mitted beam was determined by setting the instrument at "zero wave-length" so that the primary beam was directly transmitted through the crystal and collimator to the counter. Because this direct beam was far too intense to be measured by the counter directly, absorbing sheets were introduced into the beam and the absorption curve (logarithm of counting rate against thickness of absorber) was extrapolated back to zero absorption thickness to determine the true counting rate. One-half of this direct beam counting rate we associate with each of the two lines since our spectra show that they are quite closely equal in intensity. On this basis, we find for the 1.1 Mev line 4.97 counts in the diffracted beam per 10,000 counts in the direct beam, and for the 1.3-Mev line 4.12 counts in the diffracted beam per 10,000 counts in the direct beam. This is *not* the integrated reflection coefficient but it gives a basis from which the latter can be calculated. Such a calculation[14] is too involved for the scope of the present paper, but it has been shown that the integrated reflection coefficient for wave-lengths from $\lambda=200$ x.u. down to $\lambda=30$ x.u. diminishes quite closely as λ^2. The present measurements at 9 x.u. agree substantially with this law. If anything, they lie slightly higher than the λ^2 line determined at longer wave-lengths but we cannot guarantee that this is a significant deviation.

COMPARISON WITH PREVIOUS WORK

The shortest wave-lengths measured previously by direct crystal diffraction are, we believe, those observed with the photographic crystal spectrometer of Frilley.[15] The shortest of Frilley's lines was 16 x.u. or 770 kev quantum energy. From an examination of Frilley's photograph it seems doubtful whether a precision in this wave-length determination better than ±2.0 percent could be claimed. We believe it safe, therefore, to say that the present measurements of the 1.1- and 1.3-Mev lines of Co^{60}, quite independent of their high precision, set a new record for the shortest wave-lengths ever measured directly. They surpass Frilley's hardest lines as to quantum energy by a factor of nearly two. The precision is, of course, from 50 to 100 times that of Frilley's spectra.

These results are far from representing the limit of the instrument even in its present state of development. We believe it quite possible with our present collimator and crystal holder to go to 2 Mev and perhaps somewhat beyond this. With the improved tungsten partition collimator and further improvements in the crystal and the detecting system such as we have outlined above, it is probable that the limit can be pushed still further upward very substantially.

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[14] A paper by D. A. Lind on this question of the integrated reflection coefficient for the (310) planes of quartz as a function of wave-length over a very wide range of wave-lengths is now in preparation.

[15] Frilley, thesis, Paris, 1928. For reproductions of Frilley's spectra, see Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1939), p. 380.

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Penetration and Diffusion of Hard X-Rays through Thick Barriers. III. Studies of Spectral Distributions*

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Two kinds of spectral distributions of gamma-rays are discussed. These are (1) spectra found in an infinite medium with an uniformly distributed monochromatic source, and (2) equilibrium spectra obtained in an artificial penetration problem. Curves are shown for various media and for various energies of the source photons. The relations of these spectra to the general problem of γ-ray penetration is discussed.

In this note we report some results of a study of two rather simple problems in multiple γ-ray scattering.[1–4] These results may have some interest in themselves and also serve to illuminate certain features of the general problem of γ-ray penetration.

The first problem to be discussed is the following: a time-independent monochromatic source of gamma-rays is distributed uniformly in an infinitely extended, isotropic medium which is capable of absorbing gamma-rays by means of photoelectric effect and pair production, and of scattering them by Compton effect. The problem is to find the steady state spectrum in the medium, i.e., the average number of photons per cm^3 per unit energy range, as a function of the photon energy.

Solutions are to be found for various media and for various energies of the source photons. An experimental study of this problem has recently been described[5] by workers at the Naval Research Laboratory. For this reason we call this the "N.R.L." problem.

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* Work supported by an ONR contract.

[1] Bethe, Fano, and Karr, Phys. Rev. **76**, 538 (1949). (Part I of the series "Penetration and diffusion of hard x-rays through thick barriers".)

[2] U. Fano, Phys. Rev. **76**, 739 (1949). (Part II of the series.)

[3] U. Fano, Nucleonics (to be published).

[4] L. V. Spencer and F. A. Jenkins, Phys. Rev. **76**, 1885 (1949).

[5] W. R. Faust and M. H. Johnson, Phys. Rev. **75**, 467 (1949).

