam density fluctuations, the frequency of the cavity oscillations is not defined with absolute precision but has statistical uncertainty. The attending spectral width can be narrowed by using a cavity of high Q. If the driving force were strictly sinusoidal or otherwise periodic, its period would be identical with the response period, and the frequency would be defined regardless of the cavity-Q. In the other extreme, if the cavity has an extremely high Q, there would be no need for periodicity of modulation except for the purpose of securing a higher sensitivity of the cavity response. Each particle, or sharp pulse of particles, could serve individually to show deviations from the timing criterion, and the frequency of the signal would be the resonance frefiuency of the cavity. The practical case lies between the two extremes, in that strict periodicity of the beam current is marred by statistical fluctuations, and that the cavity Q is high but finite.

To analyze this case, we may consider the steady state as resulting from many transient impulses set up by the consecutive half-cycles of the modulated beam. Each impulse results in an oscillation at the natural cavity frequency which decays exponentially, but owing to the slight uncertainty $(\Delta I)_\pi$ (this suffix indicates that average has been taken over one half-cycle) the beam modulation frequency fluctuates around the due value with a probable deviation given by $(\Delta f/f)_{\text{resp.}} = 1/(2\pi)(\Delta I/I)$. Here, the steady-state value of current I and its fluctuation ΔI can be computed from the build-up process as follows:

$$\frac{\Delta I}{I} = \left(\frac{\Delta I}{I}\right)_\pi \frac{(\frac{1}{2}(1+e^{-\pi/Q}+e^{-2\pi/Q}\cdots))^{\frac{1}{2}}}{1+e^{-\pi/2Q}+e^{-2\pi/2Q}+e^{-3\pi/2Q}\cdots} = \left(\frac{\Delta I}{I}\right)_\pi\left(\frac{\pi}{8Q}\right)^{\frac{1}{2}}.$$

In the numerator, the square root represents the r.m.s. value of all random fluctuations, subject to the natural circuit decay, and the factor $\frac{1}{2}$ takes into account the fact that these fluctuations take place in the two dimensions of the a.c. vector plane. This results in

$$(\Delta f/f)_{\text{resp.}} = (\Delta I/I)_\pi 1/4(2\pi Q)^{\frac{1}{2}}. \tag{8}$$

In the present application, using current amplitudes of order 10^{-7} amp. at $f = 10^7$ c.p.s., the number of particles per half-cycle is around 5000 so statistical fluctuations amount to about 1.5 percent. With a cavity of $Q = 6000$, statistical fluctuations of signal frequency should be restricted to around 0.002 percent which is safe for the present purpose.

IV. DESIGN OF CAVITY

Choice of a relatively low modulation frequency seemed indicated on two counts. For high precision, a long drift distance is important because the fractional error in determining its length is the most stringent limitation of attainable over-all accuracy for the method. This requires that one keep the frequency low and the order number as high as possible. Secondly, by choosing the lowest frequency at which microwave techniques are still practicable, one combines the latter's advantage of high Q and shunt resistance with the advantage of high receiver sensitivity, all of which decrease at still higher frequencies. The order number n is more or less fixed by the intended beam voltage; since the length of the cavity is around 0.30λ one finds easily $ns/c \cong 0.60$. Around 2 Mev, this means $n \sim .60/0.065 \sim 9$. The frequency was chosen at 70 megacycles/sec., giving a cavity length of 125 cm.

The exact length of the cavity (Fig. 1a) is of course dependent on the gap width, about 1.5 mm in the present design. A certain freedom of choice existed between long and slender cavities and short cavities of large diameter. It was felt that the former offered the advantage of a higher accuracy (see Eq. (1), (5)), which outweighed the advantage of somewhat higher Q and R_{sh} values which might have been obtained by shortening the length. Most important, the longer cavity can be built from commercially available copper tubing which would have been impossible for larger diameters. The inner and outer diameters used were 5.0 and 15.0 cm, respectively.

