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Single Production of Mesons by Gamma-Rays Near Threshold

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The cross section for single charged meson production through absorption of a gamma-ray in a gamma-ray-nucleon collision has been calculated to order e^2g^2 in the charged scalar meson theory (scalar coupling) and charged pseudoscalar meson theory (pseudovector coupling). The nucleon was assumed to be infinitely heavy, thus restricting the validity of the results to gamma-ray energies less than a few times threshold. For a free nucleon the threshold for π-meson production is at 155 Mev in the rest system of the nucleon. Close to threshold, in the scalar theory the cross section varies as the three-halves power of the energy above threshold and the angular distribution is of the form $\sin^2\theta$, while in the pseudoscalar theory the cross section varies as the square root of the energy above threshold and the angular distribution of the mesons is approximately isotropic. At energies 50 percent above threshold, the total cross section is of order 10^{-28} cm² in the pseudoscalar theory and of order 10^{-29} cm² in the scalar theory.

THE recent completion of the 300-Mev synchrotron at the University of California has now made it possible to produce π-mesons by gamma-rays in the laboratory and thus measure the cross section for single π-meson production by a gamma-ray and the variation of the cross section with energy above threshold. It may therefore be of interest to report briefly on theoretical expectations for these quantities.[1]

The process of greatest interest here is that in which a gamma-ray of sufficient energy collides with a proton (neutron), the gamma-ray disappearing with the production of a single positive (negative) meson, the nucleon becoming a neutron (proton). For a free nucleon the threshold for this process in the system in which the nucleon is at rest is given by

$$E_\gamma=[1+(\mu/2M)]\mu c^2,$$

where μ is the mass of the meson and M is the mass of the nucleon. Using the Berkeley value[2] for the meson mass, $\mu=286m$ where m is the electron mass, the threshold is found to be 155 Mev so that energies nearly twice threshold are available with the 300-Mev synchrotron. We may note that the threshold for meson pair production in the Coulomb field of a free proton lies at 331 Mev, but in the Coulomb field of a heavy nucleus this threshold may be reduced to about 286 Mev. In the range of energies available at present, the production of meson pairs is negligible compared to the production of single mesons, except possibly in target materials of high atomic number.

In the following we calculate the cross section for single meson production by gamma-rays in the charged scalar meson theory (scalar coupling) and the charged pseudoscalar meson theory (pseudovector coupling). Quantum perturbation theory is employed retaining only terms of the lowest non-vanishing order (weak coupling approximation), and the nucleon is treated as infinitely heavy, thus restricting the validity of the results to gamma-ray energies below a few times the threshold energy. With the vector potential of the electromagnetic field and the meson potential expanded in plane waves in a volume V, the perturbation terms in the Hamiltonian are

$$H'=H_e+H_g+H_{eg}$$

with

$$H_e=e\left(\frac{4\pi}{V}\right)^{\frac{1}{2}}\sum_{\mathbf{k},\alpha}\sum_{\mathbf{K}}\sum_{\mathbf{K}'}\frac{[\lambda_{\mathbf{k}}^{\alpha}\cdot(\mathbf{K}+\mathbf{K}')]}{(8k\omega_{\mathbf{K}}\omega_{\mathbf{K}'})^{\frac{1}{2}}}\times(A_{\mathbf{k}}^{\alpha}+A_{-\mathbf{k}}^{\alpha *})(a_{\mathbf{K}'}^{*}+b_{-\mathbf{K}'})\times(a_{\mathbf{K}}+b_{-\mathbf{K}}^{*})\delta(\mathbf{K}'-\mathbf{K}-\mathbf{k}).$$

In the scalar meson theory,

$$H_g=g\left(\frac{4\pi}{V}\right)^{\frac{1}{2}}\sum_{\mathbf{K}}\frac{1}{(2\omega_{\mathbf{K}})^{\frac{1}{2}}}\left[\left(\frac{\tau_x+i\tau_y}{2}\right)(a_{\mathbf{K}}+b_{-\mathbf{K}}^{*})e^{i\mathbf{K}\cdot\mathbf{r}}+\left(\frac{\tau_x-i\tau_y}{2}\right)(a_{\mathbf{K}}^{*}+b_{-\mathbf{K}})e^{-i\mathbf{K}\cdot\mathbf{r}}\right],$$

