The Time Distribution of Decay Electrons from Mesons Stopped in Magnesium at 3500 Meter Altitude*

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The time distribution of decay electrons from the natural mixture of positive and negative mesons stopped in 13.2 g/cm² magnesium at Climax, Colorado has been measured. The apparatus recorded delayed coincidences in ten contiguous 0.6 μsec. channels covering delays from 0.5 to 6.5 μsec. The time distribution of false delayed coincidences due to G-M tube lags was also measured. These lags are shown to affect only the data from the first delay channel (0.5 to 1.1 μsec.), and an approximate correction for this effect can be made. Corrections for accidental delayed coincidences are also applied. The results (based on 2869 delayed coincidences) lie on a smooth differential decay curve, closely fitting the composite decay curve calculated on the assumption of competition between capture and decay for negative mesons. The effective mean life of the negative mesons is definitely less than 2.15 μsec., and appears to be approximately one μsec. The results can also be fitted by a single exponential of mean life 1.7±0.1 μsec.

I. INTRODUCTION

THE aim of this experiment[1] was to study the capture of stopped negative mesons through its competitive effect on the distribution in time of the decay electrons emerging from a material in which mesons are stopped. Conversi, Pancini, and Piccioni[2] had interpreted their discovery that decay electrons result from the stopping of negative mesons in graphite but not in iron by means of a capture probability depending on Z; and Wheeler[3] had pointed out that this probability would be expected to vary roughly as Z^4. Sigurgeirsson and Yamakawa[4] had shown that the yield of decay electrons from various materials agreed roughly with this prediction, the capture and decay probabilities being equal in the neighborhood of $Z=10$. Ticho[5] had pointed out that his low value for the mean lifetime of mesons stopped in aluminum at 3500-meter altitude could be explained as the effect of competition between capture and decay in the disappearance of the negative mesons. The decay curve observed with the natural mixture of positive and negative mesons should be a superposition of two exponentials of different mean life. It should appear in first approximation as a simple exponential of intermediate mean life; with greater experimental precision its composite character should become apparent. Our experiment was essentially the same as Ticho's,[5] differing only in the absorber used (magnesium rather than aluminum) and in geometrical arrangement and circuitry. The limitation of the range of measurable time delays, by G-M tube lags at short times and accidentals at long times, prevented us from establishing the composite character of the decay curve, but we did get a smooth differential decay curve of apparent mean life 1.7±0.1 microseconds (μsec.), definitely less than the 2.15±0.1 μsec.[6] obtained with absorbers of higher Z at sea-level.

Meanwhile Valley[7] and Ticho[8] have independently measured the decay curves of mesons of known charge, and the assumption of a negative meson capture probability comparable with the decay probability in the region $Z=9$–16, and increasing rapidly with Z, can now be regarded as experimentally established.[9] Our experimental points are well fitted by a curve computed for a 55–45 mixture of positive mesons of mean life 2.15 μsec. and negative mesons of mean life 1.0 μsec. (Fig. 4). Our data are in qualitative agreement with the results obtained by Ticho[8] and Valley[7] for magnesium ($Z=12$), who find respectively for the effective mean lives of the negatives 1.0±0.1 μsec. and 1.1±0.2 μsec.

II. APPARATUS

The experiment was carried out in the High Altitude Laboratory of the University of Chicago at Climax, Colorado. This laboratory is at an altitude of 3500 m; its light roof is made of one layer of pulpboard and one layer of sheet iron about 0.04 cm thick.

The arrangement of G-M tubes and absorbers is shown in Fig. 1. The meson-stopper was a 7.6 cm thick

* Assisted by the joint program of the ONR and AEC.

[1] A condensed report on this experiment was given at the Washington Meeting of the American Physical Society, May 1, 1948: R. D. Sard and A. H. Benade, Bull. Am. Phys. Soc. **23** (3), 43 (1948); Phys. Rev. **74**, 1237 (1948).

[2] Conversi, Pancini, and Piccioni, Phys. Rev. **71**, 209 (1947).

[3] J. A. Wheeler, Phys. Rev. **71**, 462 (1947); Phys. Rev. **71**, 320 (1947).

[4] T. Sigurgeirsson and A. Yamakawa, Phys. Rev. **71**, 319 (1947).

[5] H. K. Ticho, Phys. Rev. **72**, 255 (1947); H. K. Ticho and M. Schein, Phys. Rev. **72**, 248 (1947).

[6] B. Rossi and N. Nereson, Phys. Rev. **64**, 199 (1943); M. Conversi and O. Piccioni, Phys. Rev. **70**, 859 (1946); Maze, Chaminade, and Fréon, J. de phys. **6**, 202 (1945). See also references 7 and 8.

[7] G. E. Valley, Chicago Meeting of the American Physical Society, Dec. 30, 1947; G. E. Valley and B. Rossi, Phys. Rev. **73**, 177 (1948); G. E. Valley, Phys. Rev. **73**, 1251 (1948); G. E. Valley, Pasadena Cosmic Ray Symposium, June 21, 1948.

