Superconductors in Alternating Magnetic Fields

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A simple technique of determining the superconducting transitions of small and irregularly shaped samples of superconducting material is applied to the determination of the transitions and critical magnetic field curves of tin and vanadium. The zero-field transitions and critical field curve for tin agree well with previous determinations. The vanadium was found to have broad transitions in zero magnetic field and critical field slopes of over 4000 gauss per degree. The critical field curve of the vanadium was verified by measurements with a susceptibility balance.

INTRODUCTION

THE measurement of the properties of superconductors through a study of their effect on mutual inductors and self-inductors has been investigated by Shoenberg[1] and Daunt,[2] while Casimir[3] has applied the technique to the determination of the magnetic field penetration depth in superconductors.

Because of the convenience of this method for specimens of small or irregular shapes, it was felt desirable to continue the investigation further.

The apparatus used was very simple. The sample of metal was surrounded as closely as possible with a coil consisting of a few hundred turns of fine wire. This coil formed the secondary of a mutual inductor, and was located in the center of a primary solenoid which was wound on the outside of the helium flask. Using an electron tube oscillator as a source, an alternating current of frequency 250 to 2000 cycles per second was passed through the primary solenoid, and the magnitude of this current adjusted so that the peak magnetic field at the center of the primary solenoid was approximately 0.5 gauss. The signal picked up by the secondary coil was amplified, rectified, and recorded with a Brown recording potentiometer. The primary coil was connected in series with the primary of a standard mutual inductor which allowed the drift in the electronic circuits to be calibrated at intervals during the run.

When the metal sample inside the secondary coil was in the non-superconducting or "normal" state, some or all of the alternating field penetrated the metal, giving a corresponding contribution to the secondary e.m.f. which remained very nearly independent of temperature. However, as the temperature was lowered into the transition region, the alternating field was excluded from the sample, causing the secondary e.m.f. to drop sharply until it reached a small and constant value corresponding to complete superconductivity of the sample.

We define $\Delta E = (E_n - E_s)/E_n$, where E_n is the secondary e.m.f. for the sample in the normal state, and E is the secondary e.m.f. for the sample in the superconducting state. For a first approximation, which will give the maximum ΔE allowed by the geometry of the apparatus, we will assume that the effects of eddy currents in the normally conducting sample are negligible and that the normal metal has unit permeability. For a specimen of spherical shape with the secondary coil wrapped around its equator, the solution is simple, yielding $\Delta E = a^3/R^3$, where a is the radius of the sphere, and R is the coil radius. This formula proved to be correct for vanadium spheres to within the accuracy of the measurement.

A general solution for ΔE must take into account the partial shielding that will occur in the sample in the normal state due to the presence of eddy currents. An argument based on the analysis of Smythe[4] shows, for the spherical case, that the diminution of E_n (or of ΔE), due to eddy currents, will be a function of $a(\omega/\tau)^{\frac{1}{2}}$ where ω is the frequency of the applied field, τ is the resistivity of the metal, and a is the radius of the sphere. The exact nature of this relationship requires the evaluation of some hitherto untabulated modified Bessel functions of half-integral order.

The fact that the ΔE of the vanadium samples agreed quite well with the approximate formula above indicates that the normal resistivity of the metal at

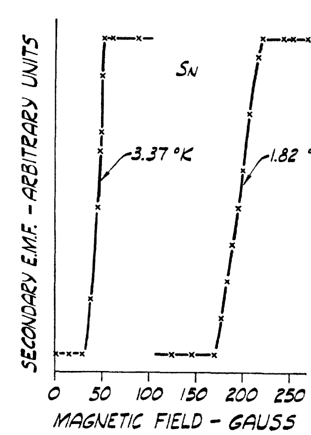

FIG. 1. Typical isothermal transitions of tin.

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** Assisted by the ONR.
[1] D. Shoenberg, Proc. Camb. Phil. Soc. **33**, 577 (1937).
[2] J. G. Daunt, Phil. Mag. **24**, 361 (1937).
[3] H. B. G. Casimir, Physica **7**, 887–896 (1940).
[4] William R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Co., Inc., New York, 1939) pp. 396 et seq.

liquid helium temperatures was quite high. The ΔE for tin, on the other hand, was almost an order of magnitude lower.