Polyethylene rings separating the end plates of the two cylinders served as insulating spacers and at the same time as vacuum gaskets between the vacuum in the drift space and the normal pressure in the cavity proper (Fig. 1b).

Two variable plate condensers provide a means for adjusting the resonance frequency and for symmetry between the two gaps. For perfect canceling of r-f inputs at the two gaps, it is important that the gap voltages be equal to each other. Beyond this, it is feasible to compensate for loss of particles between the two gaps, by increasing the second gap voltage inversely in relation to the current. Incomplete compensation of these two effects results in a minimum instead of a null, and the accuracy of the timing criterion suffers as the curvature of the response curve goes down with increasing residual signal.

Grids at the gaps, in the form of thin radial fins, served to improve the electric field configuration at the gaps. Care had to be taken to hold to a minimum the cross section offered to the approaching particles by the fins. About 2 percent of the beam particles were intercepted, and the resulting disturbance through production of secondaries or scattering proved negligible upon investigation.

The unloaded cavity Q was measured to be 6000 which compares well with a previously computed value of 9200 representing copper loss but not dielectric losses in spacers, washers, etc. Tuning to the exact resonance frequency by means of the variable condensers provided no difficulties. The final Q-value was obtained only after elimination of unsuitable spacer components. By frequency modulating the input signal and rectifying the pick-up voltage, the frequency response of the resonator could be conveniently demonstrated, and the Q measured, by means of an oscilloscope. Thus reciprocal Q contributions could be assigned to various component parts and unsuitable specimens were eliminated.

A potential disadvantage of longer drift distances is an increase in the number of particles which get lost between the two gaps by collision with gas molecules or other causes. The attending unbalance between impulses contributed at the two gaps tends to flatten the response curve from a null to a shallower minimum and thus reduces the accuracy with which the latter can be read. It is possible to compensate largely for loss of particles by introducing an asymmetry into the cavity such that the second gap executes proportionately larger voltage oscillations. Still, since the statistical nature of the loss of particles adds to the random noise, it is probably best for high accuracy to maintain a good vacuum (10^{-5} mm or better) and to improve the beam focusing, minimize stray fields, etc.

V. EXPERIMENTAL RESULTS OBTAINED WITH ELECTRONS

Preliminary to applying the speed gauge to high energy protons, test runs were performed using electron beams of relatively low energy. Since generation of the r-f signal depends on the charge and speed, not on the mass of the particles used, these preliminary runs served to test the device under conditions approximating actual operation, yet without the complications of operating the Van de Graaff generator and of the low yield of ion sources. In particular, since particle velocities remain unchanged if beam voltage is reduced in the same proportion as the rest mass, the speed gauge could be tested with electrons against known voltages of the order of 1000 volts.

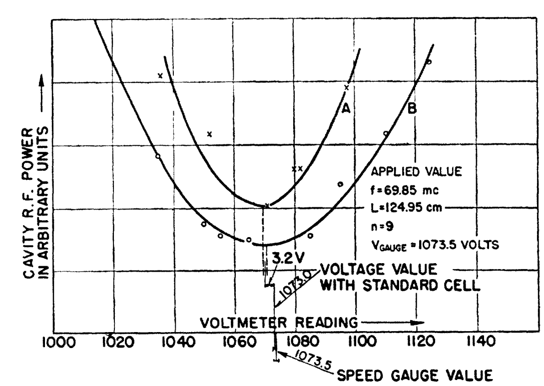

FIG. 2. Response of cavity energy build-up to beam voltage. Measured on the scale of comparing voltmeter. Curve A for more intense and better focused electron beam than in curve B. Contact potential of 3.2 volts has to be added to applied voltage for comparison value of speed gauge formula.

