for $C^{12}O^{16}O^{18}$, in good agreement with the experimental value 0.3670.

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A Comparison of the *g* Value of the Electron in Hydrogen with That in Deuterium

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The g value of the electron in the $^2S_{\frac{1}{2}}$ state of H has been compared with the g value of the electron in the corresponding state of D by the atomic beam magnetic resonance method. The frequencies, f_H, of the transition $(1, 0 \leftrightarrow 1, -1)$ in H, and f_D, of the transition $(\frac{3}{2}, -\frac{1}{2} \leftrightarrow \frac{3}{2}, -\frac{3}{2})$ in D were observed in the same magnetic field and the ratio $(g_J)_H/(g_J)_D$ was computed from the Breit-Rabi formula. The principal uncertainty is caused by the drift of the magnetic field. The average of 36 observations in 4 runs in which f_H and f_D were measured alternately is $(g_J)_H/(g_J)_D = 1.000004 \pm 0.000025$. The average of 3 runs in which the two resonances were observed simultaneously is $(g_J)_H/(g_J)_D = 0.999991 \pm 0.000010$. The uncertainty indicated in each result is the mean deviation of the residuals. We conclude that the g value of the electron when bound to the proton differs from that when bound to the deuteron by less than 0.001 percent.

INTRODUCTION

THIS experiment has as its purpose a comparison of the moment of the electron in the ground state of hydrogen with its moment in the ground state of deuterium.[1] In view of the deviations from the straightforward Dirac theory which occur in the hyperfine structure of the ground state and shift of the $2S$ level in hydrogen, and of the difference of about one part in 4000 between the reduced masses of the electron in H and D, it was considered of interest to measure the ratio of the moments.[2-4] The experiment was performed by measuring the frequency of atomic transitions in a magnetic field large enough to insure a large measure of decoupling of the electron spin from the nuclear spin. The transitions chosen were those amounting essentially to a reorientation of the electron in the external field. The atomic beam magnetic resonance method was used to detect these transitions. Since the magnetic field has not been measured with sufficient accuracy this method suffices only to measure the ratio of the moments and could detect an effect only if it were not common to the two atoms. Absolute values of the moments are not measured in this experiment.

GENERAL DISCUSSION

The ground state of an atom is split into a number of magnetic levels by the combined influence of an external magnetic field and the magnetic interaction between the nuclear spin I and the total electronic angular momentum J. In this experiment the transitions $(1, 0 \leftrightarrow 1, -1)$ in H and $(\frac{3}{2}, -\frac{1}{2} \leftrightarrow \frac{3}{2}, -\frac{3}{2})$ in D have been studied. The quantum numbers (F, m) refer to the low field designation of the states. The field dependence of these transitions is found from the Breit-Rabi[5] formula. The frequencies of the transitions are

$$f_H = \frac{\nu_H}{2}\left[(1+X_H)^{\frac{1}{2}} - (1-X_H) + 2\left(\frac{g_I}{g_J - g_I}\right)X_H\right], \tag{1}$$

$$f_D = \frac{\nu_D}{2}\left[(1-\tfrac{2}{3}X_D + X_D{}^2)^{\frac{1}{2}} - (1-X_D) + 2\left(\frac{g_I}{g_J - g_I}\right)X_D\right], \tag{2}$$

where ν_H (or ν_D) is the h.f.s. separation for H (or D) in absolute frequency units. The parameter X, which is proportional to the magnetic field intensity, is given by

$$X = (g_J - g_I)\mu_0 H/\Delta W. \tag{3}$$

In Eqs. (1)–(3) g_J is the g value of the electron in the $^2S_{\frac{1}{2}}$ state, g_I is the g value of the nucleus, ΔW is the

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h.f.s. splitting in energy units, μ_0 the Bohr magneton and H the magnetic field intensity.

If the transitions in H and D are observed in the same magnetic field the ratio of the parameters, X_H/X_D, will be proportional to the ratio $(g_J)_H/(g_J)_D$. The value of the parameter X, corresponding to a frequency f_H or f_D, may be computed from Eqs. (1) and (2) with a precision which is comparable with that of f_H or f_D. The ratio $(g_J)_H/(g_J)_D$ is calculated from X_H/X_D by

$$\frac{(g_J)_H}{(g_J)_D}=\frac{X_H}{X_D}\frac{\nu_H}{\nu_D}\left[1-\left(\frac{g_I}{g_J}\right)_D\right]\left[1-\left(\frac{g_I}{g_J}\right)_H\right]^{-1}. \quad (4)$$

The value of the ratio $(g_J)_H/(g_J)_D$ is thus obtainable from measured frequencies and is independent of any natural constants. The final result is not affected by the uncertainty in the values of the small correction terms involving the nuclear g factors.