[8] H. K. Ticho, Chicago Meeting of the American Physical Society, Dec. 30, 1947; H. K. Ticho and M. Schein, Phys. Rev. **73**, 81 (1948); H. K. Ticho, Phys. Rev. **74**, 492 (1948); H. K. Ticho, Phys. Rev. **74**, 1337 (1948).

[9] The alternative hypothesis of accelerated decay seems to be disproved by the numbers of decay electrons found by Ticho (reference 8) and Valley (reference 7) and by the discovery of neutrons resulting from the stopping of μ mesons in lead (Sard, Ittner, Conforto, and Crouch, Phys. Rev. **74**, 97 (1948)).

($13.2\ g/cm^2$) block of magnesium, of purity 99.86 percent. The three G-M tube trays, labeled $I-A$, $I-B$, and $I-C$, were used to select incoming charged particles. Each of these trays contained 5 tubes of effective length 25.4 cm and inner diameter 2.38 cm. On the underside of the absorber were placed three G-M tube trays, labelled $II-A$, $II-B$, and $II-C$, whose function was to detect decay electrons emerging downward from the magnesium. These trays each contained seven tubes, identical with those above the absorber except for having an effective length of 50.8 cm. A group of ten 50.8-cm tubes, labeled B, was placed around the sides of the absorber; this group was connected in the circuit so that it reduced the frequency of accidental delayed coincidences. Underneath the tubes of group II was a 2.9-cm lead slab, below which was a group of five 25.4-cm G-M tubes, labeled C. The lead slab and tray C were needed for the determination of the frequency of long lags in the discharges of the G-M tubes of groups I and II.

Each group of G-M tubes connected in parallel was held in a box made of 0.16-cm aluminum sheet. The tubes had 0.079-cm brass walls, and were filled to a pressure of 10 cm Hg with a 9:1 mixture of argon and ethyl alcohol. They were operated at about 70 volts above threshold, the circuit responding to pulses greater than 1 volt.

Figure 2 is a functional diagram of the circuit, which is based essentially on an earlier design of M. L. Sands.[10] The pulses from trays $I-A$, $I-B$ and $I-C$ went to a special coincidence circuit, α, which gave an output 0.5 μsec. after the time of occurrence of the *earliest* of the three pulses, provided that all three occurred within 0.5 μsec. of one another. This output will be referred to as pulse I. This timing from the first (rather than the last) pulse had the aim of reducing the effect of fluctuating time delays in the G-M tube discharges. In effect, the delay depends on the initial positions of the ions, and is least when the ions are formed close to the wire and near its central portion.[11] By using three trays of counters, we increased the probability of obtaining a short delay in at least one of the three tubes traversed. The staggering of the trays, both cross-wise and lengthwise, should have somewhat further increased this probability.

The same technique was used for the pulses of trays $II-A$, $II-B$, and $II-C$. Here, too, the special coincidence circuit, β, gave an output 0.5 μsec. after the time of occurrence of the earliest of the three pulses provided that all three occurred within 0.5 μsec. We shall refer to this output as pulse II.

Pulse I, delayed an additional 0.5 μsec. by delay unit γ, and pulse II, delayed an additional 1.0 μsec. by delay unit δ, entered an eleven-channel double-coincidence unit, ϵ. In this unit, pulse I produces a sequence of eleven "gate" pulses, one on the heels of the other. The first lasted 1.0 μsec., while the succeeding ten were each about 0.6 μsec. long. Each of the eleven gate pulses went to one input of eleven corresponding double-coincidence circuits. Pulse II was applied as a narrow "spike" to the other (paralleled) inputs of the eleven double-coincidence circuits. Only that coincidence circuit gave an output whose input gate was "on" at the time of arrival of pulse II. The counts of the eleven double-coincidence circuits were recorded on eleven electro-mechanical counters ("message registers"). Consideration of the time relationships in the complete circuit shows that the first message register recorded "prompt" coincidences, in which counter group II discharged between 0.5 μsec. before and 0.5 μsec. after counter group I; the second message register recorded delayed coincidences in which II discharged between 0.5 and 1.1 μsec. after I; the third message register recorded delayed coincidences in which II discharged between 1.1 and 1.7 μsec. after I; and so on, out to a maximum delay of about 6.5 μsec. The apparatus thus recorded continuously a differential time-distribution curve, the time scale being divided up into segments of about 0.6 μsec.

The essential features of the circuits just described are due to M. L. Sands.[10] We added several refinements designed to reduce spurious effects. The two main sources of false delayed coincidences were accidental delayed coincidences of two unrelated particles, and the occasional large delays of the counters of group II with respect to those of group I when actuated simultaneously. The special coincidence circuits had a feature (pointed out in reference 10) which reduced the frequency of accidental coincidences. A pulse, on any of its three inputs, paralyzed the circuit for about 14 μsec. Thus, if the particle which triggered group I also triggered any one of the counters of group II, a second particle which triggered group II could not give rise to an accidental delayed coincidence (the maximum measurable delay,

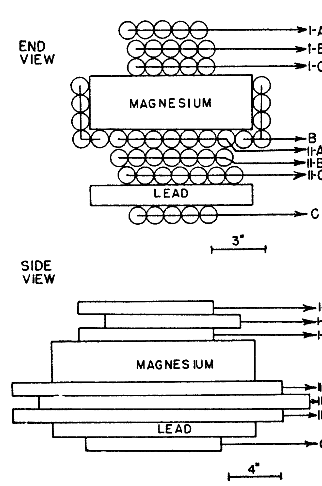

FIG. 1. Geometrical arrangement of the absorbers and G-M tubes.

