however, the lifetime of this state exceeds 10^{-2} sec. and is less than the lifetime of Hf^{181}, it would be very difficult to detect. This would certainly be the case if the transition gives rise to a magnetic 2^3-pole or electric 2^4-pole radiation. Correcting the theoretical half-life for $l=4$ for the probability of internal conversion, the result comes out as a few seconds. A probable working hypothesis is therefore to assume that the 0.134-Mev transition is of magnetic 2^3-pole and/or electric 2^4-pole character.

Finally the β-transition from the ground state of Hf^{181} to the higher metastable state in Ta^{181} will be considered. From the values of the disintegration constant and the maximum energy of the β-particles, we find that the position of the point in the Sargent diagram for the β-emitters in the heavier elements[12] corresponds to a once-forbidden transition.

Collecting the results of the above discussion as to the nature of the different transitions, we arrive at the most probable spin and parity assignments to the different nuclear states as indicated in Fig. 2. We have tried a number of alternative models for the decay scheme, but all of them seem to be in disagreement with experiments. It should be remarked that the transition between the two metastable states is very rare owing to the special selection rule operating when the centers of mass and charge of a system coincide. If this transition had not been forbidden it had been necessary to assign a spin 13/2 or larger to the first excited state of Ta^{181}.

[12] N. Feather and E. Bretscher, Proc. Roy. Soc. **A165**, 545 (1938). N. Feather, Nature **161**, 451 (1948).

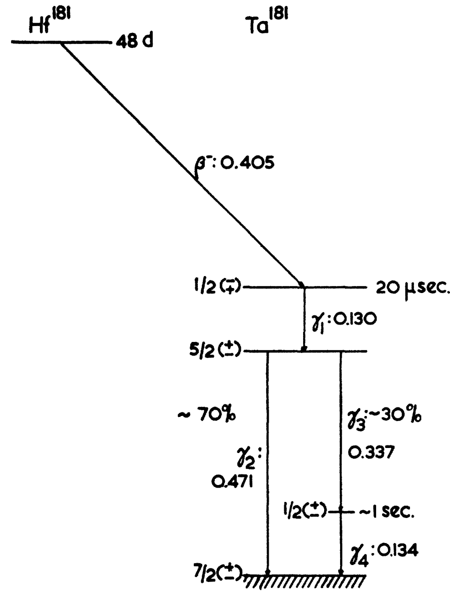

FIG. 2. The decay scheme of Hf^{181}.

Note added in proof: Comparison with recent discussions of nuclear shell structures (e.g., M. G. Mayer, Phys. Rev. **75**, 1969 (1949)) shows that the energy-levels of Ta^{181} all appear within the same shell. The states in order of increasing excitation energy are then supposed to be formed by the odd proton moving in $g_{7/2}$, $s_{1/2}$, $d_{3/2}$ or $d_{5/2}$ and $h_{11/2}$ orbits. The problem still arises as to why the direct transition from metastable to ground state does not occur. Special selection rules must apparently operate in this case.

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Multiple Scattering of Neutrons. II. Diffusion in a Plate of Finite Thickness

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The diffusion of neutrons in an infinite plate of finite thickness is studied. Analytic expressions are derived for the density and current of the returning and transmitted neutrons at the boundaries. Density and current distribution inside the material at sufficiently large distances from the boundaries are also calculated.

INTRODUCTION

IN I we succeeded[1] in obtaining a rigorous analytical solution for the density and current distributions of neutrons which have been impinging with an arbitrary velocity distribution upon a plate of infinite thickness; the neutrons were assumed to undergo elastic isotropic scattering processes and capture inside the material. We here extend the treatment under the same physical assumptions to the case of a plate of finite thickness; in the limit of very large thickness the results, of course, will have to agree with those of I. The solutions given here will be *asymptotically* valid if the thickness of the plate is large compared with the mean free path of the neutrons inside the material. They will, therefore, become rigorous for the limiting case of infinitely large thickness.

