gravitational field, that is, a field whose metric is of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \quad (\mu, \nu = 1, \cdots, 4), \tag{3}$$

with $\partial g_{\mu\nu}/\partial x^4 \equiv 0$.

The energy momentum tensor is, as before, given by

$$\left.\begin{aligned} &T^{\mu\nu} = {}'T^{\mu\nu} + {}''T^{\mu\nu}, \\ &{}'T^{\mu\nu} = -g^{\mu\nu}p_0, \\ \text{where} \\ &{}''T^{\mu\nu}\begin{cases} = (\rho_{00}+p_0)(dx^4/ds)^2 = (\rho_{00}+p_0)/g_{44}, & (\mu=\nu=4) \\ = 0 \text{ (all other components)}, \end{cases} \end{aligned}\right\} \tag{4}$$

since $dx^i/ds = 0$; note that now $g^{44} \neq 1/g_{44}$. The equation expressing the vanishing of the covariant derivative of $T_\mu{}^\nu$ may be written

$$T^\nu{}_{i;\nu} = {}'T^\nu{}_{i;\nu} + {}''T^\nu{}_{i;\nu} = -\frac{\partial p_0}{\partial x^1} + \tfrac{1}{2}{}''T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i} - \frac{1}{(-g)^{\frac{1}{2}}}\frac{\partial[(-g''T_i{}^\nu)^{\frac{1}{2}}]}{\partial x^\nu} = 0. \tag{5}$$

It follows immediately from (4) that the last term of (5) vanishes, while the second term gives $[(\rho_{00}+p_0)/2g_{44}][\partial g_{44}/\partial x^i]$, so that (5) reduces to

$$\frac{\partial p_0}{\partial x^i} + \tfrac{1}{2}(\rho_{00}+p_0)\frac{\partial \log g_{44}}{\partial x^i} = 0. \tag{6}$$

But (6) is identical with Eq. (26) in *TE*; so that the relation (2) follows as before.

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Apparent Straggling of $_{16}S^{35}$ Beta-Particles in Glass*

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When a beam of particles having kinetic energies in unit range of E, enters a solid normally to its surface, the particles suffer scattering and collisional losses of energy at fluctuating rates while traversing the solid; consequently the projected (on the normal direction) ranges of the particles, r, are more or less dispersed about some mean value R in accord with a suitable statistical frequency distribution. For alpha-particles in air, for example, scattering does not appreciably affect the distribution function and it has the Gaussian form[1]

$$W = \exp-[(r-R)/\rho]^2 \Big/ \int_0^\infty \exp-[(r-R)/\rho]^2, \tag{1}$$

where ρ is the coefficient of straggling. The object of the present investigation is to find out whether Eq. (1) can apply to the passage of beta-particles through a thin slab of glass; in this case scattering is large, so that ρ might be expected to be far greater than for alpha-particles. Equation (1) leads to a coefficient of transmission given by

$$\tau(E) = \int_{(t-R)/\rho}^\infty e^{-x^2}dx \Big/ \int_{-R/\rho}^\infty e^{-x^2}dx, \tag{2}$$

for a slab of thickness t; and for ρ comparable to or greater than R,

$$\rho \cong (t-kE^2)/\text{erfi}^{-1}(1-2\tau). \tag{3}$$

In getting Eq. (3), R was replaced by kE^2 according to the energy-range relation; k is a constant.

To obtain data for use in Eq. (3), the beta-particle spectrum from the decay of $_{16}S^{35}$ was measured, using a glass Geiger-Müller counter with a nominal window thickness of 0.001 cm and superficial mass of 2.5 mg/cm². This radioactive material was the separated, reactor-made isotope, and was obtained from the Isotope Division of the AEC's Oak Ridge Operations, at Oak Ridge, Tennessee. Spectra from several active sources, prepared[2] under the guidance of Dr. Arthur Roe, were measured with an electrostatic analyzer designed (after Backus)[3] by Mr. S. J. Bame. Column 2 of Table I shows the energy distribution in this spectrum as observed through the 0.001 cm of glass; because of the weakness of sources used, these entries may be uncertain by several percent. Column 3 presents the true spectrum as computed from the theory of Fermi[4] according to the experimental studies of Cook, Langer, and Price,[5] and of Albert and Wu.[6] Column 4 lists the effective transmission at the respective energies.

Table I. Data on straggling of beta-particles.