[10] Rossi, Sands, and Sard, Phys. Rev. **72**, 120 (1947). It is to be hoped that the designer of these ingenious and reliable circuits will find time to publish a detailed description of them.

[11] See, for references, D. R. Corson and R. R. Wilson, Rev. Sci. Inst. **19**, 207 (1948).

ca. 6.5 μsec., being less than 14 μsec.). In our experiment we strengthened this protection by adding the G-M tubes of group B. These were connected to the special coincidence circuit, β, in such a way that any pulse from B paralyzed the circuit for the next 14 μsec. but did not give rise to an output pulse II unless it was followed within 0.5 μsec. by pulses from $II-A$, $II-B$, and $II-C$. A coincidence (within 0.5 μsec.) of $II-A$, $II-B$, and $II-C$ produced, on the other hand, an output pulse II whether or not there was a pulse from B. The expected rate of accidental coincidences in the i-th channel is, therefore,

$$\{(I-A\cdot I-B\cdot I-C) \\ -(I-A\cdot I-B\cdot I-C{:}II-A+II-B+II-C+B)\} \\ \times(II-A\cdot II-B\cdot II-C)\tau_i,$$

where the first term in curly brackets represents the rate of triple coincidences of $I-A$, $I-B$, and $I-C$, the second term in curly brackets represents the rate of triple coincidences of $I-A$, $I-B$, and $I-C$ accompanied by a pulse from $II-A$ or $II-B$ or $II-C$ or B; the second factor is the rate of triple coincidences of $II-A$, $II-B$, and $II-C$, while τ_i is the width of the channel in question. Addition of the counters B reduced the magnitude of the expression in curly brackets. Even so, the latter was about one-third of $(I-A\cdot I-B\cdot I-C)$, a result which can be ascribed to the large amount of soft radiation present at 3500 m altitude. By means of a toggle switch, circuit β could be altered to give a pulse II whenever a pulse from $II-A$ or $II-B$ or $II-C$ or B entered it. The prompt $(I{:}II)$ coincidence rate in this case gave the second term in curly brackets.

The effect of fluctuating counter lags on the delayed coincidence rates is discussed in the next section. It is shown there that the effect can be calculated with good accuracy if one knows how often a particle (or simultaneous particles) traversing $I-A$, $I-B$, $I-C$, $II-A$, $II-B$ and $II-C$ produces a coincidence in each of our delayed coincidence channels. In order to determine this frequency experimentally, we added the lead slab, G-M tubes C, the gate generating circuit ζ, and the coincidence gating circuit η. The lead slab was of ample thickness to stop any decay electron coming from above. Thus, when there was a prompt coincidence of group I and C, group II must have been traversed simultaneously, and any delayed coincidence recorded must have been due to the shortest counter lag in group II exceeding by this recorded delay time the shortest counter lag in group I. In the gate generating circuit, ζ, pulses from C produced 0.5 μsec. (or 1.0 μsec., depending on the throw of a switch) rectangular pulses which entered the gating circuit, η. The narrow pulse I, delayed an additional 0.5 μsec. by circuit γ (for a total delay of 1.0 μsec. from the earliest of the pulses of group I) also entered the gating circuit. It could only pass through to the multiple coincidence unit ϵ if the gate from C was on. Thus ϵ could only function if there was a pulse from C within 0.5 μsec. (or 1.0 μsec.) after the earliest pulse from group I. The readings of ϵ in this case gave us the frequency of delayed coincidences due to counter lags from prompt events. When we wished to measure true delayed coincidences due to meson disintegrations, we permanently opened the gate η by throwing a switch. Measurements of true delays and of counter lags were alternated.

The circuit also contained a scaler and message-register for monitoring various counting rates. The scaler could be connected to any one of six points, enabling us to determine the following rates: $(I-A\cdot I-B\cdot I-C{:}C)$, $(I-A\cdot I-B\cdot I-C)$, $(I-A+I-B+I-C)$, $(II-A\cdot II-B\cdot II-C)$, $(II-A+II-B+II-C+B)$, C. The first expression represents the rate