$$H_{eg}=0,$$

while in the pseudoscalar theory,

$$H_g=i\frac{g}{\mu}\left(\frac{4\pi}{V}\right)^{\frac{1}{2}}\sum_{\mathbf{K}}\frac{(\boldsymbol{\sigma}\cdot\mathbf{K})}{(2\omega_{\mathbf{K}})^{\frac{1}{2}}}\times\left[\left(\frac{\tau_x+i\tau_y}{2}\right)(a_{\mathbf{K}}+b_{-\mathbf{K}}^{*})e^{i\mathbf{K}\cdot\mathbf{r}}-\left(\frac{\tau_x-i\tau_y}{2}\right)(a_{\mathbf{K}}^{*}+b_{-\mathbf{K}})e^{-i\mathbf{K}\cdot\mathbf{r}}\right],$$

[1] Single meson production has been treated in the following papers: W. Heitler, Proc. Roy. Soc. **166**, 529 (1938); M. Kobayashi and T. Okayama, Proc. Phys. Math. Soc. Japan **21**, 1 (1939); H. S. W. Massey and H. C. Corben, Proc. Camb. Soc. **35**, 84 (1939) and **35**, 463 (1939); L. Nordheim and G. Nordheim, Phys. Rev. **54**, 254 (1938).

[2] A. S. Bishop, Phys. Rev. **75**, 1468 (1949).

$$H_{eg}=i\frac{eg}{\mu}\left(\frac{4\pi}{V}\right)\sum_{\mathbf{k},\alpha}\sum_{\mathbf{K}}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_{\mathbf{k}}{}^{\alpha})}{(4k\omega_{\mathbf{K}})^{\frac{1}{2}}}$$
$$\times\left[\left(\frac{\tau_x+i\tau_y}{2}\right)(A_{\mathbf{k}}{}^{\alpha}+A_{-\mathbf{k}}{}^{\alpha *})(a_{\mathbf{K}}+b_{-\mathbf{K}}{}^{*})e^{i(\mathbf{K}+\mathbf{k})\cdot\mathbf{r}}\right.$$
$$\left.-\left(\frac{\tau_x-i\tau_y}{2}\right)(A_{\mathbf{k}}{}^{\alpha *}+A_{-\mathbf{k}}{}^{\alpha})(a_{\mathbf{K}}{}^{*}+b_{-\mathbf{K}})e^{-i(\mathbf{K}+\mathbf{k})\cdot\mathbf{r}}\right].$$

In the above, g is the coupling constant of the meson field to the nucleon, $a_{\mathbf{K}}{}^{*}$, $a_{\mathbf{K}}$, $b_{\mathbf{K}}{}^{*}$, $b_{\mathbf{K}}$ are creation and destruction operators for positive and negative mesons of momentum $\mathbf{K}$, $A_{\mathbf{k}}{}^{\alpha *}$, $A_{\mathbf{k}}{}^{\alpha}$ are creation and destruction operators for photons of momentum $\mathbf{k}$, $\boldsymbol{\lambda}_{\mathbf{k}}{}^{\alpha}$ ($\alpha=1, 2$) are unit polarization vectors of the photons, $\mathbf{r}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$ are the position, spin, and isotopic spin operators for the nucleon, and $\omega_{\mathbf{K}}=(\mu^2+K^2)^{\frac{1}{2}}$ with μ the mass of the meson. Units have been used in which $\hbar$ and c are unity.

The term H_{eg} in the pseudoscalar case is necessitated by considerations of gauge invariance and is present in any charged meson theory with gradient coupling. In spite of the fact that it leads to the dominant contribution to the cross section, it has been overlooked in some past calculations.

From the above expressions one may readily calculate the transition matrix element from a state in which there is a photon of momentum $\mathbf{k}$ and a nucleon in the spin state u_0 to a final state in which there is a meson of momentum $\mathbf{K}$ and the nucleon is in the spin state u. In the scalar theory the transition can only take place through an intermediate state, while in the pseudoscalar theory, because of the term H_{eg} in the perturbation, a direct transition is also possible. The results for the transition matrix element in the scalar and pseudoscalar theories, respectively, are

$$H_s'=-\frac{4\pi egV^{-1}}{(k\omega_{\mathbf{K}})^{\frac{1}{2}}\omega_{\mathbf{K}-\mathbf{k}}{}^{2}}(u^*|1|u_0),$$

$$H_{ps}'=-\frac{4\pi iegV^{-1}}{2\mu(k\omega_{\mathbf{K}})^{\frac{1}{2}}}\left[(u^*|\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_{\mathbf{k}}{}^{\alpha}|u_0)\right.$$
$$\left.-\frac{2(\boldsymbol{\lambda}_{\mathbf{k}}{}^{\alpha}\cdot\mathbf{K})(u^*|\boldsymbol{\sigma}\cdot(\mathbf{K}-\mathbf{k})|u_0)}{\omega_{\mathbf{K}-\mathbf{k}}{}^{2}}\right].$$