E (kev)	$O(E)$ (arbitrary units)	$I(E)$	$\tau(E)$	ρ (cm)
35	12	132	0.09	5.1×10^{-4}
40	25	133	0.19	5.2
42.5	31	133	0.24	4.6
45	43	132	0.33	4.5
47.5	66	132	0.50	—
50	75	130	0.57	4.8
52.5	93	128	0.73	4.0
55	114	126	0.91	3.1
57.5	123	123	1.00	4.1
60	120	121	0.99	3.2
62.5	113	118	0.97	5.0×10^{-4}

From a curve of τ *vs.* E, E was found to be 48.5 kev for $\tau = 50$ percent; hence from the energy-range relation, 0.001 cm $= k$(48.5 kev),[2] k was found to be 4.255×10^{-7} cm/kev.[2] This enabled the computation of ρ from the data, by Eq. (3); the results are shown in column 5 of Table I. The test of the applicability of Eq. (1) in this case, is in the values of ρ. Although there is evidence of a trend in the ρ-values, it does not seem significant in view of the appreciable uncertainties in $O(E)$; therefore this study indicates that $\rho = 4.4\times10^{-4}$ cm, with an A.D. of 0.6×10^{-4} cm. The sensible constancy of ρ-values as here obtained, is rather surprising when the complexity of the process of apparent straggling is considered; but it does indicate that the simple Eq. (1) can describe the phenomenon, at least for energies barely appropriate to penetration.

* The experimental work here reported was carried out in the research laboratories of The University of North Carolina in Chapel Hill, North Carolina, during the academic years 1947–1948 and 1948–1949. The authors are particularly indebted to Messrs. N. Di Costanzo and R. Morris for their excellent shop-work in construction of the electrostatic analyzer.

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[2] The preparation of active sources was carried out in the new laboratory for radio-biological research at The University of North Carolina. The Division of Biology and Medicine of the AEC provided, through the ONR, a portion of the funds for this laboratory.

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Energy Release in Beryllium and Lithium Reactions with Protons

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Measurements have been made on the energy released in the reactions:

$$Be^9 + H^1 \rightarrow Li^6 + He^4 + Q_1, \tag{1}$$

$$Be^9 + H^1 \rightarrow Be^8 + D^2 + Q_2, \tag{2}$$

$$Be^8 \rightarrow 2He^4 + Q_2', \tag{2'}$$

$$Li^6 + H^1 \rightarrow He^3 + He^4 + Q_3. \tag{3}$$

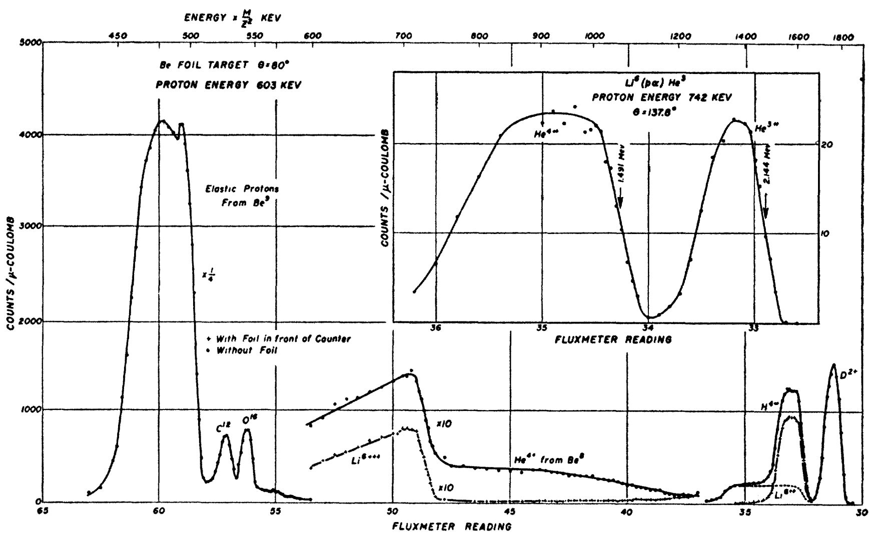

FIG. 1. Momentum analysis of the ions produced in the bombardment of Be^9 by protons. Inset: momentum analysis of the products of the reaction $Li^6(p\alpha)He^3$.

Protons monoenergetic to better than 0.03 percent were obtained from an electrostatic accelerator and analyzer previously described.[1] The analyzer was calibrated by measurements on the strong gamma-ray resonance in $F^{19}(p\alpha',\gamma)O^{16}$ which has been carefully standardized at 873.5 kev.[2] The targets used in studying reactions (1) and (2) were beryllium foils which were 10–20 kev thick for fast protons. The target for reaction (3) was produced by evaporating a thin lithium layer on copper in vacuum.

The energy of the ions produced in the reactions was accurately determined by a double-focusing magnetic spectrograph.[3] Reactions (1) and (2) were studied at two angles of observation in laboratory coordinates, 80.0° and 137.8°, while reaction (3) was studied only at 137.8°. The angles were measured directly and the angular aperture in each case was 6.6° in the plane in which the particles were deflected. The spectrograph field was determined by a null reading fluxmeter[4] and could be held constant to 0.05 percent and reproduced to ±0.15 percent. The fluxmeter was calibrated by observing protons of known energy scattered elastically from copper, beryllium, carbon, and oxygen. The energy of the scattered particles varies with angle for the light elements, and thus the consistency of the measurements was a verification of the direct measurement of the angle.