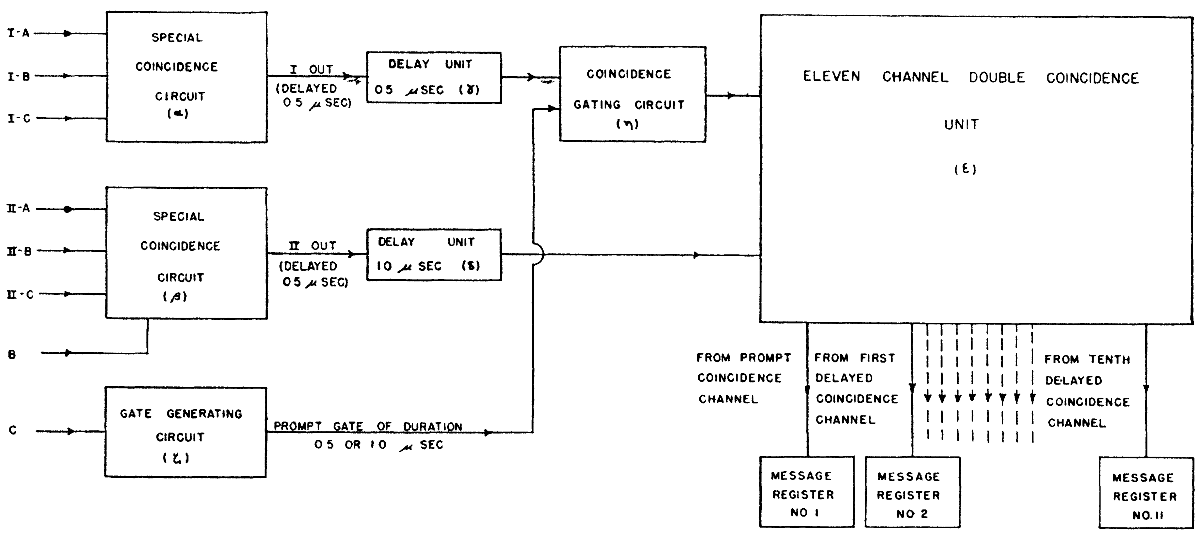

FIG. 2. Functional diagram of the circuit.

of events in which $I-A$, $I-B$, and $I-C$ discharge within 0.5 μsec. and C discharges within 0.5 (or 1.0) μsec. of the earliest of them; the second the rate at which $I-A$, $I-B$, and $I-C$ discharge within 0.5 μsec. of each other; the third the rate at which $I-A$ and/or $I-B$ and/or $I-C$ discharge, being recorded through a mixer of 14 μsec. insensitive time; the fourth the rate at which $II-A$, $II-B$, and $II-C$ discharge within 0.5 μsec. of each other; the fifth the rate at which $II-A$ and/or $II-B$ and/or $II-C$ and/or B discharge, as recorded through a mixer of 14 μsec. insensitive time; and the sixth the rate at which C discharges. The monitor was switched between the six positions during the experiment. The reading in the first position had the meaning stated only when the circuit η was switched to perform its gating function; when the gate was thrown open, the readings in the first and second positions coincided.

Periodic calibrations of the circuit were made, using three artificial pulses whose time of occurrence with respect to a trigger pulse could be continuously and independently adjusted by fifteen-turn helical potentiometers ('helipots') whose settings were reproducible to within ± 0.01 μsec. This "sliding pulser" was copied from an unpublished design of Sands'; we added a circuit for synchronizing the trigger with a 2.500-Mc crystal-controlled oscillator which was also used to provide marker pips 0.4 μsec. apart. These marker pips were used to calibrate the helipots. The pulses were shaped to simulate G-M tube pulses, and their amplitudes were adjusted to equal those of the smallest pulses from the particular G-M tube groups. We feel that the determination of time differences was very accurate; the zero point of our time scale may, however, be somewhat off, because of the varying rise rates of the G-M tube pulses as contrasted with the uniformity of the artificial pulses.

III. THEORY OF THE EFFECT OF COUNTER LAGS

As has been pointed out by Rossi and Nereson,[12] the effect of variable G-M tube delays and of an error in the location of the zero of the time scale is to make each measured delay differ from the true delay by a variable amount t' independent of the delay. Let $f(t')dt'$ be the probability of an error in the interval dt' at t', $N(t)$ be the number of coincidences in which counter group II is actuated by a particle arriving more than t seconds after its parent has actuated counter group I, and $N_{\mathrm{obs}}(t)$ be the measured number of coincidences in which the pulse from group II occurs more than t seconds after the pulse from group I. The smearing of the time distribution is given by[12]

$$N_{\mathrm{obs}}(t)=\int_{t'=-\infty}^{+\infty} N(t-t')f(t')dt'. \tag{1}$$

As illustrated in Fig. 3a, $N(t)$ has the large constant

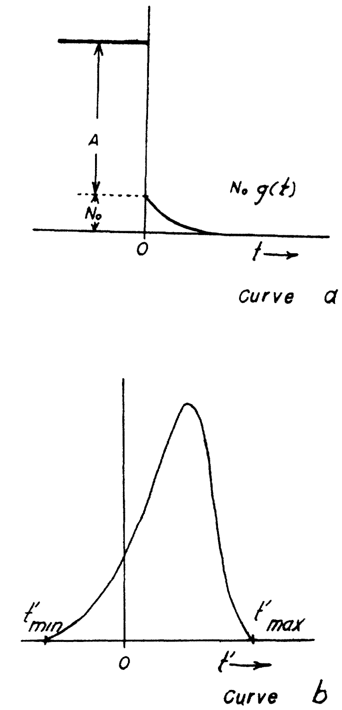

FIG. 3. a. Plot of $N(t)$, the true distribution of coincidences with delay greater than t. A is the number of prompt coincidences due to penetrating particles and showers; N_0 is the number of delayed coincidences due to decay electrons. b. Plot of $f(t')$, the differential probability distribution of the difference between the measured delay and the true delay. For no distortion of the time distribution, $f(t')$ would have to be a delta-function.