The treatment is here extended considerably farther than in the first paper. We obtain information not only

[1] This second paper (see Halpern, Luneburg, and Clark, Phys Rev. **53**, 173 (1938), referred to as I) appears belatedly due to reasons beyond the control of the authors; much of its content has been presented orally at an earlier opportunity (Phys. Rev. **56**, 1068 (1939)).

about the distribution functions at both boundaries but also about significant regions in the interior of the plate. It was, furthermore, possible to put the new transcendental function which represents the main part of the solution into a form which permits easy use in the evaluation of individual problems. Tables for the solution and auxiliary functions are given.

1. FORMULATION OF THE PROBLEM

The distribution function $w(x, \xi)$ satisfies the well-known transport equation (see I)

$$\xi(\partial w/\partial x)+Aw=B\bar{w}(x), \quad \bar{w}=\int_{-1}^{+1} wd\xi. \tag{1}$$

If the plate is located between $x=0$ and $x=l$, then we want to find a solution of (1) with the boundary conditions

$$w(0, \xi)=(f\xi), \quad \xi>0, \tag{1.1}$$

$$w(l, \xi)=0, \quad \xi<0. \tag{1.2}$$

Here, $f(\xi)$ is a known function.

It has been found useful in I to introduce a Laplace transformation of $w(x, \xi)$ or $\bar{w}(x)$. We here generalize the transformation by writing

$$u(z)=-B/z\int_0^l \bar{w}(x)e^{Ax/z}dx, \tag{1.3}$$

the upper limit being given by the actual thickness of the plate. Since we are in particular interested in the boundary values $w(0, \xi)$ and $w(l, \xi)$, it is important to notice that they are already determined by $u(z)$ and that we need to invert the Laplace transformation only if we are interested in values of w in the interior of the plate. This statement can be proven as follows: We first notice that (1) is solved by the integral equation

$$w(x, \xi)=e^{-(Ax/\xi)}\left[w(0, \xi)+B/\xi\int_0^x \bar{w}(x)e^{Ax/\xi}dx\right], \tag{1.4}$$

which gives, in particular,

$$w(l, \xi)=e^{-\lambda/\xi}\left[w(0, \xi)+B/\xi\int_0^l \bar{w}(x)e^{Ax/\xi}dx\right] \tag{1.41}$$

with the abbreviation $\lambda=Al$.

Introducing (1.3) into (1.41) one obtains

$$w(l, \xi)=e^{-(\lambda/\xi)}[w(0, \xi)-u(\xi)], \tag{1.5}$$

and specializing (1.5) for $\xi>0$ and $\xi<0$,

$$w(0, \xi)=u(\xi), \quad \xi<0 \tag{1.61}$$

$$w(l, \xi)=e^{-\lambda/\xi}[f(\xi)-u(\xi)], \quad \xi>0. \tag{1.62}$$

The relations (1.61) and (1.62) prove the statement made.

It may be noted that $u(z)$ is regular in the whole complex plane with the exception of the origin, which constitutes an essential singularity.

2. AN INTEGRAL EQUATION FOR $u(z)$

Introducing the function $v(\xi, z)$ by the relation

$$v(\xi, z)=-B/z\int_0^l w(x, \xi)e^{Ax/\xi}dx \tag{2.1}$$

so that

$$u(z)=\int_{-1}^{+1} v(\xi, z)d\xi, \tag{2.11}$$

one obtains from (2.1) and (1)

$$v(\xi, z)+\sigma\frac{z}{\xi-z}u(z)=-\frac{\sigma\xi}{\xi-z}[w(l, \xi)e^{\lambda/z}-w(0, \xi)] \tag{2.2}$$

with the abbreviation $\sigma=B/A$.

If we now put

$$\rho(z)=1+\sigma z\int_{-1}^{+1}\frac{d\xi}{\xi-z}=1+\sigma z\log\frac{z-1}{z+1}, \tag{2.3}$$

one obtains by integrating (2.2) over ξ the relation

$$\rho(z)u(z)=-\sigma\int_{-1}^{+1}\frac{\xi}{\xi-z}[w(l, \xi)e^{\lambda/z}-w(0, \xi)]d\xi. \tag{2.4}$$