The scintillation counter used to detect the particles was arranged so that thin aluminum foils of known stopping power could be introduced directly in front of the ZnS screen. This made it possible to separate ions of different charge and energy that appeared at the same value of the magnetic field. For particles of energy greater than 100 kev, a plateau is exhibited in the integral spectrum of pulses from the scintillation counter. It was possible to set the discriminator bias high enough to eliminate all the dark current pulses, and still be operating on the plateau. It was verified with an ionization chamber that the plateau represented a counting efficiency of close to 100 percent.

Figure 1 shows data obtained by bombarding a 15-kev thick beryllium foil with 603-kev protons. The angle of observation was 80.0°. The peak at a fluxmeter reading of 60 mv* is elastically scattered protons from the beryllium. At readings of 57 mv and 58 mv the peaks due to elastically scattered protons from O^{16} and C^{12}, respectively, appear. Very thin carbon and oxygen layers were found on both the beryllium and lithium targets. Using the heights of the peaks it is possible to estimate the thickness of the layers from the cross sections for Rutherford scattering. The energies of all particles involved in any reaction were corrected for energy loss in these layers. The total carbon and oxygen layer in the curve shown is 0.44 kev thick normal to the target for 603-kve protons.

The alpha-particles from reaction (1) appear at 33 mv. They are superimposed on the doubly ionized Li^6 ions from the same reaction.** These two groups were separated by means of foils inserted in front of the ZnS screen. The triply ionized Li^6 ions from this reaction are observed at about 50 mv. The singly ionized He^4 and Li^6 ions could not be observed in the spectrograph with the maximum available field. The three groups of ions that were measured give three independent determinations of Q_1.

The deuterons from reaction (2) appear at 32 mv and Q_2 is calculated, using the energy of these particles. The Be^8 residual nucleus formed in reaction (2) recoils with a momentum equal and opposite to that of the deuteron in the center-of-mass system. Because of its short half-life, it disintegrates before it can travel far enough to make collisions and hence slow down. Since it disintegrates while in motion, the resultant two alpha-particles generate a continuous energy distribution. This is shown extending from 37 mv to 48 mv. The doubly ionized Li^6 peak gave some interference and foils were again used to separate the particles. The spectrum $N(E_\alpha)$ plotted against E_α is shown in Fig. 2. The

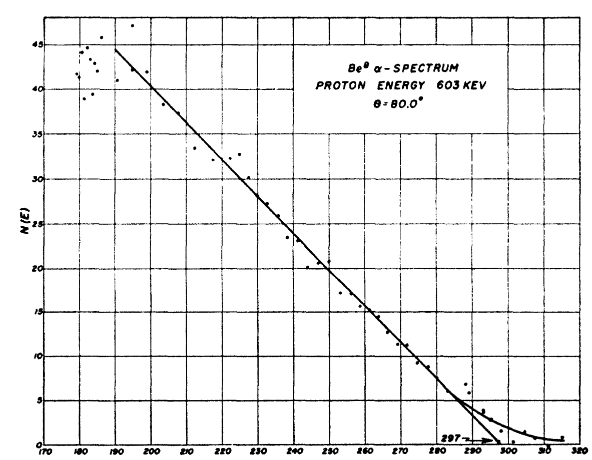

FIG. 2. Distribution in energy of the alpha-particles resulting from the disintegration of Be^8 produced in the reaction $Be^9(pd)Be^8$.

linearity of the curve within 150 kev of the end point is in accordance with theoretical expectations. The "fillet" near the end point can be accounted for by the angular window of the spectrograph. The extrapolated end point of this spectrum was used along with Q_2 to calculate Q_2'.

The curves obtained from reaction (3) are shown in the insert in Fig. 1. Both the doubly ionized He^4 and He^3 ions were detected and both groups were used in the energy determination. The reaction was studied at 742- and 1237-kev bombarding energy.

In the calculations, the midpoint of the front edge of the peaks was taken as corresponding to particles leaving the front surfaces of the target at the mean entrance angle of the spectrograph. Small corrections for relativity and surface layers have been made in each case. The errors were calculated by compounding the estimated systematic errors with the observed statistical errors which were very small. The probable systematic errors assumed were 0.3° in the angle of observation, 0.3 percent in observed energy, and 0.2 percent in the bombarding energy.