Squaring, summing over final spin states and averaging over initial spin states for the nucleon, and averaging over the direction of polarization of the gamma-ray, one obtains

$$\sum\langle|H_s'|^2\rangle_{\mathrm{Av}}=\frac{8\pi^2e^2g^2V^{-2}K^2\sin^2\theta}{k\omega_{\mathbf{K}}\omega_{\mathbf{K}-\mathbf{k}}{}^{4}},$$

$$\sum\langle|H_{ps}'|^2\rangle_{\mathrm{Av}}=\frac{4\pi^2e^2g^2V^{-2}}{\mu^2k\omega_{\mathbf{K}}}\left[1-\frac{2\mu^2K^2\sin^2\theta}{\omega_{\mathbf{K}-\mathbf{k}}{}^{4}}\right],$$

with θ the angle between the direction of emission of the meson and the direction of the incident gamma-ray.

From conservation of energy, one has $K=(k^2-\mu^2)^{\frac{1}{2}}$. The differential cross section for the process is then given by

$$d\sigma=2\pi\sum\langle|H'|^2\rangle_{\mathrm{Av}}\frac{V^2K\omega_{\mathbf{K}}d\Omega_{\mathbf{K}}}{8\pi^3},$$

and substituting the above results, we obtain

$$d\sigma_s=\frac{2e^2g^2}{(\mu c^2)^2}\left(\frac{k^2-\mu^2}{\mu^2}\right)^{\frac{3}{2}}\left(\frac{\mu^5}{k\omega_{\mathbf{K}-\mathbf{k}}{}^{4}}\right)\sin^2\theta d\Omega_{\mathbf{K}},$$

$$d\sigma_{ps}=\frac{2e^2g^2}{(\mu c^2)^2}\left[\frac{1}{2}\left(\frac{k^2-\mu^2}{\mu^2}\right)^{\frac{1}{2}}\frac{\mu}{k}\right.$$
$$\left.-\left(\frac{k^2-\mu^2}{\mu^2}\right)^{\frac{3}{2}}\left(\frac{\mu^5}{k\omega_{\mathbf{K}-\mathbf{k}}{}^{4}}\right)\sin^2\theta\right]d\Omega_{\mathbf{K}}$$

Integrating over all directions of emission for the meson we obtain, for the total cross sections,

$$\sigma_s=\frac{2\pi e^2g^2}{(\mu c^2)^2}\left(\frac{k^2-\mu^2}{\mu^2}\right)^{\frac{3}{2}}\frac{\mu}{k}$$
$$\times\left\{\frac{\mu^4}{k(k^2-\mu^2)^{\frac{1}{2}}}\ln\frac{k+(k^2-\mu^2)^{\frac{1}{2}}}{k-(k^2-\mu^2)^{\frac{1}{2}}}-\frac{2\mu^4}{k^2(k^2-\mu^2)}\right\},$$

$$\sigma_{ps}=\frac{4\pi e^2g^2}{(\mu c^2)^2}\left(\frac{k^2-\mu^2}{\mu^2}\right)^{\frac{1}{2}}\frac{\mu}{k}-\sigma_s.$$

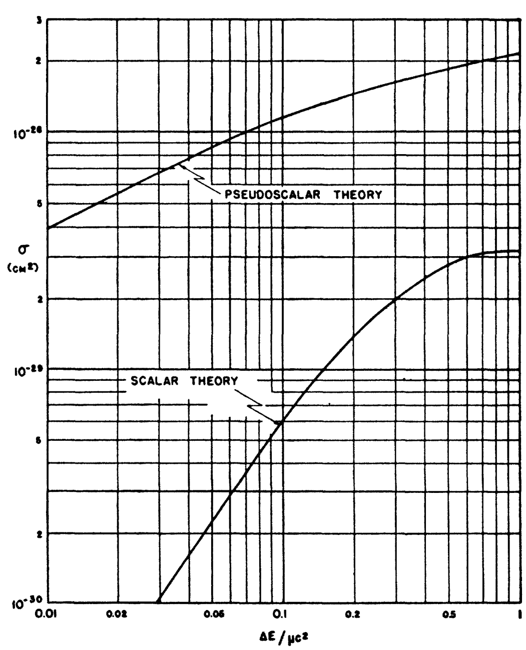

FIG. 1. Total cross section for single charged meson production by gamma-rays on nucleons as a function of the energy ΔE above threshold. Values assumed for the constants are: $g^2/\hbar c=\frac{1}{6}$, $\mu=286$ electron masses.

