

The accuracy will be limited principally by the uncertainty in the gyromagnetic ratio. Work is in progress here which should considerably decrease the uncertainty in the gyromagnetic ratio. The agreement of our result with that of the voltmeter gives added support to the value of e/m for the electron^{6,7} resulting from our value of γ .

If our value of the ratio ν_c/ν_n is combined with the measurement of Gardner and Purcell,⁷ one obtains a value of the ratio of the mass of the proton to the mass of the electron of greatly improved accuracy. This new result is

$$M_p/m_e = 1835.979 \pm 0.056.$$

Several possible applications of this instrument suggest themselves. In the first place, it looks very promising for the measurement of packing fractions. Its simplicity, high sensitivity, and variable resolution should make it useful in many other research and analytical applications. Since this device measures ω , it is suggested that it be called the *omegatron*.

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⁴ J. W. M. DuMond and E. R. Cohen, *Rev. Mod. Phys.* **20**, 82 (1948).
⁵ S. J. Bates and G. W. Vinal, *J. Acous. Soc. Am.* **36**, 916 (1914); G. W. Vinal and S. J. Bates, *Bull. Bur. Stand.* **10**, 425 (1914); G. W. Vinal, *Comptes Rendus* **3**, 95 (1932).
⁶ Thomas, Driscoll, and Hipple, *Phys. Rev.* **75**, 992 (1949).
⁷ J. H. Gardner and E. M. Purcell, *Phys. Rev.* **76**, 1262 (1949).
⁸ F. G. Dunnington, *Phys. Rev.* **52**, 475 (1937).

Theory of the Electric Resistivity of Polycrystalline Graphite*

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IT is possible to explain the observed temperature dependence of electric resistivity in commercial polycrystalline graphite by an extension of the theory of single crystal graphite as developed by Wallace.¹ There are two mechanisms required. First, one assumes that the conduction in the polycrystalline sample takes place in the planes of cleavage of the individual crystals. The large anisotropy (of the order of 10^4) prevents any appreciable conduction across the planes. This confinement of the current to the direction of orientation of the crystals, which will not necessarily be in the direction of gross current flow, increases the resistivity of the polycrystalline material over that of a single

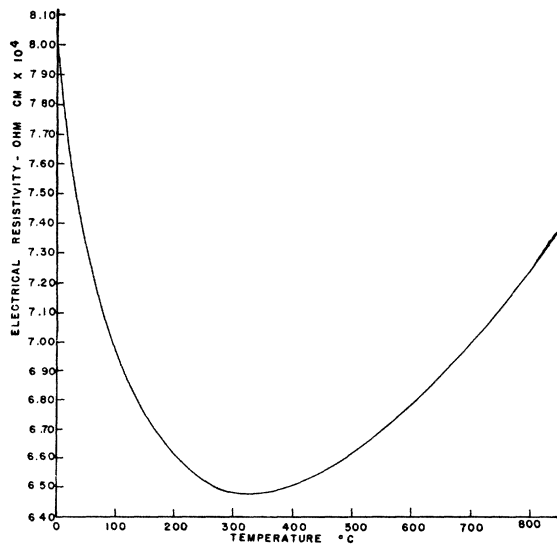


FIG. 1. Typical plot showing the variation of resistivity with temperature for polycrystalline graphite.

crystal in the plane of cleavage by a geometrical factor, s , depending on the ratio of the principal dimensions of the crystals, and their orientation with respect to gross current flow. For completely random orientation and a common ratio of crystal dimensions of 10, this factor is about 7. Second, it is assumed that the electron waves are scattered off the crystallite boundaries. Since these boundaries are regions of disorder, there is almost a certainty of scattering at these regions. Thus the probability of scattering per unit time can be estimated from the size of the crystallites, and the velocity of the waves in the crystal. Using estimates of crystallite size based on the width of x-ray diffraction lines, one computes the scattering probability per second to be 10^{14} . Furthermore, this probability should be temperature independent since the percentage change in crystallite dimension with temperature will be small. Adding this scattering probability to the thermal lattice scattering probability in Wallace's formula and multiplying by the geometrical factor s , one has

$$\rho = \frac{h^2 cs}{16\pi^2 k T} \left(\frac{1}{\tau_T} + \frac{1}{\tau_b} \right),$$

where the τ 's are the inverse scattering probabilities.

By computing τ_T from the original Wallace equation, and measurements on single crystals, a typical plot showing the variation of resistivity with temperature for polycrystalline graphite has been reproduced in Fig. 1.