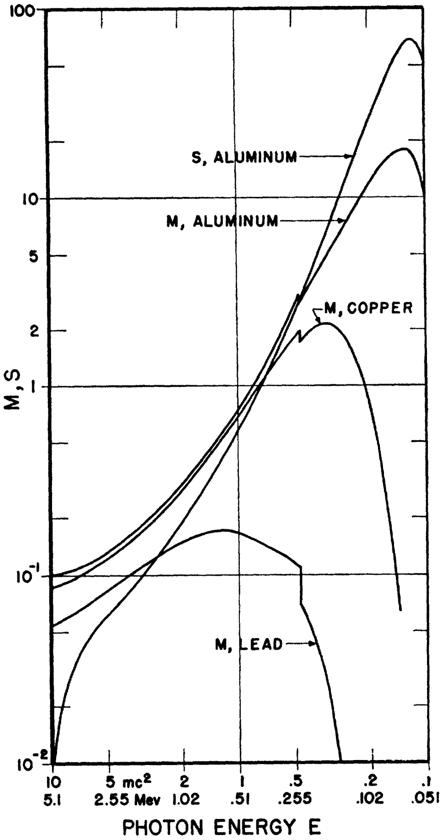

FIG. 1. "N.R.L." spectra M and "equilibrium" spectrum S for $E_0=10$ mc² (5.1 Mev). The ordinate M represents the number of photons per Mev per cm³ for a source density of 1 photon per cm³. S is plotted in arbitrary units.

The problem is governed by the following integral equation:

$$\mu(E)M(E)=k(E_0,E)+\int_E^{E_0} k(E',E)M(E')dE', \quad (1)$$

where μ is the total "narrow beam" absorption coefficient, E_0 is the energy of the source photons, E is the energy of the scattered photons, $k(E',E)dE$ is the probability per unit path length (which is proportional to the probability per unit time) that a photon of energy E' will be Compton-scattered to the interval between E and $E+dE$, and MdE is the density of photons in the energy range dE per unit density of the source photons.

In Eq. (1) the term $\mu(E)M(E)$ represents the rate of disappearance of photons of energy E. Under steady-state conditions this must be balanced by the rate of *creation* of photons of the same energy. The term $k(E_0,E)$ represents the rate of creation of photons by direct scattering from the source photons, and the integral represents the rate of creation by scattering from secondary photons.

The term $k(E_0,E)$ drops discontinuously to zero at an energy E such that $(mc^2/E)-(mc^2/E_0)=2$, since by virtue of the Compton relation, a primary photon cannot be degraded to a point below this energy. For this reason there is a discontinuity in M amounting to $k(E_0,E)/\mu(E)$ at $E=[1/E_0+2/mc^2]^{-1}$.

Solutions of Eq. (1) have been obtained by numerical integration. Figure 1 is a plot of M *vs.* E for various media, with the primary energy $E_0=10$ mc^2. One notes that M rises with decreasing E until a certain point when a rather rapid decrease sets in. The rise is due, roughly speaking, to the fact that successive Compton scatterings of a photon decrease its energy, on the average, by decreasing amounts, thus tending to create a piling-up effect at lower energies. The rather sudden drop is due to the rapid increase of μ at low energies because of the photoelectric effect. The discontinuity discussed above is well in evidence.

Figure 2 illustrates the effect of different primary energies in the same medium. It has been found that when we plot M for the same medium with different primary energies, the curves all have the same shape except for those energies which are rather close to the primary energy, where a "transient" effect occurs. In fact when we plot, as in Fig. 2, the photon density per Mev per unit rate of creation of source photons, which is $M/[\mu(E_0)c]$, (c being the velocity of light), the curves very nearly coincide, except for the "transient." The remaining difference is considerable only when E_0 is in the region where pair production accounts for a sizable fraction of the total absorption coefficients.

In the above problem, the spectrum does not depend on the position in space. The problem, nevertheless,

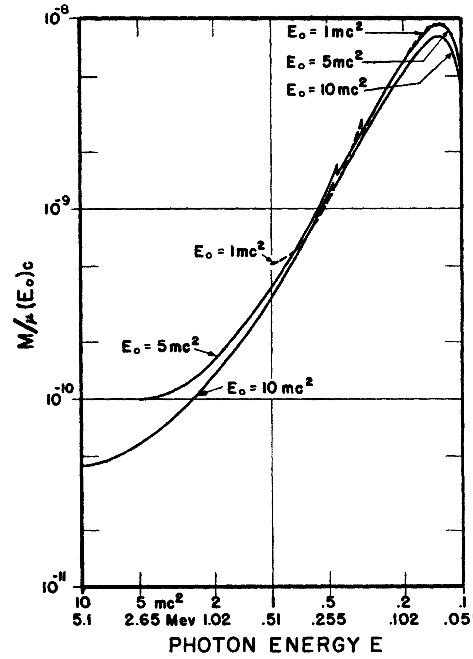

FIG. 2. "N.R.L." spectra for aluminum for several values of E_0. The ordinate $M/\mu(E_0)c$ represents the number of photons per Mev per cm³ for a rate of generation of 1 source photon per sec. per cm³.