Here k now represents the energy of the gamma-ray divided by c^2.

We note that in the scalar theory the angular distribution of the mesons produced close to threshold has the form $\sin^2\theta$, and that the cross section increases as $(\Delta E)^{\frac{3}{2}}$ where $\Delta E=(k-\mu)c^2$ is the energy above threshold of the gamma-ray. In the pseudoscalar theory, on the other hand, the dominant term gives an isotropic angular distribution close to threshold and a cross section increasing as $(\Delta E)^{\frac{1}{2}}$. This latter behavior is a consequence of the term H_{eg} in the pseudoscalar Hamiltonian. It has been pointed out to the author[3] that if the anomalous magnetic moments of the proton and neutron are described by Pauli terms in the Hamiltonian, then this latter behavior will occur with any meson theory. In view of the great present difficulties of meson theory, we do not wish to imply much faith in the details of the present calculations beyond their usefulness for orientation purposes; however, if the anomalous moments of the neutron and proton are due to the interaction of these particles with the meson field, as is commonly assumed, then it is reasonable to expect that these two features of our result in the pseudoscalar theory, the isotropic angular distribution, and the $(\Delta E)^{\frac{1}{2}}$ variation of the cross section close to threshold, will be verified by experiment. In passing, we may also note the remarkable similarity between the results for the pseudoscalar theory and the results obtained for the cross section for photodisintegration of the deuteron.[4] In fact, our calculation may be considered as that of the photo-disintegration of the proton into a neutron and positive meson and of the neutron into proton and negative meson in a generalized sense.

The numerical magnitude of the cross sections may be in error both due to the inadequacies of the present theory and because of a lack of a reliable value for the meson coupling constant. The order of magnitude is probably reliable, however, and if we assume $g^2/\hbar c=\frac{1}{6}$, we obtain the values plotted in Fig. 1. Results for other meson theories may be expected to be comparable. It need hardly be pointed out that an accurate determination of the cross section at some known energy will yield a value for the coupling constant g which is likely to be more accurate than any estimated value from the meson theory of nuclear forces. In particular, the cross section of hydrogen would be of especial value.

From the above formulas one may readily calculate capture cross sections of nucleons for mesons with gamma-ray emission, but since this question is treated in detail in another paper[5] it will not be discussed here.

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Note added in proof: Since the submission of the present paper for publication, another paper[6] on this same subject has come to the author's attention. While it has not yet been possible for the author to see this paper, the results as quoted in another publication[7] appear to be in substantial agreement with the results published here.

Recently the production of π-mesons by 335 Mev gamma-rays has been announced by the group working with the synchrotron at Berkeley.[8] The target is essentially glass (nuclear plates) in one case and carbon in another and a cross section of approximately 3×10^{-30} cm^2 per nucleon was obtained though this result is admittedly very rough. Since only mesons with energies less than 20 Mev could be counted, the cross section may be considerably larger and the effect of binding of the nucleons into nuclei also makes any comparison with the cross section calculated in this paper unwarranted. It was also observed in this experiment that many more negative than positive mesons were produced (or at least detected in the emulsions). If this result is correct it would be extremely improbable that it could be reconciled with the theory we have presented which would predict at best only small differences in production probability between positive and negative mesons on the basis of the effect of the Coulomb field of the nucleons.

[3] Private communication from R. E. Marshak; see also reference 5.

[4] H. Bethe, *Elementary Nuclear Theory* (John Wiley & Sons, Inc., New York, 1947), p. 56.

[5] R. E. Marshak and A. S. Wightman, Phys. Rev. **76**, 114 (1949).

[6] J. Hamilton and H. W. Peng, Proc. Roy. Ir. Ac. **49A**, 197 (1944); see also C. Morette and H. W. Peng, Nature **160**, 59 (1947).

[7] W. Heitler, Rev. Mod. Phys. **21**, 113 (1949).

[8] E. M. McMillan and J. M. Peterson, Science **109**, 438 (1949).