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¹ P. R. Wallace, *Phys. Rev.* **71**, 622 (1947).

A Note on the Variational Method for the Scattering Problem

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AS we have already pointed out,¹ Hulthén's formulation of the variational principle² has the advantage over other formulations³ in that a more flexible trial wave function can be used. But in his practical applications he adopts a method which leads to a very inconvenient numerical procedure. Moreover, the manner of his application does not utilize the variational principle in its strict form: one of the equations he uses is not logically connected with the principle. In this note we shall indicate how we can improve Hulthén's method by making it more rigorous and at the same time much simpler for practical purposes. We shall illustrate the proposed method by considering S -scattering by a potential field of the Yukawa type.

Considering for the sake of simplicity only S -scattering, we define, following Hulthén

$$\mathcal{Q} = \int_0^\infty \psi(H - k^2) \psi dr, \quad (1)$$

where

$$H = -d^2/dr^2 + V(r). \quad (2)$$

It can then be shown that

$$\delta\mathcal{Q} = 2 \int_0^\infty \delta\psi(H - k^2) \psi dr + k\delta\eta, \quad (3)$$

where η represents the phase shift. After obtaining this equation Hulthén suggested a variational method in which

$$\mathcal{Q} = 0 \quad (4)$$

is used to determine η while

$$\delta\mathcal{Q} = 0 \quad (5)$$

is used for obtaining a set of equations for the variational parameters. In this manner with a trial function of the form

$$\psi(r) = f(r) \cos\eta \sin kr + g(r) \sin\eta \cos kr, \quad (6)$$

Hulthén obtains a system of equations each of which is of the second degree in $\tan\eta$ and solves them by a method of successive approximations.

Now Eq. (4) is not essential in the formulation of variational principle for the *free state*, because it can be satisfied even if ψ is not an exact solution of the wave equation. Moreover in deriving Eq. (3) the condition $\xi=0$ is nowhere used. In fact, instead of using Eq. (4), we might with equal justification have used any integral involving $(H-k^2)\psi$ as the equation for $\tan\eta$. The use of $\xi=0$ would therefore seem to be arbitrary and not inherent for the variational principle derived from Eq. (3). The fact that Hulthén finds by his method two solutions for $\tan\eta$ is a consequence of this ambiguity resulting from the use of the condition $\xi=0$; he actually rejects one of the two solutions as not "good" though there is no way of deciding as to which of the two solutions is "good," except by comparison with results obtained by other methods. A rigorous method for computing the wave function for the continuous spectrum should, therefore, be based solely on Eq. (3) or its equivalent:

$$\delta\mathcal{L} = k\delta\eta. \quad (7)$$

We now assume a trial wave function of the form

$$\psi(r) = f(r) \sin kr + g(r) \cos kr, \quad (8)$$

subject to the boundary conditions

$$\left. \begin{aligned} f(0) &= \text{finite}, & \lim_{r \rightarrow \infty} f(r) &= 1; \\ g(0) &= 0, & \lim_{r \rightarrow \infty} g(r) &= \lambda = \tan\eta. \end{aligned} \right\} \quad (9)$$

For ψ given by (8)–(9), the variational integral becomes

$$\mathcal{L} \equiv \int_0^\infty [f_1^2 \sin^2 kr + g_1^2 \cos^2 kr + k(fg_1 - gf_1) + V(f^2 \sin^2 kr + g^2 \cos^2 kr) + (g_1 f_1 + Vgf) \sin 2kr] dr, \quad (10)$$

where f_1 and g_1 denote the derivatives of f and g .

Equation (7) then becomes

$$\delta\mathcal{L} = k\delta\lambda \quad (11)$$

when \mathcal{L} is computed with the wave function (9). Now we determine λ and all the parameters by the equations

$$\partial\mathcal{L}/\partial c_i = 0, \quad i=1, 2, \dots, n \quad (12)$$

and

$$\partial\mathcal{L}/\partial\lambda = k. \quad (13)$$

Equations (12) express the stationary property of the phase shift, while Eq. (13) follows from Eq. (11) directly. Now if we assume for $f(r)$ and $g(r)$ the forms

$$\begin{aligned} f(r) &= 1 + \sum_{n=1} P_n(r) e^{-nr}, \\ g(r) &= (1 - e^{-r}) \left[\lambda + \sum_{n=1} Q_n(r) e^{-nr} \right], \end{aligned} \quad (14)$$

where $P_n(r)$ and $Q_n(r)$ may, for example, be polynomials in r , involving a set of coefficients c_1, c_2, \dots, c_n which we subject to the variation, Eqs. (12)–(14) will then give a set of $(n+1)$ linear equations for λ, c_1, \dots, c_n . The coefficients of this set of equations form a symmetrical determinant, which can be readily solved.