value $A+N_0$ for negative t, and the small diminishing value $N_0 g(t)$ for positive t. A is the true number of "prompt" coincidences caused by penetrating particles and showers and N_0 is the true number of delayed coincidences due to meson decays; $g(t)$ would be $\exp(-t/\tau)$ for a simple exponential decay. As illustrated in Fig. 3b, $f(t')$ is expected to be a peaked but not necessarily symmetrical function, differing appreciably from zero only for several tenths of a microsecond to either side of the origin. It is clear physically that $f(t')$ is essentially zero everywhere outside a finite interval $t'_{\mathrm{min}}<t<t'_{\mathrm{max}}$. If the shortest delay measured, t, is greater than the longest lag, t'_{max}, (this is the case for all delayed channels after the first), (1) becomes

$$N_{\mathrm{obs}}(t)=N_0\int_{t'_{\mathrm{min}}}^{t'_{\mathrm{max}}} g(t-t')f(t')dt'. \tag{2}$$

For a mixture of positive and negative mesons, $g(t)$ is a linear combination of two exponentials

$$g(t)=a\exp(-t/\tau_+)+b\exp(-t/\tau_-)$$

and

$$N_{\mathrm{obs}}(t)=N_0 a\exp(-t/\tau_+)\int_{t'_{\mathrm{min}}}^{t'_{\mathrm{max}}}\exp(t'/\tau_+)f(t')dt'$$
$$+N_0 b\exp(-t/\tau_-)\int_{t'_{\mathrm{min}}}^{t'_{\mathrm{max}}}\exp(t'/\tau_-)f(t')dt'. \tag{3}$$

The number of counts in a delay channel extending from

[12] B. Rossi and N. Nereson, Phys. Rev. 62, 417 (1942).

TABLE I. Calibration of delayed coincidence channels.*

		Nov. 17	Dec. 3	Dec. 16	Dec. 30	Jan. 4	Mean value, weighted according to duration of decay measurements between successive calibrations
First channel	Delay to midpoint of channel	0.75	0.74	0.80	0.80	0.80	0.76
	Width of channel	0.71	0.73	0.64	0.69	0.59	0.67
Second channel	Delay to midpoint of channel	1.37	1.36	1.38	1.39	1.35	1.37
	Width of channel	0.63	0.62	0.58	0.59	0.63	0.61
Third channel	Delay to midpoint of channel	1.94	1.92	1.98	1.98	1.96	1.95
	Width of channel	0.60	0.62	0.63	0.64	0.63	0.62
Fourth channel	Delay to midpoint of channel	2.51	2.51	2.55	2.59	2.56	2.54
	Width of channel	0.61	0.63	0.66	0.66	0.66	0.64
Fifth channel	Delay to midpoint of channel	3.08	3.10	3.15	3.19	3.15	3.13
	Width of channel	0.58	0.57	0.66	0.65	0.64	0.61
Sixth channel	Delay to midpoint of channel	3.68	3.68	3.76	3.81	3.78	3.72
	Width of channel	0.65	0.63	0.67	0.65	0.67	0.65
Seventh channel	Delay to midpoint of channel	4.27	4.29	4.37	4.42	4.39	4.33
	Width of channel	0.60	0.61	0.65	0.62	0.63	0.62
Eighth channel	Delay to midpoint of channel	4.87	4.88	4.95	5.01	4.97	4.92
	Width of channel	0.66	0.67	0.66	0.64	0.66	0.66
Ninth channel	Delay to midpoint of channel	5.45	5.46	5.55	5.64	5.61	5.51
	Width of channel	0.64	0.65	0.68	0.70	0.67	0.66
Tenth channel	Delay to midpoint of channel	6.05	6.05	6.18	6.24	6.21	6.11
	Width of channel	0.60	0.61	0.65	0.62	0.63	0.62

* The numbers are times in microseconds.

t_i to t_{i+1} ($t_i > t'_{\max}$) is then,

$$N_{\rm obs}(t_i)-N_{\rm obs}(t_{i+1})$$
$$=N_0 a[\exp(-t_i/\tau_+)-\exp(-t_{i+1}/\tau_+)]\times\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_+)f(t')dt'+N_0 b[\exp(-t_i/\tau_-)-\exp(-t_{i+1}/\tau_-)]\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_-)f(t')dt'. \quad (4)$$

The relative weights in the linear combination are distorted by the lags, but by a factor independent of t_i and t_{i+1}. A series expansion of the exponential in the integrands shows that the distortion is small if, as in the present experiment, the mean and root-mean-square values of t' are small compared to both τ_+ and τ_-. For the limiting case of a single exponential ($\tau_+=\tau_-$) the distortion vanishes, the only effect of the lags being to change the number of decays observed.[12]