If we now replace $w(l)$ and $w(0)$ in (2.4) by the expressions given for them in (1.61) and (1.62), one obtains an integral equation for $u(z)$,

$$\rho(z)u(z)=\sigma\int_0^1\frac{\xi b(\xi)}{\xi-z}(1-e^{\lambda/z-\lambda/\xi})d\xi+\sigma\int_{-1}^0\frac{\xi u(\xi)}{\xi-z}d\xi$$
$$+\sigma e^{\lambda/z}\int_0^1\frac{\xi u(\xi)e^{-\lambda/\xi}}{\xi-z}d\xi. \tag{2.5}$$

We assume without restriction of generality that $f(\xi)=\delta(\xi-z_0)$ and write (2.4) in the form

$$\rho(z)u(z)=\sigma\int_{-1}^0\frac{\xi w(0, \xi)}{\xi-z}d\xi-\sigma e^{\lambda/z}\int_0^1\frac{\xi w(l, \xi)}{\xi-z}d\xi$$
$$+\sigma\int_0^1\frac{\xi f(\xi)}{\xi-z}d\xi$$
$$=\sigma\int_{-1}^0\frac{\xi w(0, \xi)}{\xi-z}d\xi-\frac{\sigma z_0}{z-z_0}$$
$$-\sigma e^{\lambda/z}\int_0^1\frac{\xi w(l, \xi)}{\xi-z}d\xi\cdots. \tag{2.6}$$

If we define a pair of new functions $U(z)$ and $V(z)$ by the equations

$$U(z)=1+\frac{z_0-z}{z_0}\int_{-1}^{0}\frac{\xi w(0,\xi)}{\xi-z}d\xi, \qquad (2.61)$$

$$V(z)=\frac{z_0-z}{z_0}\int_{0}^{1}\frac{\xi w(l,\xi)}{\xi-z}d\xi, \qquad (2.62)$$

we can write (2.6) in the form

$$\rho(z)u(z)=\frac{\sigma z_0}{z_0-z}[U(z)-e^{\lambda/z}V(z)]. \qquad (2.7)$$

The functions U and V can also be expressed directly as integrals containing $u(z)$ by introducing (1.61) and (1.62) into (2.61) and (2.62)

$$U(z)=1+\frac{z_0-z}{z_0}\int_{-1}^{0}\frac{\xi u(\xi)}{\xi-z}d\xi, \qquad (2.71)$$

$$V(z)=e^{-\lambda/z_0}-\frac{z_0-z}{z_0}\int_{0}^{1}\frac{\xi u(\xi)e^{-\lambda/\xi}}{\xi-z}d\xi. \qquad (2.72)$$

Inspection of (2.71) and (2.72) proves that the function U and V are regular, respectively, in the right and left half of the complex plane.

Letting $z=0$, we obtain in (2.61) and (2.62)

$$U(0)=1+\int_{-1}^{0}w(0,\xi)d\xi, \qquad (2.81)$$

$$V(0)=\int_{0}^{1}w(l,\xi)d\xi. \qquad (2.82)$$

Equations (2.81) and (2.82) show that the neutron density at the near and far boundaries are determined, respectively, by $U(0)$ and $V(0)$.

Similarly, by letting $z\to\infty$, one obtains the relations

$$z_0[U(\infty)-1]=\int_{-1}^{0}\xi w(0,\xi)d\xi, \qquad (2.83)$$

$$z_0V(\infty)=\int_{0}^{1}\xi w(l,\xi)d\xi. \qquad (2.84)$$

Equations (2.83) and (2.84) express the returning and transmitted current in terms of $U(\infty)$ and $V(\infty)$.

3. REGULARITY CONDITIONS FOR U AND V

In analogy with the treatment of I we shall here show that the functions U and V are uniquely determined by the regularity conditions. For this purpose we rewrite (2.7) as follows:

$$\frac{1}{\rho(z)}[U(z)e^{-(\lambda/2z)}-V(z)e^{\lambda/2z}] =\frac{z_0-z}{\sigma z_0}U(z)e^{-(\lambda/2z)}\equiv K(z). \qquad (3.1)$$

