Letters to the Editor

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Anomalous Magnetic Moments of Nucleons

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May 31, 1949

IN this note we report an extension of previous work[1] to the calculation of the anomalous magnetic moments of nucleons. The interaction between quantized Dirac nucleon fields, quantized scalar and pseudoscalar meson fields, and an applied magnetic field is investigated by third-order perturbation theory in a manner similar to that recorded in reference 1. With the same notation, we have for the term in the Hamiltonian density which describes the first-order interaction of the meson-nucleon fields with an applied vector potential, **A**,

$$^{e}\mathcal{H}=ie\mathbf{A}\cdot(\psi^*\mathbf{grad}\psi-\psi\mathbf{grad}\psi^*)-e\Psi_P{}^*\boldsymbol{\alpha}\cdot\mathbf{A}\Psi_P.$$

The meson-nucleon coupling terms appear in Eqs. (6) and (7) of reference 1 for a scalar meson field. The β-matrix there is replaced by $i\beta\gamma_5$ for a pseudoscalar field with pseudoscalar coupling.

We summarize our results as follows:

$$\mu_P-1=(g^2/4\pi)(\tfrac{1}{2}\pi)\mu_{\mathrm{I}};\ \mu_N=(g^2/4\pi)(\tfrac{1}{2}\pi)(-\mu_{\mathrm{I}}+\mu_{\mathrm{II}});\qquad \text{(charged meson theory)}$$

$$\mu_P-1=(g'^2/4\pi)(\tfrac{1}{2}\pi)(2\mu_{\mathrm{I}}+\mu_{\mathrm{II}});\ \mu_N=(g'^2/4\pi)(1/\pi)(-\mu_{\mathrm{I}}+\mu_{\mathrm{II}});\qquad \text{(symmetric meson theory)}$$

$$\mu_P-1=(g''^2/4\pi)(\tfrac{1}{2}\pi)\mu_{\mathrm{II}};\ \mu_N=0;\ \text{neutral meson theory)}$$

where

$$\mu_{\mathrm{I}}=\left[(\tfrac{3}{2})-a^2-(1-3a^2+a^4)\ln 1/a-(5-5a^2+a^4)\frac{a\cos^{-1}a/2}{(4-a^2)^{\frac{1}{2}}}\right]\pm\left[1-(1-a^2)\ln 1/a-(3-a^2)\frac{a\cos^{-1}a/2}{(4-a^2)^{\frac{1}{2}}}\right]$$

represents that part of the moment contributed by a virtual intermediate meson cloud of positive electric charge, and

$$\mu_{\mathrm{II}}=\left[\tfrac{1}{2}-a^2+(2a^2-a^4)\ln 1/a-(2-4a^2+a^4)\frac{a\cos^{-1}a/2}{(4-a^2)^{\frac{1}{2}}}\right]\pm\left[1+a^2\ln 1/a-(2-a^2)\frac{a\cos^{-1}a/2}{(4-a^2)^{\frac{1}{2}}}\right]$$

is the contribution of the virtual intermediate charged nucleon states. The plus sign is for scalar mesons, the minus sign for pseudoscalar ones, and a is the ratio of meson mass to nucleon mass.

With proper adjustments in notation these results agree with those obtained by different means by Luttinger,[2] Slotnick[3] and Case[4] for pseudoscalar theories, and by Case for a scalar meson field. Numerical results appear below for mesons of mass 300 m_e.

	Scalar	*Pseudoscalar*
Charged meson theory	$\mu_N=+.49(g^2/4\pi)$ $\mu_P-1=-.30(g^2/4\pi)$	$\mu_N=-.13(g^2/4\pi)$ $\mu_P-1=+.054(g^2/4\pi)$
Symmetric meson theory	$\mu_N=+.98(g'^2/4\pi)$ $\mu_P-1=-.42(g'^2/4\pi)$	$\mu_N=-.26(g'^2/4\pi)$ $\mu_P-1=+.034(g'^2/4\pi)$.

Moments obtained from the scalar calculation are of wrong sign and too small in magnitude[1] for $g^2/4\pi=0.30$ and $g'^2/4\pi=0.30$. The pseudoscalar theory yields moments of appropriate sign. If one chooses $g^2/4\pi\approx 36$, on the basis of questionable nuclear force arguments,[5] the anomalies are of proper magnitude for the protons, but too large in the neutron case because of exaggerated contributions from the charged intermediate nucleon states. Investigation of fourth-order interactions between nucleons which are important in the pseudoscalar theory has led Bethe[6] to suggest $g^2/4\pi\approx 4$. This has the effect that magnetic moment anomalies resulting from a pseudoscalar theory are also too small. It would appear to be necessary to investigate higher order effects in order to ascertain if contributions from a fifth-order calculation are large compared with these third-order results, as are the fourth-order effects in Bethe's calculation, relative to the second-order ones.