Q_1 is found to be 2.121 ± 0.012 Mev, Q_2 is 0.558 ± 0.003 Mev, while Q_3 is 4.017 ± 0.022. These are both somewhat higher than the measurements of Rosario,[5] and Perlow.[6] Q'_2 is 89 ± 5 kev, which is to be compared with Hemmendinger's value of 102 ± 10 after his value is corrected for the gamma-ray recoil.[7] A combination of the above Q values with those of $D(dp)H^3$ and $D(dn)He^3$, which have Q-values of 4.036 ± 0.022 Mev and 3.265 ± 0.018 Mev, respectively,[8] and the binding energy of the deuteron, 2.237 ± 0.007 Mev[9] makes it possible to calculate the energy release in the reactions $Li^6(nH^3)He^4$ and $Be^9(\gamma n)Be^8$. We find for the first reaction $Q=4.788\pm0.023$ Mev and for the second, $Q=-1.679\pm0.008$ Mev. The ratio of the Q-value of $H^2(\gamma n)H^1$ to that of $Be^9(\gamma n)Be^8$ is 1.332 ± 0.010, which is in agreement with the measurement of Waldman and Miller[10] who find 1.338 ± 0.004. The recently determined ranges of the particles in the Li^6 reaction[11] give the following two points on the range energy relations: 0.912-Mev protons have a range of 2.00 ± 0.02 cm and 2.058-Mev alpha-particles have a range 1.04 ± 0.02 cm in air at N.T.P.

We should like to thank Mr. Robert G. Thomas for his help in obtaining the data on beryllium. This work was assisted by the joint program of the ONR and the AEC.

* Fluxmeter readings are proportional to the (charge)/(momentum) of the observed particles.

** The particles and residual nuclei produced in a reaction at a given angle have approximately the same momentum. Hence, the ions of the two products with the same charge overlap in a momentum analysis made by a magnetic field. This is quite different than in an analysis by range measurements.

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A Revaluation of the Gamma-Radiations from Co^{60} and Zn^{65}

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In a recent paper[1] we described briefly the construction of an iron-free magnetic lens spectrometer and discussed the character of the corrections to be applied to gamma-ray measurements made with an instrument of that type. Energy values for the gamma-radiations from Co^{60} and Zn^{65}, obtained by measurement of photoelectric conversion lines, were reported. Calibration of the spectrometer was effected through the use of photoelectrons produced by annihilation radiation and of conversion electrons from ThB (*F*-line).

It has since become possible for us to obtain sources which would permit comparative measurements of the internal conversion line corresponding to the 2.62 Mev gamma-ray of ThC″ and the *F*-line from ThB. Such a comparison was considered desirable, since it would afford a definitive check of the presumed proportionality between the focused momenta and the currents in the spectrometer. Upon learning from Dr. T. Lauritsen that spectrometer work[2] at the California Institute of Technology indicated energies for the Co^{60} gamma-radiations approximately 15 kev higher than those cited by us,[1] we undertook to perform this check of our instrument *in conjunction with* a remeasurement of the photoelectric conversion lines from Co^{60}. This work has confirmed the linearity of the instrument but has led to a revised value of the spectrometer constant which is almost one percent greater than that found in our earlier work.

Based on momentum values of 1385 and 10,000 gauss-cm for the internal conversion electrons from ThB and ThC″, respectively,[3] and with the corrections for the photoelectric radiator made in the manner previously described,[1] our recent measurements lead to energies of 1.16_9 and 1.33_1 Mev for the gamma-radiations from Co^{60}. It is believed that these new values are more reliable than those given by us previously, as the stability of the spectrometer has been improved since the earlier data were obtained. These revised values are consistent with those determined by Hornyak *et al.*,[2] and with the precision wave-length measurements which have now been reported by Lind, Brown, and DuMond.[4] The values we have now obtained, when used in conjunction with previously reported[5] data obtained from a *composite* source in which *both* Co^{60} and Zn^{65} activities were present, leads to a revised value of 1.11_8 Mev for the Zn^{65} gamma-ray.

It should be mentioned that our corrections for the energy loss experienced by the electrons in the photoelectric converter are based[1] on the *average* energy loss by collision and, in the energy range with which we are concerned here, disregard complicating scattering effects. It is understood that the results of a more elaborate analysis have been used in the work of Hornyak *et al.*,[2] and lead to corrections appreciably smaller than those which we have been led to apply.

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Ferro-electric Properties of WO_3

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We have recently grown single crystals of WO_3 and have found that these crystals appear to show ferro-electric behavior. The crystal structure of WO_3 may be looked at as being similar to a perovskite structure with the omission of the cations at the cube corners of the unit cell. That is, we may view WO_3 as derived from $BaTiO_3$ by the complete omission of the Ba-ions and the substitution of W for Ti. The ionic radius[1] of W^{+6} is 0.62 A and we therefore see that the WO_6 octahedra obey the empirical rule for the occurrence of ferro-electricity in perovskite-type structures