The right side of (3.1) is everywhere regular except at the origin; it follows that

$$\frac{1}{\rho(z)}[U(z)e^{-(\lambda/2z)}-V(z)e^{\lambda/2z}]$$

can *only be singular at the origin*; since, furthermore, $K(z_0)=0$, it follows with the aid of (2.71) and (2.72),

$$U(z_0)e^{\lambda/2z_0}=V(z_0)e^{\lambda/2z_0}, \qquad (3.11)$$

$$U(z_0)=1, \qquad (3.12)$$

$$V(z_0)=e^{-(\lambda/z_0)}. \qquad (3.13)$$

Since $K(z)$ is everywhere regular except possibly at the origin, the same will hold true for its even and odd parts. Now, from (3.1) we have

$$K(z)+K(-z)=\frac{1}{\rho}[\{U(z)-V(-z)\}e^{-(\lambda/2z)} +\{U(-z)-V(z)\}e^{\lambda/2z}], \qquad (3.21)$$

$$K(z)-K(-z)=\frac{1}{\rho}[\{U(z)+V(-z)\}e^{-(\lambda/2z)} -\{U(-z)+V(z)\}e^{\lambda/2z}]. \qquad (3.22)$$

We introduce a pair of auxiliary functions which are both regular in the right hand of the complex plane $F(z)$ and $G(z)$,

$$F(z)=U(z)-V(-z), \qquad (3.31)$$

$$G(z)=U(z)+V(-z) \qquad (3.32)$$

and can, therefore, write (3.21), (3.22) as

$$K(z)+K(-z)=\frac{1}{\rho}[F(z)e^{-(\lambda/2z)}+F(-z)e^{\lambda/2z}], \qquad (3.41)$$

$$K(z)-K(-z)=\frac{1}{\rho}[G(z)e^{-(\lambda/2z)}-G(-z)e^{\lambda/2z}]. \qquad (3.42)$$

We thus obtain the following conditions for F and G:

(1) F and G are both regular in the right-half plane.

(2) The expressions given by the right sides of (3.41) and (3.42) are regular everywhere except possibly at the origin.

(3) $$\tfrac{1}{2}[F(z_0)+G(z_0)]=1, \qquad (3.43)$$

$$\tfrac{1}{2}[G(-z_0)-F(-z_0)]=e^{-(\lambda/2z_0)}. \qquad (3.44)$$

The conditions (3.43) and (3.44) are a consequence of (3.12), (3.13) and (3.31), (3.32).

The regularity conditions will turn out to be sufficient to determine uniquely F and G. Before proceeding to establish this result, we want to obtain a relation between F and G. Multiplying (3.41) by $Ge^{-(\lambda/2z)}$ and (3.42) by $Fe^{-(\lambda/2z)}$ and subtracting, one obtains the result that

$$1/\rho[G(z)F(-z)+F(z)G(-z)]$$

is regular in the right-half plane. Since it is an even function of z, it must, therefore, be regular in the whole z plane and thus equal to a constant C

$$G(z)F(-z)+F(z)G(-z)=C\rho(z). \tag{3.5}$$

4. INTEGRAL EQUATIONS FOR F AND G

We shall first find two functions F and G which satisfy the first two regularity conditions but not (3.43), (3.44). In place of (3.43), (3.44) we require that

$$\frac{F(\infty)}{[\rho(\infty)]^{\frac{1}{2}}}=\frac{G(\infty)}{[\rho(\infty)]^{\frac{1}{2}}}=1. \tag{4.1}$$

Let us denote these special functions by φ and ψ.

If we now split (as in I) ρ into factors,

$$\rho(z)=P(z)P(-z), \tag{4.2}$$

$P(z)$ being a regular function in the right-half plane, then we obtain by multiplying (3.41), (3.42) with $P(z)$ the result that the expressions

$$\frac{\varphi(z)}{P(-z)}e^{-(\lambda/z)}+\frac{\varphi(-z)}{P(-z)}-1, \tag{4.21}$$

$$\frac{\psi(z)}{P(-z)}e^{-(\lambda/z)}-\frac{\psi(-z)}{P(-z)}+1, \tag{4.22}$$

are regular in the right-half plane. The function

$$\rho(z)=1+\sigma z\log(z+1)/(z-1) \tag{4.23}$$

vanishes at the two points $z=\pm\alpha$. If $\sigma<\frac{1}{2}$ then α is real; if $\sigma>\frac{1}{2}$ then α is purely imaginary.