As recently discussed by French and Weisskopf[7] in the analogous calculations for electrons and a quantum field, one encounters a set of terms which may be written as $c[\boldsymbol{\alpha}\cdot\mathbf{A}]_{\mathrm{av}}$. In our calculations contributions to c come from interactions of **A** with both charged mesons and nucleons. By a careful integration of the individually diverging terms which contribute to it, we have also found that this term vanishes, as it must for covariance.

Of the four possible charge and mass renormalizations for protons and neutrons, only the proton mass renormalization must be taken into account in the calculation of the moments. Schemes according to which a proton pair is created, and then the same pair annihilated, contribute nil. They are analogous to the vacuum polarization terms of quantum electrodynamics. The proton mass renormalization is handled similarly to Feynman's[8] treatment for the quantum field. Neutron mass renormalization is a higher order effect since a neutron core has no direct interaction with electromagnetic fields.

Details of this calculation are to be found in the University of Illinois Ph.D. thesis of S. D. Drell, 1949.

We wish to thank Professor S. M. Dancoff for his continued guidance and interest.

[1] S. M. Dancoff and S. D. Drell, Phys. Rev. **76**, 205 (1949).
[2] J. M. Luttinger (Helv. Phys. Acta **21**, 483 (1948)) has performed a second-order perturbation calculation, treating nucleons and mesons in a uniform magnetic field as his zero order stationary states.
[3] M. Slotnick, Phys. Rev. **75**, 1295 (1949); Slotnick and Heitler, Phys. Rev. **75**, 1645 (1949) has calculated the spin-orbit coupling in an electrostatic field.
[4] K. M. Case (Phys. Rev. **74**, 1884 (1948); **76**, 1 (1949)) has performed this calculation in the framework of the Schwinger-Tomonaga-Dyson formalism.
[5] F. Villars, Helv. Phys. Acta **20**, 476 (1947).
[6] H. A. Bethe, Phys. Rev. **76**, 191 (1949).
[7] J. B. French and V. F. Weisskopf, Phys. Rev. **75**, 1240 (1949).
[8] R. P. Feynman, Phys. Rev. **74**, 1430 (1948).

Temperature Equilibrium in a Stationary Gravitational Field

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June 13, 1949

IT has been shown by Tolman and Ehrenfest[1] that the distribution of proper temperature T_0 of matter in thermodynamic equilibrium situated in a static gravitational field whose metric is

$$ds^2=g_{ik}dx^idx^k+g_{44}(dx^4)^2,\quad (i,k=1,2,3),\tag{1}$$

is described by the relation

$$T_0(g_{44})^{\frac{1}{2}}=\text{const.}\tag{2}$$

In (1) x^4 is the 'time,' and $\partial g_{ik}/\partial x^4=\partial g_{44}/\partial x^4\equiv 0$. By the method of *TE* it is very easily shown that (2) continues to hold in a *stationary* gravitational field, that is, a field whose metric is of the form

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu, \quad (\mu, \nu=1, \cdots, 4), \tag{3}$$

with $\partial g_{\mu\nu}/\partial x^4\equiv 0$.

The energy momentum tensor is, as before, given by

$$\left.\begin{aligned} &T^{\mu\nu}={}'T^{\mu\nu}+{}''T^{\mu\nu}, \\ &{}'T^{\mu\nu}=-g^{\mu\nu}p_0, \\ \text{where}\quad &{}''T^{\mu\nu}\begin{cases}=(\rho_{00}+p_0)(dx^4/ds)^2=(\rho_{00}+p_0)/g_{44}, & (\mu=\nu=4)\\ =0 \text{ (all other components)},\end{cases}\end{aligned}\right\} \tag{4}$$

since $dx^i/ds=0$; note that now $g^{44}\neq 1/g_{44}$. The equation expressing the vanishing of the covariant derivative of $T_\mu{}^\nu$ may be written

$$T^\nu{}_{i;\nu}={}'T^\nu{}_{i;\nu}+{}''T^\nu{}_{i;\nu}=-\frac{\partial p_0}{\partial x^1} +\tfrac{1}{2}{}''T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i}-\frac{1}{(-g)^{\frac{1}{2}}}\frac{\partial[(-g''T_i{}^\nu)^{\frac{1}{2}}]}{\partial x^\nu}=0. \tag{5}$$

It follows immediately from (4) that the last term of (5) vanishes, while the second term gives $[(\rho_{00}+p_0)/2g_{44}][\partial g_{44}/\partial x^i]$, so that (5) reduces to

$$\frac{\partial p_0}{\partial x^i}+\tfrac{1}{2}(\rho_{00}+p_0)\frac{\partial \log g_{44}}{\partial x^i}=0. \tag{6}$$

But (6) is identical with Eq. (26) in *TE*; so that the relation (2) follows as before.

[1] R. C. Tolman and P. Ehrenfest. Phys. Rev. **36**, 1791 (1930). This will be referred to as 'TE'.