It is essential that the transitions in H and D be induced in the same magnetic field or that correction be made for the drift of the field. The erratic nature of the drift of the field limits the precision obtainable in this experiment.

APPARATUS AND PROCEDURE

The atomic beam apparatus used in this experiment is the one employed in our measurement of the h.f.s. of H and D.[2] In this apparatus the atoms are deflected through a collimating slit by an inhomogeneous field and then focused on a Pirani detector by a second inhomogeneous field whose gradient is oppositely directed with respect to the first field. Between the deflecting and focusing fields the atom traverses a uniform magnetic field where a radiofrequency magnetic field may induce transitions between the magnetic components of the h.f.s. levels. If the magnetic moments of the atom in the initial and final h.f.s. levels differ by an amount of the order of one Bohr magneton the beam will not be focused by the second inhomogeneous field. The intensity of the beam is observed as a function of the frequency of the r-f field inducing the transitions. The frequency associated with the minimum intensity of the beam is assumed to be the center of the resonance line.

The r-f currents, at frequencies near 1000 and 1200 mc, were supplied by coaxial line oscillators which use Lighthouse tubes. The frequency was measured by heterodyning the signals with harmonics of a 240 mc crystal controlled frequency standard. The precision of the measurement of the ratio $(g_J)_H/(g_J)_D$ was not limited by the stability of the oscillators or the accuracy in the measurement of frequency.

In the first series of measurements correction was made for the drift in the magnetic field intensity by taking alternate observations of f_H and f_D. The time required for each observation was about 10 min. The drift in the magnetic field intensity between successive observations of f_H and f_D was about 1 part in 20,000. The drift was not associated with changes in the magnet current, which was held constant by manual adjustment to better than 1 part in 20,000. Each value of f_H was paired with a value of f_D obtained by linear interpolation and conversely.

A technique for observing the transitions in H and D simultaneously was developed to avoid the error introduced by the drift in the intensity of the magnetic field. In this method a beam containing approximately equal parts of H and D atoms was employed. Two oscillators were connected to the r-f loop in which the transition field is produced. On varying the magnetic field the H and D resonances were observed in general to occur at different magnet currents. Holding the frequency of one oscillator fixed, that of the other oscillator was changed stepwise in such a direction as to cause the resonances to coalesce. The depth of the composite line was then investigated by varying the magnet current for each pair of values of frequency to find the minimum beam intensity. The reduction in beam intensity was plotted as a function of the varied frequency. The resulting curve has a maximum when the transitions in H and D occur in the same field. The half-widths of the composite line is approximately the sum of the half-widths of the individual lines. Thus the increased precision due to the simultaneous measurement of f_H and f_D is offset, in part, by the increased width of the composite line.

RESULTS

(A) Alternate Measurement of f_H and f_D

A sample of the data from a run of alternate observations of f_H and f_D is given below,

$$f_H=1000.371\text{ mc},$$
$$f_D=1216.924\text{ mc},$$
$$X_H=0.999292,$$
$$X_D=4.329941,$$
$$H_0=506.1\text{ gauss},$$
$$(g_J)_H/(g_J)_D=1.000020,$$
$$\delta=20\times10^{-6},$$

where δ is given by $(g_J)_H/(g_J)_D=1+\delta$. The mean value of δ obtained from 36 observations (pairs of frequencies) made in 4 different runs is $\delta=(4\pm25)\times10^{-6}$. The average deviation ($\pm25\times10^{-6}$) of the individual values from the mean is to be compared with the fractional half-width of the lines, $\Delta f/f\sim100\times10^{-6}$, and the average drift of the magnetic field intensity between successive observations, $\Delta H/H\sim50\times10^{-6}$.

(B) Simultaneous Observations of f_H and f_D

A sample curve obtained in the simultaneous measurement of f_H and f_D is given in Fig. 1. The total reduction in beam intensity is plotted as a function of the frequency required to induce the H transition. The