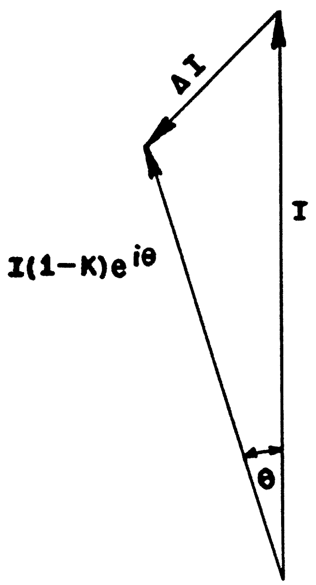

Fig. 3. Diagram for computing the theoretical curves for Fig. 2.

An electron gun of the type used in the RCA cathode-ray tube 5BP1A was carefully lined up along the axis of the speed gauge. Shielding against the earth's magnetic field is necessary with electrons to avoid curvature of the paths. This was effected by wrapping $\frac{1}{32}$-in. Hypernik ribbons around the outer cylinder.

Modulation was provided in the preliminary runs by sweeping the electron beam transversely back and forth by means of a 70-mc voltage applied to one pair of deflecting plates of the electron gun. The gun was first adjusted carefully to be coaxial with the gauge cylinders so that in the absence of transverse fields a focused electron beam impinged on a fluorescent screen mounted at the far end of the cavity. A biasing d.c. voltage equal to the amplitude of the r-f voltage was next applied to the deflecting plates, permitting the beam to enter the cavity through a centered aperture only at the extreme end of the swing. This has the advantage of highest modulated yield for given aperture size, because of the relatively long time intervals spent by the beam at positions near sweep reversal. The aperture diameter was chosen 2.8 mm, a compromise between conflicting requirements of high modulated yield and of good focusing. The number of particles lost between grids was measured by means of an ammeter connected between the inner and the grounded outer cylinder. The total beam current was measured with the same ammeter by means of magnetic deflection of all beam particles into the inner conductor. The fraction of particles lost was thus determined to be about 30 percent of the total. This can only in a small part be attributed to scattering and dispersion of the beam. The air pressure inside the cavity, estimated to be 5×10^{-5} mm Hg, implies a mean free path for 1kv electrons which must be around 20 times the drift length[5] so gas collisions account for only a fraction of the observed particle loss.

Dispersion caused the beam to spread to twice its diameter at the exit diaphragm of the cavity, and the first diaphragm was small enough to keep this source of particle loss in small bounds. Inaccurate focusing as well as defocusing by stray fields must thus be held accountable for most of the loss of particles, and will have to be avoided in future experiments aiming at a better accuracy. With the yield from the gun already cut down to 2 or 3 microamps., it seemed inappropriate with the present limited equipment to try for better focusing conditions in these electron experiments. It would have necessitated replacing our simple crystal rectifier with a detector-amplifier stage.

Figure 2 shows r-f response of the cavity in arbitrary power units as a function of the kinetic energy of the electrons. The theoretical shape of these curves was derived for parameter values k indicating the fraction of particles lost between gaps. Experimental points were then analyzed for a determination of k to give the best fit with a theoretical curve.

R-f power can be expressed in terms of beam current, shunt resistance, transit angle and characteristic resonator impedance $(L/C)^{\frac{1}{2}}$

$$\text{Power}=[(\Delta I)^2/2r_{sh}]L/C$$

where ΔI is the length of the vector difference shown in Fig. 3 between two vectors of length I and, respectively, $(1-k)$I embracing the net transit angle θ between them. This leads to

$$\text{power}\sim k^2+2(1-k)(1-\cos\theta).$$

For best fit, the values $k=0.32$ and $k=0.37$ had to be assigned to the curves A and B (Fig. 2), respectively. With these high k-values a good fit was obtained; also, they are well consistent with the direct determination of k by means of the ammeter. With this procedure, one ascertains the minimum of the theoretical curve with an accuracy far exceeding that obtainable from individual points themselves.