Apparent Straggling of $_{16}S^{35}$ Beta-Particles in Glass*

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May 26, 1949

WHEN a beam of particles having kinetic energies in unit range of E, enters a solid normally to its surface, the particles suffer scattering and collisional losses of energy at fluctuating rates while traversing the solid; consequently the projected (on the normal direction) ranges of the particles, r, are more or less dispersed about some mean value R in accord with a suitable statistical frequency distribution. For alpha-particles in air, for example, scattering does not appreciably affect the distribution function and it has the Gaussian form[1]

$$W=\exp-[(r-R)/\rho]^2\bigg/\int_0^\infty \exp-[(r-R)/\rho]^2, \tag{1}$$

where ρ is the coefficient of straggling. The object of the present investigation is to find out whether Eq. (1) can apply to the passage of beta-particles through a thin slab of glass; in this case scattering is large, so that ρ might be expected to be far greater than for alpha-particles. Equation (1) leads to a coefficient of transmission given by

$$\tau(E)=\int_{(t-R)/\rho}^\infty e^{-x^2}dx\bigg/\int_{-R/\rho}^\infty e^{-x^2}dx, \tag{2}$$

for a slab of thickness t; and for ρ comparable to or greater than R,

$$\rho\cong(t-kE^2)/\text{erfi}^{-1}(1-2\tau). \tag{3}$$

In getting Eq. (3), R was replaced by kE^2 according to the energy-range relation; k is a constant.

To obtain data for use in Eq. (3), the beta-particle spectrum from the decay of $_{16}S^{35}$ was measured, using a glass Geiger-Müller counter with a nominal window thickness of 0.001 cm and superficial mass of 2.5 mg/cm². This radioactive material was the separated, reactor-made isotope, and was obtained from the Isotope Division of the AEC's Oak Ridge Operations, at Oak Ridge, Tennessee. Spectra from several active sources, prepared[2] under the guidance of Dr. Arthur Roe, were measured with an electrostatic analyzer designed (after Backus)[3] by Mr. S. J. Bame. Column 2 of Table I shows the energy distribution in this spectrum as observed through the 0.001 cm of glass; because of the weakness of sources used, these entries may be uncertain by several percent. Column 3 presents the true spectrum as computed from the theory of Fermi[4] according to the experimental studies of Cook, Langer, and Price,[5] and of Albert and Wu.[6] Column 4 lists the effective transmission at the respective energies.

TABLE I. Data on straggling of beta-particles.

E (kev)	$O(E)$ (arbitrary units)	$I(E)$	$\tau(E)$	ρ (cm)
35	12	132	0.09	5.1×10^{-4}
40	25	133	0.19	5.2
42.5	31	133	0.24	4.6
45	43	132	0.33	4.5
47.5	66	132	0.50	—
50	75	130	0.57	4.8
52.5	93	128	0.73	4.0
55	114	126	0.91	3.1
57.5	123	123	1.00	4.1
60	120	121	0.99	3.2
62.5	113	118	0.97	5.0×10^{-4}

From a curve of τ *vs.* E, E was found to be 48.5 kev for $\tau=50$ percent; hence from the energy-range relation, 0.001 cm$=k$(48.5 kev),[2] k was found to be 4.255×10^{-7} cm/kev.[2] This enabled the computation of ρ from the data, by Eq. (3); the results are shown in column 5 of Table I. The test of the applicability of Eq. (1) in this case, is in the values of ρ. Although there is evidence of a trend in the ρ-values, it does not seem significant in view of the appreciable uncertainties in $O(E)$; therefore this study indicates that $\rho=4.4\times10^{-4}$ cm, with an A.D. of 0.6×10^{-4} cm. The sensible constancy of ρ-values as here obtained, is rather surprising when the complexity of the process of apparent straggling is considered; but it does indicate that the simple Eq. (1) can describe the phenomenon, at least for energies barely appropriate to penetration.

* The experimental work here reported was carried out in the research laboratories of The University of North Carolina in Chapel Hill, North Carolina, during the academic years 1947–1948 and 1948–1949. The authors are particularly indebted to Messrs. N. Di Costanzo and R. Morris for their excellent shop-work in construction of the electrostatic analyzer.

[1] E. Rutherford, J. Chadwick, and C. D. Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1930), pp. 112 *et seq.*

[2] The preparation of active sources was carried out in the new laboratory for radio-biological research at The University of North Carolina. The Division of Biology and Medicine of the AEC provided, through the ONR, a portion of the funds for this laboratory.

[3] J. Backus, Phys. Rev. **68**, 59 (1945).

[4] E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943).

[5] Cook, Langer, and Price, Jr., Phys. Rev. **73**, 1395 (1948).

[6] R. D. Albert and C. S. Wu, Phys. Rev. **74**, 847 (1948).

Energy Release in Beryllium and Lithium Reactions with Protons

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June 9, 1949

MEASUREMENTS have been made on the energy released in the reactions:

$$Be^9+H^1\rightarrow Li^6+He^4+Q_1, \tag{1}$$

$$Be^9+H^1\rightarrow Be^8+D^2+Q_2, \tag{2}$$

$$Be^8\rightarrow 2He^4+Q_2', \tag{2'}$$

$$Li^6+H^1\rightarrow He^3+He^4+Q_3. \tag{3}$$