When t can be smaller than $t'_{\max}$, (1) becomes

$$N_{\rm obs}(t)=N_0\int_{t'=t'_{\min}}^{t} g(t-t')f(t')dt'+(A+N_0)\times\int_{t'=t}^{t'_{\max}} f(t')dt'. \quad (2')$$

For our first delay channel, $t_1<t'_{\max}<t_2$, and

$$N_{\rm obs}(t_1)-N_{\rm obs}(t_2)$$
$$=N_0 a\exp(-t_1/\tau_+)\int_{t'_{\min}}^{t_1}\exp(t'/\tau_+)f(t')dt' -N_0 a\exp(-t_2/\tau_+)\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_+)f(t')dt' +N_0 b\exp(-t_1/\tau_-)\int_{t'_{\min}}^{t_1}\exp(t'/\tau_-)f(t')dt' -N_0 b\exp(-t_2/\tau_-)\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_-)f(t')dt' +(A+N_0)\int_{t_1}^{t_2}f(t')dt', \quad (4')$$
$$=N_0 a[\exp(-t_1/\tau_+)-\exp(-t_2/\tau_+)]\times\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_+)f(t')dt' +N_0 b[\exp(-t_1/\tau_-)-\exp(-t_2/\tau_-)]\times\int_{t'_{\min}}^{t'_{\max}}\exp(t'/\tau_-)f(t')dt' +(A+N_0)\int_{t_1}^{t_2}f(t')dt'. \quad (4'')$$

In going from (4′) to (4″), we have used the fact that only a very small fraction of the lags (of the order of 1.6×10^{-4}) occurs beyond t_1. The last term of (4″) is the number of prompt coincidences thrown into the first delay channel by the lags. This can be calculated with the help of the data obtained when group I is required to be in prompt coincidence with tray C. In effect, the integral is very closely the ratio of the number of counts in the first delay channel to the number in the prompt channel when C is in control, while $(A+N_0)$ is very closely the number of counts in the prompt channel when C is not controlling.

The effect of variable lags and of an error in the location of the zero of the time scale is seen to be twofold: a small distortion of the relative weight of the two exponential components (smearing of the true decay curve) and a throwing of some truly prompt coincidences into the first delay channel. As we shall show in the next section, only the second effect is serious in our experiment.

IV. RESULTS

The meson decay measurements (C not controlling), which were made at intervals between November 22, 1947 and January 1, 1948, had a total duration of 466.6 hours. Lag measurements (C controlling) were interspersed throughout the period December 3 to January 3, and lasted 214.65 hours. Time calibrations of the circuit were made on November 17, December 3, December 16, December 30, and January 4. Tests of the individual G-M tubes were made at more frequent intervals, and during the runs the counting rates were checked two or three times a day.

The calibration data relative to the delayed coincidence channels are shown in Table I. While the calibrations could be made with good precision and repeatability (± 0.01 μsec.), there were appreciable variations in the results obtained on different days. These are thought to be due in large part to bias changes resulting from changes in the room temperature and aging of circuit components. The widths of the different channels are not expected to be exactly the same, each being determined by the effective length of its own section of delay line.

In an effort to take into account the systematic changes in the positions and widths of the delay channels, we have divided the decay data into four groups, each comprising the data obtained between successive calibrations. The data in each group are assumed to refer to channel widths and midpoints obtained by averaging the results of the calibrations which began and ended the period. The weight of each group in the final result is taken to be proportional to the duration of the decay measurements included in it. In this way, we arrive at the numbers in the last column of Table I, the weighted means of the widths of the channels and the delays to their centers.

TABLE II. Delayed coincidence data.

Delayed coincidence channel	Raw data	Corrected for effect of lags	Reduced to standard channel width of 0.63 μsec.	Corrected for accidental coincidences	Final corrected data (number for first channel multiplied by 1.959)	Estimated statistical standard error
	Counts in 466.6 hours (Nov. 22–Jan. 1)					
First	2241 (849 in the 238.2 hours between Dec. 3 and Jan. 1)	— 474 ± 40 (in 238.2 hours)	— 445 ± 40 (in 238.2 hours)	— 421 ± 40 (in 238.2 hours)	825	$\pm 83(=[849+78^2]^{\frac{1}{2}})$
Second	571	571	590	549	549	± 24
Third	374	374	380	339	339	± 19
Fourth	275	275	271	230	230	± 17
Fifth	212	212	219	178	178	± 15
Sixth	174	174	169	128	128	± 13
Seventh	136	136	138	97	97	± 12
Eighth	108	108	103	62	62	± 10
Ninth	92	92	88	47	47	± 10
Tenth	78	78	79	38	38	± 9

TABLE III. Coincidences with lag-measuring arrangement (C controlling).

Prompt coincidences	Delay channels: First	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth	Ninth	Tenth
Counts in 261.77 hours in the interval Dec. 3–Jan. 3										
523,104	83	1	0	1	0	1	0	1*	1*	0*

* For only 174.45 hours, in which the number of prompt coincidences was 410,704.