It is seen from Fig. 2 that the minimum can be read with 0.1 percent accuracy. Several runs of these curves taken on various days gave a total spread of 0.2 percent for the position of the minima. The average reading of the two minima shown is 1069.8 ± 1.0 volts as measured on the scale of the calibrated comparing instrument. The contact potential of the copper of the cavity and the BaO-surface of the gun had to be added to this value. This correction amounts to 3.2 volts and was arrived at in two ways with an accuracy sufficient for these measurements. First, the contact potential equals the difference of the work functions of the two materials concerned, except for a negligible contribution due to thermal effects. The work function of copper is 4.3

[5] Knoll, Ollendorf, and Rompe, *Gasentladungstabellen* (Verlag. Julius Springer, Berlin, 1935), pp. 115 and 116.

volts,[6] that of the BaO surface 1.1 volts,[7] yielding a contact potential of 3.2 volts. Secondly, the difference of speed gauge measurements of electron energy taken at other points and the corresponding voltage drop was found consistently to be 3.9 ± 1.0 volts. The only conceivable error might have been the lengthening of the electron path due to curvature in a magnetic field. The latter had, however, been effectively shielded, and such an effect could have increased the difference only by a very small amount. The total energy applied to the electrons therefore amounts to $1069.8 + 3.2 = 1073.0 \pm 2.0$ volts. The absolute voltage value as determined by Eq. (1), with $L = 124.95$ cm, $f = 69.85$ megacycles, $n = 9$ and an $e/m_0 = 1.7592 \times 10^7$ (emu) g^{-1} is 1073.5 ± 0.3 volts in good agreement with the value above. However, the real test for the accuracy of the method lies in the precision with which the minimum can be located, and this is determined by the factor k, not by Eq. (1) in which all variables were measured to far better than 0.1 percent. The voltage was compared to a standard cell through precision resistances. The frequency was heterodyned against a secondary frequency standard.

VI. APPLICATION TO ION BEAMS

The accuracy attained in the electron runs was taken to give reassurance that at least equal accuracy would be attained with ion beams of several million volts. As described in the companion paper,[2] the ion beam which enters the cavity is already well focused, the small current strength being largely due to elimination of all badly focused beam portions, and the stiffness of the HV beam guards against much further particle loss within the cavity proper so k should be very small. This seems to be born out by the appearance of the minima in Fig. 4 of reference 2 which shows much stronger curvature at the minima.

[6] J. A. Becker, Rev. Mod. Phys. **7**, 95 (1935).
[7] J. B. Blewett, J. App. Phys. **10**, 831 (1939).

VIII. POSSIBILITY OF REVISING FUNDAMENTAL CONSTANTS

The best existing determinations of e/m have a probable error of 0.02 percent but other determinations are in poor agreement with this, giving a total spread of 0.2 percent of values used for determining the weighted average now accepted.[8,9] If the accuracy of our method is improved by a factor of 10, it will be possible to determine the electronic charge-to-mass ratio with greater accuracy, and more directly, than has been possible so far. To this end, the following improvements will be necessary:

1. For sharper definition of the gap spacing L, it will be advisable to reduce all gap dimensions to a fraction of the present values. Gap fields may be sharply localized between conical electrodes or other shapes designed for low capacity and high Q. Provision will have to be made for measurement of L after the cavity has been assembled.
2. Focusing and absence of stray fields will be very important to reduce particle loss between the gaps to a minimum. To obtain stiff electron beams, the accelerating voltage should have the highest possible value that can still be conveniently compared with a standard cell.
3. Modulation by sideways deflection will be even more important than in the present tests for keeping the forward velocity of the particles unchanged.

It will also be possible to determine h/e more accurately by measuring the electron energy exactly just before hitting an x-ray target. Once e/m has been determined, there is no further limitation on magnitude of accelerating voltage except the constancy with which it can be maintained.

Several discussions with Dr. J. W. Coltman proved exceedingly helpful in the course of this investigation, and our appreciation of his cooperation is gratefully acknowledged.

[8] J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. **20**, 82 (1948).
[9] *Note added in proof.*—A very accurate value has been published recently by Thomas, Driscoll, and Hipple (Phys. Rev. **75**, 992 (1949)) using yet another indirect method.