has a relation to the more difficult problem of penetration through a medium of a monochromatic broad beam of γ-rays. In the latter problem the spectral density is a function of E and two additional variables, z the penetration distance, and θ the direction of the photons with respect to the z direction. If we average over-all values of z and θ we obtain $M(E)$, the solution of the N.R.L. problem. Now in the penetration problem, one is particularly interested in great depths of penetration, i.e., large z. The N.R.L. function $M(E)$ is a *biased* estimate of the spectral density for the penetration problem for large z and θ averaged out: the spectral density function for large z is steeper on the high energy side than $M(E)$ because those photons that penetrate large thicknesses tend to have been scattered many times with little energy degradation.

We turn now to a discussion of another spectrum which is biased in the *opposite* sense relative to the penetration problem. The problem in which this spectrum arises, which we call the "equilibrium" problem, is a modification of the penetration problem in the following sense: the photons are supposed to be Compton scattered and to lose energy according to the usual probability functions, but the angular deflections are disregarded and all photons are assumed to continue forward in the positive z direction. Furthermore the source is not monochromatic but is supposed to have a spectrum so chosen that the *same* spectrum obtains for all values of z with attenuation according to $\exp[-\mu(E_0)z]$ where E_0 is now the peak energy in the spectrum. The equation for this problem is:

$$[\mu(E)-\mu(E_0)]S(E)=\int_E^{E_0} k(E',E)S(E')dE', \qquad (2)$$

where $S(E)$ represents the spectral distribution. Solutions of (2) have been obtained by a combination of methods. We find, just as in the N.R.L. case, that the curves for a given medium but with different values of E_0 all have the same shape except for an initial transient. The "equilibrium" spectra resemble qualitatively the N.R.L. spectra except for E near E_0. The portion of an "equilibrium" curve near E_0 is much steeper than for the N.R.L. curve for the same E_0 and medium. This portion of the curve starts as $(E_0-E)^{K_0-1}$ where K_0 is a parameter discussed in reference 1. In fact the entire rising portion of an equilibrium curve is steeper than the corresponding N.R.L. spectrum. (An equilibrium curve which illustrates this is given in Fig. 1, for $E_0=10\ mc^2$, in aluminum; it is to be compared with the corresponding N.R.L. curve in the figure.) Furthermore the equilibrium problem has been so formulated that its spectra are somewhat steeper than the corresponding spectra of the penetration problem for any finite value of z, with θ averaged out.

Thus the two problems treated above give some insight into the penetration problems since the spectra obtained bracket the penetration problem spectra. It may be noted in this connection that in a penetration problem with monochromatic source and no angular deflection, the Laplace transform $y(E, p)$ of the spectral distribution $Y(E, x)$ [2] satisfies an equation intermediate between (1) and (2), in that $\mu(E)-p$ appears on the left-hand side of the equation, rather than $\mu(E)$ as in (1) and $\mu(E)-\mu(E_0)$ as in (2). Here p is the transform variable and $\mu(E_0)>p>0$.

A more complete analysis of these problems will be published at a later date. We wish to thank Dr. U. Fano for much valuable advice and assistance.

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Pressure Dependence of Line Width in Infra-Red Spectra

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The effect of coupling of molecular vibrations on line width in infra-red spectra is dealt with. Line widths due to this effect are proportional to the square root of pressure. Numerical discussion indicates that at least part of the line width in water-vapor spectra can be ascribed to this effect.

1. INTRODUCTION

RECENT discussions[1] of experiments on the dependence of absorption in infra-red band spectra on pressure have indicated that the usual assumption of proportionality of the line width to pressure, which is based on Lorentz' theory of damping by impacts, might not always be correct. A line width proportional to the square root of pressure would be in better agreement with the observations, at least for water vapor. Indeed, already in 1925, Holtsmark[2] pointed out that a coupling effect should yield such a law for the line width.

The interaction of the molecules, considered as coupling of oscillators with essentially the same eigenfrequency, gives rise to a splitting of the frequencies the amount of which depends on the distance and the

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[1] F. Matossi and E. Rauscher, Zeits. f. Physik **125**, 418 (1949); W. M. Elsasser, Harvard Meteorological Studies Nr. 6 (1942), p. 46 f. (there also other references are to be found).

[2] J. Holtsmark, Zeits. f. Physik 34, 722 (1925).