The data obtained in the meson-decay measurements (C not controlling) are shown in the second column of Table II. The numbers given there have to be corrected for the effect of lags and for accidental delayed coincidences.

The data obtained in the lag measurements (C controlling) are totalized in Table III. Almost all of the measurements were made with the gate pulse from circuit ζ 0.5 μsec. long, requiring that C discharge between 0 and 0.5 μsec. after the earliest discharge of counter group I. For 26.03 of the 261.77 hours, the gate was made 1.0 μsec. long; as this change had no significant effect on the results, the data from this period are included in the total. The lag counts in the delay channels beyond the first are suspected to be of accidental origin. Unfortunately no lag measurements were made with the switch in circuit β thrown to give $(I-A\cdot I-B\cdot I-C{:}II-A+II-B+II-C+B)$, so that we are unable to calculate the expected number of accidental coincidences (which would have been produced by side showers striking IA, IB, IC and C but missing counter group II). Whether or not the counts in these later channels are true lags or accidentals has, at any rate, a negligible effect on the results.

The ratio of the lag counts in the first delay channel to the prompt counts is seen to be $83/523{,}104=(1.59\pm 0.17)\times 10^{-4}$. This is the value of the integral in the last term on the right-hand side of Eq. (4″), representing the probability of a truly prompt coincidence being thrown into the first delay channel. In this connection it is worth noting that the starting time of the first delay channel ranged from 0.37 μsec. (December 3) to 0.50 μsec. (January 4).

As the number of counts obtained in the first delay channel in the decay-measuring arrangement was large, it is thought wiser to use only those (numbering 849)

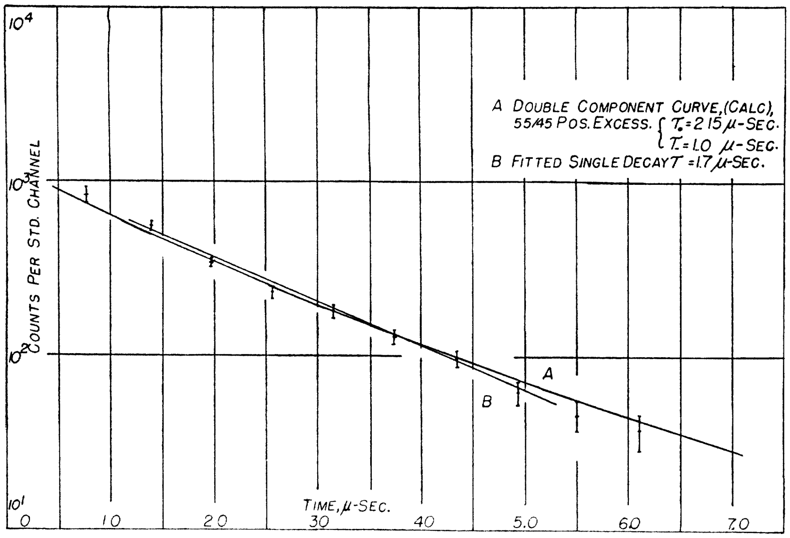

FIG. 4. The differential decay curve from mesons stopped in magnesium. The number of counts per 0.63 μsec. time interval is plotted against the delay to the center of the interval. The experimental points are estimates of the mean numbers, derived from our data, the vertical lines representing estimated statistical standard deviations. Also plotted for comparison is a calculated composite curve, as well as a single exponential fitted to the data by the method of Peierls (see text).

which were obtained in the period (December 3–January 1) during which frequent lag measurements were made. In this period the number of prompt coincidences (A in (4″)) was 2,358,672, so that the last term in (4″) becomes 375±40. Subtracting this correction, we obtain the figure in the third column of Table II, 474±40.

It is seen that the lag correction is appreciable for the first delay channel. Unfortunately, the frequency of the lags is so small that the correction itself has a considerable statistical inaccuracy. An even greater uncertainty results from the fact that the C tray did not completely cover the cone defined by counter groups I and II. Hence the lag distribution measured refers only to cases in which particles passed through the central portion of counter group II.

It remains to correct for the accidental delayed coincidences. The number of these as well as of true delays (to a good approximation) is proportional to the channel width. We therefore now reduce the number of counts in each channel to a standard channel width of 0.63 μsec. The results are given in the fourth column of Table II. Measurements with the switch in circuit β thrown so as to give $(I-A\cdot I-B\cdot I-C:II-A+II-B+II-C+B)$ were made on December 12–13, simultaneously with measurements of the $(I-A\cdot I-B\cdot I-C)$ rate. The difference of these two was 99.9/min. (i.e. 433.9/min.–334.0/min.). Immediately afterwards (December 13–14) the $(II-A\cdot II-B\cdot II-C)$ rate was measured as 1483/min. The expression in Section II for the mean accidental rate gives then 9.33×10^{-2}/hr. for a 0.63 μsec. channel. The average $(I-A\cdot I-B\cdot I-C)$ rate in the period November 22–January 1 was 412.5/min. Assuming that the accidental rate is proportional to this rate, we use 8.87×10^{-2}/hr. for the mean accidental rate, giving 41 as the expected number of accidentals per standard channel width in 466.6 hours. A similar calculation for the 238.2 hour period between December 3 and January 1 gives 24 as the expected number of accidentals in the first delay channel (reduced to standard width). The fifth column of Table II shows the corrected numbers of counts per standard channel, the expected number of accidentals having been subtracted off. The sixth column gives the final results: our estimate of the mean number of true delayed coincidences in each channel during 466.6 hours. The number of counts in the first delay channel has been scaled up to correspond to a duration of 466.6 hours. It is seen from Table II that the result for the first delay channel is strongly affected by the rather uncertain lag correction, while the correction for accidentals has a large effect on the results for the last delay channels. In fitting curves to the data we have, therefore, ignored the first, ninth, and tenth delay channels.