EXPERIMENTAL RESULTS

(a) Tin

As a check on the alternating field method, a determination was made of the transition and critical fields of a small spheroidal single crystal of spectroscopically pure tin, supplied by Johnson Mathey and Company, London (purity 99.992 percent). The midpoint of the transition in zero steady field occurred at 3.71°K, slightly higher than the accepted value of 3.69°K.[5] The transition width was approximately 0.02°K.

Steady magnetic fields transverse to the direction of the alternating field were supplied by a large pair of Helmholtz coils. Typical isothermal transitions are shown in Fig. 1. When the fields at which the secondary e.m.f. is restored to its normal value were plotted against temperature, we obtained a critical field curve which is identical with that of de Haas and Engelkes.[5]

(b) Vanadium

The superconductivity of vanadium was discovered in 1930 by Meissner and Westerhoff,[6] who observed that the electrical resistance of a sintered rod of this metal vanished at about 4.3°K. Since that time very little further information has been published regarding the properties of this metal at low temperatures, due largely to the difficulty in securing pure samples of a form suitable for measurements.

The samples of vanadium used by us were obtained from the A. D. McKay Company (Sample No. 1, purity 99.8 percent) and from the Vanadium Corporation of America (Sample No. 2, purity 99.7 percent). These samples were polycrystalline in the form of nearly exact spherical solid globules about $\frac{1}{8}$ inch in diameter.

The experimental arrangement was the same as that for the tin sample, with the exception that the steady fields used to obtain the critical field curves were obtained with the use of a large Weiss electromagnet capable of producing fields up to 10,000 gauss.

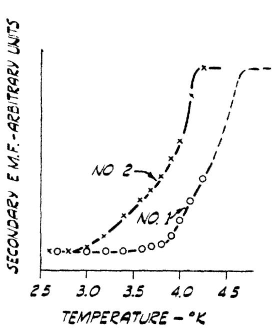

FIG. 2. Zero field transitions of two samples of vanadium.

[5] W. J. de Haas and A. D. Engelkes, Physica **4**, 325 (1937).

[6] W. Meissner and H. Westerhoff, Zeits. f. Physik **87**, 206 (1934).

The transitions of these specimens into superconductivity in zero steady magnetic field are shown in Fig. 2. Both samples exhibited transition ranges approaching a degree of temperature, as compared with the transition ranges of about 0.001° that have been observed in single crystals of pure tin.[7]

Because it exhibited a sharper transition at higher temperatures, Sample No. 1 was chosen for study of the critical field properties. A typical isothermal transition is illustrated in Fig. 3. The rising part of this graph is more or less what might be expected. The secondary e.m.f. remained at a low and constant value until the steady magnetic field reached a magnitude P (which we will call the "penetration field"). Increasing the steady field above P, the secondary e.m.f. rises as more and more of the alternating magnetic field penetrates the metal sample until a field S ("saturation field") is reached at which the sphere is entirely normally conducting. For fields higher than S, the secondary e.m.f. remains a constant.

On lowering the steady field below S, an interesting hysteresis appeared, which had the net effect of leaving the sample as a slightly better shield of the alternating magnetic field than it had been prior to the quenching of the superconductivity by the steady magnetic field. This hysteresis was observed in the vanadium samples at all temperatures for which the critical fields were determined, but was not observed in the tin sample.

The penetration and saturation fields at various temperatures are shown in Fig. 4. The slope of the saturation curve ($dH_c/dT=4100$ gauss per degree) is much larger than found in any other pure non-alloy conductor. A similar, but less thorough study of the critical fields of our vanadium Sample No. 2 yielded a saturation field slope of about 5000 gauss per degree.

As a further check on the transitions into superconductivity of the vanadium, Sample No. 1, the critical fields were obtained by measuring the magnetic susceptibility of the sample. A modified form of a Hutchison-Reekie[8] balance was used. The so-called Faraday body force was employed in making the measurement,

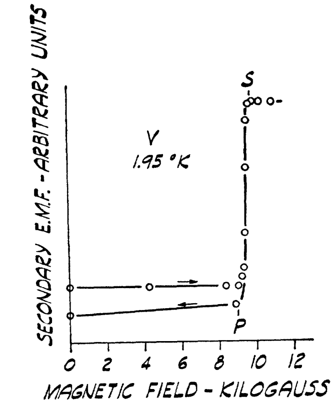

FIG. 3. Typical isothermal transition of vanadium (Sample No. 1).

[7] W. J. de Haas and J. Voogd, Leiden Comm. 214c (1931).