The method derived in the foregoing paragraph differs from Hulthén's in that by using Eq. (13) instead of Eq. (4) we have made the method more rigorous by conforming to the variational principle, Eq. (11); also by introducing the λ -formulation (as we may call it) we have essentially facilitated the actual calculations.

As an illustration of the foregoing method we shall consider the scattering by the potential

$$V(r) = l e^{-r}/r, \quad (15)$$

where l is a constant. In conformity with Eqs. (14) we write

$$\begin{aligned} f(r) &= 1 + (c_1 + c_2 r) e^{-r}, \\ g(r) &= (1 - e^{-r}) \left[\lambda + (c_3 + c_4 r) e^{-r} \right]. \end{aligned} \quad (16)$$

After some elementary calculations, a set of linear equations in λ, c_1, \dots, c_4 are finally obtained. The coefficients in these equations can best be expressed in terms of certain standard functions of k .⁴

In order to compare our results with Hulthén's, we have computed the phase shift for two values of l and k , namely $l = -1.5$,

TABLE I. The phase shifts and the constants for the expansion of the wave function at $k=0.8$.

$-l$	Approx.	λ	c_1	c_2	c_3	c_4	η
1.5	(c_1, c_2, c_3, c_4)	1.11469	0.02278	0.01492	0.92077	-0.06414	0.83958
	(c_1, c_3, c_4)	1.11524	0.05205	0.89689	-0.06512	0.83982
	(c_1, c_2, c_3)	1.11188	0.59566	-0.28842	0.44184	0.83832
	Hulthén	1.10430	0.03493	0.90190	0.83492
2.1	(c_1, c_2, c_3, c_4)	3.31023	0.61034	0.02359	3.48932	-0.49419	1.27742
	(c_1, c_3, c_4)	3.31111	0.65660	3.45157	-0.49574	1.27749
	(c_1, c_2, c_3)	3.28028	4.86017	-2.22788	-0.08688	1.27489
	Hulthén	3.22313	0.52837	3.47190	1.26996

$k=0.8$ and $l=-2.1$, $k=0.8$ respectively. The results are tabulated below together with Hulthén's best values of the phase shift. In Table I in addition to the results for the 4-parameter (c_1, c_2, c_3, c_4) trial wave functions (Eqs. (16)) we have also included the results derived for 3-parameter (c_1, c_3, c_4) and (c_1, c_2, c_3) and 2-parameter (c_1, c_2) trial functions.

The present method can be extended to electron scattering by the hydrogen atom, and to allow also for exchange effects. The calculations relating to these extensions are now in progress.

Finally, I should like to express my sincere thanks to Professor S. Chandrasekhar for his interest in this problem and also for his valuable discussions.

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Photo-Disintegration of the Deuteron at Intermediate Energies

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IN the photo-disintegration process, all but a small fraction of the γ -ray energy appears as relative energy of neutron and proton, while in N - P scattering half of the incident neutron energy appears as kinetic energy of the center of mass of the N - P system. Consequently, for energies well above the threshold, photo-disintegration experiments should yield as much information about the N - P interaction as N - P scattering experiments performed at twice the energy. Therefore, energies up to 20 Mev, which are of particular interest because of the large number of electrostatic generators and betatrons operating in this range, are of considerable theoretical importance. (A copy of a letter by Fuller¹ describing preliminary photo-disintegration experiments in this energy region arrived when this report was in preparation.)

In this energy range computations are relatively simple. Only a small number of multipoles are involved (electric dipole, magnetic dipole, and electric quadrupole). For well radii below 2.8×10^{-13} cm, range-corrected Bethe-Peierls formulas are essentially correct, and the total electric cross section is given by:

$$\sigma_e = \frac{8\pi e^2 \hbar^2}{3 \hbar c M} \frac{W_1^3 (\hbar\omega - W_1)^3 \sin^2(kr_1) \exp(2\alpha r_1)}{(\hbar\omega)^3 (1 + \alpha r_1)}, \quad (1)$$

where W_1 is the binding energy of the deuteron. (The photo-magnetic contribution can be neglected except for very low energies.) For a 50-50 mixture for which the outgoing nucleons may be treated as free, expression (1) is only slightly modified by the inclusion of tensor forces. The total cross section including tensor forces is given by (we use the approximation² in which contributions from inside the well are neglected. This approximation gives the right order of magnitude for our case, the