If the mean accidental rate were known with high precision, the standard error of one of our estimates, e.g. 230 for the fourth channel, could be fairly estimated as simply the square root of the number of observed delays,[13] e.g. $(275)^{\frac{1}{2}}=17$. These estimated standard errors, which are purely statistical, are given in the last column of Table II. In the first delay channel the statistical error in the lag correction (±78) is compounded with the statistical error in the decay measurement $(\pm(849)^{\frac{1}{2}})$ to give ±83.

There were, of course, systematic errors over and above the statistical uncertainties. There were changes of as much as 20 percent in the prompt coincidence rates, which seemed to be correlated with weather conditions. These could affect only the accidental correction (41 per channel), so that the corresponding systematic errors in the corrected numbers of delayed coincidences would be negligible compared to the statistical uncertainties, though approximately equal to the estimated statistical standard errors in the last three delayed channels. The shifting of the channel widths and positions (Table I)

[13] A. Sard and R. D. Sard, to be published.

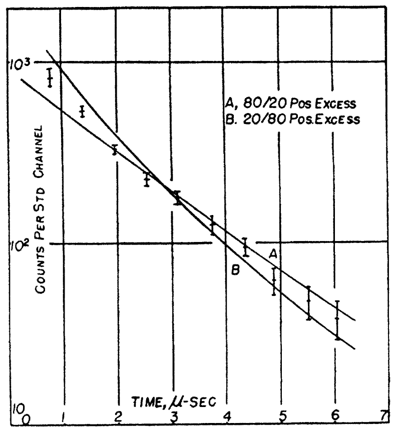

FIG. 5. Comparison of the results with composite decay curves calculated for positive-negative stopped meson ratios of 80–20 and 20–80. The effective mean lives of the positives and negatives are assumed to be 2.15 and 1.0 μsec. respectively.

introduced systematic errors in both the corrected numbers of counts per standard channel width and in the weighted mean values of the delays of the channel midpoints.

The results (last two columns of Table II, last column of Table I) are plotted semilogarithmically in Fig. 4. These are points of the differential decay curve of the mixture of positive and negative mesons stopped in magnesium at 3500 m. altitude. As the systematic errors are not large enough to have an appreciable effect on the fitting of curves, we have not indicated them in Fig. 4.

We have calculated the differential decay curves to be expected on the assumption that competition between capture and decay results in a lower effective mean life for the negative meson and a smaller number of decay electrons from negative mesons. The curve for a 55–45 mixture of positives and negatives and a mean life for the negatives of 1.0 μsec. is shown in Fig. 4. It is seen to fit the experimental results. Even with our relatively small experimental uncertainties, it is, unfortunately, impossible to pin down these two parameters within narrow limits. Figure 5 shows that it is necessary to go to such extreme values of the positive-negative ratio as 80–20 and 20–80 in order to get a poor fit. This justifies, incidentally, our neglect of the effect of counter lags on the relative weight of positives and negatives in the mixture.

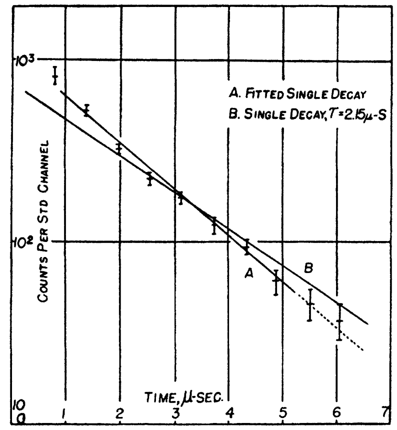

FIG. 6. Comparison of data and its equivalent single decay curve of mean life 1.7 μsec. with a single decay curve of 2.15 μsec.

The expected composite curve can be approximated by a pure exponential of intermediate mean life. By applying the procedure of Peierls[14] to the results for the second through eighth delay channels, we obtain 1.7 ± 0.1 μsec. for this apparent mean life. The 1.7 μsec. pure exponential is drawn in Figs. 4 and 6 and is seen to give essentially as good a fit as the composite curve. It is clear from Fig. 6 that a pure exponential of 2.15 μsec. mean life would give a poor fit.

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We are indebted to Mr. J. D. Miller for his fine work in building and testing the electronic equipment used in this experiment.

[14] R. Peierls, Proc. Roy. Soc. **A149**, 467 (1935).