We now consider a closed path of integration L in the z plane which shall enclose the real axis between 0 and 1 as well as the point $+\alpha$. Replacing z in (4.21), (4.22) by s and multiplying by $[1/(2\pi i)][1/(s+z)]$, we integrate along the closed path L in the s-plane. It shall be assumed that the point $s=-z$ lies outside the closed path of integration. Then the integral is zero and we obtain the relations

$$\frac{\varphi(z)}{P(z)}=1-\frac{1}{2\pi i}\int_L\frac{\varphi(s)e^{-(\lambda/s)}}{P(-s)}\frac{ds}{s+z}, \tag{4.31}$$

$$\frac{\psi(z)}{P(z)}=1+\frac{1}{2\pi i}\int_L\frac{\psi(s)e^{-(\lambda/s)}}{P(-s)}\frac{ds}{s+z}. \tag{4.32}$$

By making

$$\varphi(z)=a(z)P(z), \tag{4.41}$$

$$\psi(z)=b(z)P(z), \tag{4.42}$$

we obtain

$$a(z)=1-\frac{1}{2\pi i}\int_L\frac{P(s)}{P(-s)}e^{-(\lambda/s)}\frac{a(s)}{s+z}ds, \tag{4.51}$$

$$b(z)=1+\frac{1}{2\pi i}\int_L\frac{P(s)}{P(-s)}e^{-(\lambda/s)}\frac{b(s)}{s+z}ds. \tag{4.52}$$

We now contract the path of integration to a small loop around the section (0, 1) of the real axis, leaving α outside. This gives us

$$a(z)=1+\frac{P^2(\alpha)}{\rho'(\alpha)}e^{-(\lambda/\alpha)}\frac{a(\alpha)}{\alpha+z}-\int_0^1\frac{\tau(s)e^{-(\lambda/s)}a(s)}{s+z}ds, \tag{4.61}$$

$$b(z)=1-\frac{P^2(\alpha)}{\rho'(\alpha)}e^{-(\lambda/\alpha)}\frac{b(\alpha)}{\alpha+z}+\int_0^1\frac{\tau(s)e^{-(\lambda/s)}b(s)}{s+z}ds \tag{4.62}$$

in which $\tau(s)$ stands for the discontinuity of $P(s)/P(-s)$ on the real axis.

The functions a and b thus satisfy two regular integral equations; we know from general theorems that there exists *only one* solution if the associated homogenous equations do not happen to have any other but the trivial solutions $a=b=0$.

We can now construct with the aid of

$$\varphi(z)=P(z)a(z),\quad \psi(z)=P(z)b(z),$$

the pair of functions F and G satisfying (3.43), (3.44) as follows: The first two regularity conditions are satisfied by

$$F(z)=C_1P(z)a(z), \tag{4.71}$$

$$G(z)=C_2P(z)b(z), \tag{4.72}$$

with arbitrary constants C_1 and C_2. These two constants can now be used to satisfy (3.43) and (3.44) also. For this purpose we insert (3.31) and (3.32) into (3.12) and (3.13) and obtain, after adding and subtracting,

$$C_1\varphi(z_0)+C_2\psi(z_0)=2, \tag{4.73}$$

$$-C_1\varphi(-z_0)+C_2\psi(-z_0)=2e^{\lambda/z_0}. \tag{4.74}$$

These equations give for C_1 and C_2 the expressions

$$C_1=\frac{1}{\rho(z_0)}[\psi(-z_0)-\psi(z_0)e^{-(\lambda/z_0)}], \tag{4.75}$$

$$C_2=\frac{1}{\rho(z_0)}[\varphi(-z_0)+\varphi(z_0)e^{-(\lambda/z_0)}]. \tag{4.76}$$