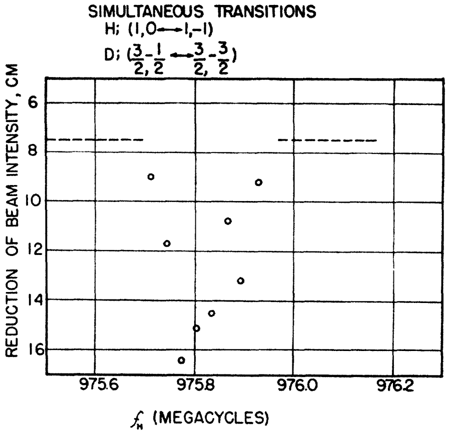

FIG. 1. The total reduction in the intensity of an atomic beam of H and D is plotted as a function of the frequency inducing the H transition when the frequency inducing the D transition was held fixed. The dashed lines indicate the reduction in beam intensity caused by the deuterium resonance. The frequency inducing the deuterium resonance was held constant at 1187.329 mc.

frequency of the r-f field inducing the D transition was set at 1187.329 mc. The reduction in beam intensity caused by the D transition was 7.5 cm, independent of the frequency inducing the H transition. When the frequency inducing the H transition is such that both the H and D transition occur at the same intensity of the magnetic field the two resonance lines coincide. The frequency inducing the H transition was 975.810 mc. The result obtained from three measurements of this type is $\delta=(-9\pm10)\times10^{-6}$. The mean deviation, $(\pm10\times10^{-6})$ is to be compared with the fractional half-width of the combined lines, $\Delta f/f\sim150\times10^{-6}$, which is the limiting factor in the precision of the measurement.

As a test of the accuracy of the method of superposition a measurement was made of the h.f.s. separation of the ground state of D. The transitions $(\frac{3}{2}, \frac{3}{2}\leftrightarrow\frac{1}{2}, \frac{1}{2})$ and $(\frac{3}{2}, -\frac{1}{2}\leftrightarrow\frac{3}{2}, -\frac{3}{2})$ were observed simultaneously. The frequencies were 1150.670±0.020 mc and 720.000 mc respectively, in a field of about 325 gauss. From these observations the value of ν_D is 327.395±0.020 mc which is to be compared with $\nu_D=327.384\pm0.003$ mc which was obtained previously by direct measurement. The precision to be expected in the value of the ratio $(g_J)_H/(g_J)_D$ is about four times that found in the measurement of ν_D for the ratio of the g factors depends almost directly on the ratio of the measured frequencies, while the value of the h.f.s., determined by the superposition method, depends on the difference between the frequencies of the two transitions.

DISCUSSION OF SYSTEMATIC ERRORS

The observed transitions in H and D are those which, in high field, would correspond to a reorientation of the electronic moment with respect to the external magnetic field; thus the frequencies f_H and f_D are approximately the Larmor precession frequency of the electron. The ratio, f_H/f_D, differs from the ratio, $(g_J)_H/(g_J)_D$, by about 20 percent. This difference is a consequence of the h.f.s. interaction of the electronic and nuclear moments. The validity of the Breit-Rabi formula to describe the field dependence of the magnetic components of the h.f.s. doublet has been demonstrated by Kusch, Millman and Rabi[6] to ±0.005 percent in K^{39} and Li^7 for magnetic fields ranging from a few gauss up to 4000 gauss. The uncertainty introduced in the ratio $(g_J)_H/(g_J)_D$ by the use of the Breit-Rabi formula is, at most, ±0.001 percent.

The calculation of X_H and X_D from observed transition frequencies and of $(g_J)_H/(g_J)_D$ from these values of x involve small correction terms containing the ratio of the nuclear to electronic g factors. The term $(g_I/(g_J-g_I))X$ in Eqs. (1) and (2) contributes only +0.179 percent to the calculated value of X_H and the similar term contributes +0.024 percent to X_D. The terms $(g_I/g_J)_H$ and $(g_I/g_J)_D$ in Eq. (4) contribute −0.152 percent and −0.023 percent, respectively, and the sign of their contribution is opposite to that of $(g_I/(g_J-g_I))X$ entering X. The final result is quite independent of $(g_I/g_J)_D$ and the total dependence on $(g_I/g_J)_H$ is +0.027 percent. It is concluded that the uncertainty in the ratio $(g_I/g_J)_H$ and $(g_I/g_J)_D$ has a negligible effect on the ratio of the electronic g factors.

In the fields of intermediate strength used in this experiment, the decoupling of the nuclear and electronic moments is such that a 1 percent error in ν_H or ν_D would change the calculated value of $(g_J)_H/(g_J)_D$ by 0.17 percent and 0.13 percent, respectively. As ν_H and ν_D are known to ±0.001 percent their uncertainty introduces no error into the ratio.

CONCLUSION

The g factors of the electron in the ground state of H and D have been found to be equal to within ±0.001 percent. The precision of measurement is limited by the half-width of the resonance lines and the uncertainty in the value of the magnetic field due to the erratic drift of the field intensity in time.

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