[8] T. S. Hutchison and J. Reekie, J. Sci. Inst. **23**, 209 (1946).

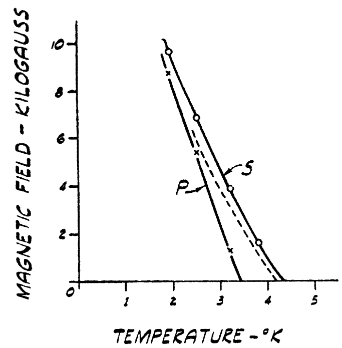

FIG. 4. Critical fields of vanadium (Sample No. 1).

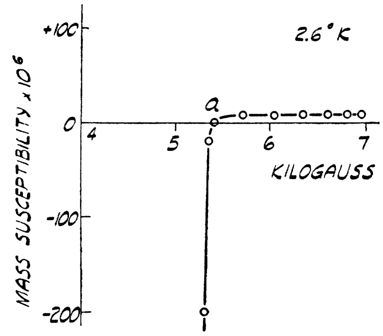

FIG. 5. The susceptibility of vanadium obtained from body force measurements.

wherein the vanadium sample was placed in an inhomogeneous magnetic field (H). The force (F) on the specimen of volume (V) is then given by $\mathbf{F}=[(k-k_0)/2]v\nabla H^2$, where k and k_0 are, respectively, the magnetic susceptibility per unit volume of the specimen and the surrounding medium (the latter being zero in these measurements). The force obtained for different values of H was compared to a standard substance (spectroscopically pure bismuth, purity 99.998 percent).

In Fig. 5, the susceptibility of vanadium at 2.6°K is plotted as a function of the magnetic field. It will be noted that when the field is decreased below 5450 gauss, the sample becomes highly diamagnetic. This is in accordance with the "Meissner effect" in which the field inside a superconductor becomes zero ($B=0$). Consequently, from the relation

$$B=H+4\pi(1-\tfrac{1}{3})I,$$

we obtain

$$I/H=K=-3/8\pi.$$

Thus, we should approach a value $-3/8\pi$ for the volume susceptibility of a pure superconductor of spherical shape. Unfortunately, the measurements could not be carried out to this limit as the balance was not built to measure forces over 100 dynes.

The critical field curve obtained from these susceptibility measurements is shown in Fig. 4 by the dashed line. This curve was obtained by taking the points of inflection of the susceptibility curves (point a, Fig. 5) at temperatures 3.5°, 3°, 2.8°, and 2.6°. It is seen to be in fair agreement with those obtained from the a.c. measurements.

DISCUSSION OF RESULTS

It is evident that the purest vanadium now available shows many of the characteristics of alloy-type superconductors,[9] such as broad transitions in the absence of magnetic field, very high critical fields, and hysteresis. It is quite possible that extremely pure, strain-free specimens of this metal will exhibit much lower critical fields and will otherwise more nearly conform to the behavior of the other pure metal superconductors that have been studied thus far.

It is also apparent that the alternating field technique yields a different view of the intermediate state than do measurements of the resistance and of the susceptibility. It can be easily shown[10] that for a superconducting sample of spherical shape, the magnetic field begins to penetrate the metal when the external applied magnetic field equals $\frac{2}{3}$ the critical field (H_c). As the field is raised from $\frac{2}{3}H_c$ to H_c, the penetration of the metal by the field should increase uniformly. This has been confirmed experimentally by the susceptibility measurements of Shoenberg.[11] Examination of Figs. 1 and 3 shows that the alternating field does not penetrate the metal to any great extent until fields considerably larger than $\frac{2}{3}H_c$ are reached. A similar distortion of the isothermal transitions in hollow cylinders in alternating fields was observed by Daunt.[2] A reasonable explanation of this distortion is that there exist in the intermediate state closed current loops which are still for the most part superconducting, having only a very small section which is resistive.

It has been suggested[12] that such low resistance current loops may exist at the boundary surface between regions in the superconducting and in the normal state.

A general investigation of time effects in the intermediate state is being continued and will be reported in a later issue of this Journal.

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[9] D. Shoenberg, *Superconductivity* (Cambridge University Press, London, 1938), Chapter 6.
[10] See reference 9, p. 27.
[11] D. Shoenberg, Proc. Roy. Soc. **155**, 712 (1936).
[12] K. Mendelssohn and R. B. Pontius, Physica **3**, 327 (1936).