In (4.75) and (4.76) use has been made of (3.5) which now reads

$$\psi(z)\varphi(-z)+\varphi(z)\psi(-z)=2\rho(z). \tag{4.77}$$

The explicit expressions for F and G can now be written down:

$$F(z)=\frac{P(z)a(z)}{\rho(z_0)}[P(-z_0)b(-z_0)-P(z_0)b(z_0)e^{-(\lambda/z_0)}], \tag{4.81}$$

$$G(z)=\frac{P(z)b(z)}{\rho(z_0)}[P(-z_0)a(-z_0)+P(z_0)a(z_0)e^{-(\lambda/z_0)}], \tag{4.82}$$

$$\frac{2U(z)}{P(z)}=a(z)\left[\frac{b(-z_0)}{P(z_0)}-\frac{b(z_0)e^{-(\lambda/z_0)}}{P(-z_0)}\right]+b(z)\left[\frac{a(-z_0)}{P(z_0)}+\frac{a(z_0)e^{-(\lambda/z_0)}}{P(-z_0)}\right], \quad (4.83)$$

$$\frac{2V(z)}{P(-z)}=-a(-z)\left[\frac{b(-z_0)}{P(z_0)}-\frac{b(z_0)e^{-(\lambda/z_0)}}{P(-z_0)}\right]+b(-z)\left[\frac{a(-z_0)}{P(z_0)}+\frac{a(z_0)e^{-(\lambda/z_0)}}{P(-z_0)}\right]. \quad (4.84)$$

The formulas (4.83) and (4.84) together with (2.7) express $u(z)$ as function of $a(z)$ and $b(z)$; a simple substitution would lead to an explicit though somewhat lengthy equation for $u(z)$. Later applications make it appear advisable to express $u(z)$ with the aid of two auxiliary functions $f(z)$ and $g(z)$ which in turn are defined with the aid of $a(z)$ and $b(z)$. We write:

$$f(z)=\frac{a(-z)}{P(z)}e^{\lambda/2z}+\frac{a(z)}{P(-z)}e^{-(\lambda/2z)}, \quad (4.85)$$

$$g(z)=\frac{b(-z)}{P(z)}e^{\lambda/2z}-\frac{b(z)}{P(-z)}e^{-(\lambda/2z)}. \quad (4.86)$$

The combination of Eqs. (4.83) to (4.86) and (2.7) leads directly to the wanted expression for $u(z)$:

$$u(z)=\frac{\sigma z_0}{2(z_0-z)}e^{\lambda/2z-\lambda/2z_0}[f(z)g(z_0)-f(z_0)g(z)]. \quad (4.9)$$

5. ASYMPTOTIC EXPRESSIONS

Equations (4.61) and (4.62) can immediately be solved for a plate of infinite thickness; in that case, the integral disappears on account of $e^{-\lambda/s}\rightarrow 0$. We can also derive an asymptotic solution of (4.61) and (4.62) valid for all cases in which

$$e^{-\lambda}\ll e^{-(\lambda/\alpha)}. \quad (5.1)$$

In this latter case, the integral becomes small and we obtain asymptotically

$$a(z)=1+\frac{2\alpha S(\alpha)e^{-(\lambda/\alpha)}}{\alpha+z}a(\alpha), \quad (5.11)$$

$$b(z)=1-\frac{2\alpha S(\alpha)e^{-(\lambda/\alpha)}}{\alpha+z}b(\alpha) \quad (5.12)$$

with the abbreviation

$$S(\alpha)=\frac{1}{2\alpha}\frac{P^2(\alpha)}{\rho'(\alpha)}. \quad (5.13)$$

$a(\alpha)$ and $b(\alpha)$ are in this approximation given by

$$a(\alpha)=1/[1-S(\alpha)e^{-(\lambda/\alpha)}], \quad (5.14)$$

$$b(\alpha)=1/[1+S(\alpha)e^{\lambda/\alpha}], \quad (5.15)$$

so that we finally can write

$$a(z)=1+\frac{2\alpha S(\alpha)e^{-(\lambda/\alpha)}}{1-S(\alpha)e^{-(\lambda/\alpha)}}\frac{1}{\alpha+z}, \quad (5.21)$$

$$b(z)=1-\frac{2\alpha S(\alpha)e^{-(\lambda/\alpha)}}{1+S(\alpha)c^{-(\lambda/\alpha)}}\frac{1}{\alpha+z}. \quad (5.22)$$

We are primarily interested in $w(0, \xi)$ and $w(l, \xi)$, which are given by $u(\xi)$ and $u(\xi)e^{-(\lambda/\xi)}$, respectively, (cf. (1.61) and (1.62)) and, therefore, need to construct asymptotic expressions for

$u(z)$ in the case $-1<z<0$,
$u(z)e^{-(\lambda/z)}$ in the case $0<z<1$,

or, referring to (4.9),

$f(z)e^{\lambda/2z}$ and $g(z)e^{\lambda/2z}$ for $-1<z<0$,
$f(z)e^{-(\lambda/2z)}$ and $g(z)e^{-(\lambda/2z)}$ for $0<z<1$.

Combining (4.85), (4.86), (5.21), and (5.22) we obtain $-1<z<0$:

$$b(z)e^{\lambda/2z}\sim\frac{a(z)}{P(-z)}\sim\frac{1}{P(-z)}\left[1+\frac{2\alpha S(\alpha)e^{-(\lambda/\alpha)}}{1-S(\alpha)e^{-(\lambda/\alpha)}}\frac{1}{\alpha+z}\right], \quad (5.31)$$

$$g(z)e^{\lambda/2z}\sim-\frac{b(z)}{P(z)}\sim-\frac{1}{P(-z)}\left[1-\frac{2\alpha Se^{-(\lambda/\alpha)}}{1+Se^{-(\lambda/\alpha)}}\frac{1}{\alpha+z}\right]. \quad (5.32)$$

$0<z<1$:

$$b(z)e^{-(\lambda/2z)}\sim\frac{a(-z)}{P(z)}\sim\frac{1}{P(z)}\left[1+\frac{2\alpha Se^{-(\lambda/\alpha)}}{1-Se^{-(\lambda/\alpha)}}\frac{1}{\alpha-z}\right], \quad (5.41)$$

$$g(z)e^{-(\lambda/2z)}\sim\frac{b(-z)}{P(z)}\sim\frac{1}{P(z)}\left[1-\frac{2\alpha Se^{-(\lambda/\alpha)}}{1+Se^{-(\lambda/\alpha)}}\frac{1}{\alpha-z}\right]. \quad (5.42)$$

If we introduce these last asymptotic expressions into (4.9), we obtain the final expressions

$$w(0,z)=\frac{\sigma z_0}{(z_0-z)P(-z)P(z_0)}-\frac{2\sigma z_0\alpha S^2(\alpha)e^{-(2\lambda/\alpha)}}{1-S^2e^{-(2\lambda/\alpha)}}\times\frac{1}{(\alpha-z_0)(\alpha+z)}\frac{1}{P(z_0)}\frac{1}{P(-z)}, \quad (5.5)$$

$$w(l,z)=\frac{2\sigma z_0\alpha Se^{-(\lambda/\alpha)}}{1-S^2e^{-(2\lambda/\alpha)}}\frac{1}{(\alpha-z_0)\cdot(\alpha-z)}\frac{1}{P(z_0)}\frac{1}{P(z)}. \quad (5.6)$$

Equations (5.5) and (5.6) permit a simple interpretation. For $\lambda\rightarrow\infty$ the second term in (5.5), disappears and we obtain the result derived in I. Equation (5.6) gives an explicit expression for the transmitted density distribution.

6. LIMITING CASE OF VERY SMALL CAPTURE

In the case of $\sigma\rightarrow\frac{1}{2}$, as given by (cf. 4.23)

$$\sigma=\frac{1}{\alpha\log(\alpha+1)/(\alpha-1)},\qquad(6.1)$$

α becomes very large; we, therefore, shall look for an expansion in negative powers of α. We find for $S(\alpha)$ (cf. (5.13)), treating the denominator first:

$$\alpha^3\rho'(\alpha)=\frac{\alpha^2}{(\alpha/2)\log(\alpha+1)/(\alpha-1)}\times\left[\frac{1}{1-(1/\alpha^2)}-(\alpha/2)\log\frac{\alpha+1}{\alpha-1}\right],\qquad(6.2)$$

or up to terms of order $1/\alpha^2$.

$$\alpha^3\rho'(\alpha)=\frac{2}{3}\left[1+\frac{13}{15a^2}\right].\qquad(6.21)$$

To expand $\alpha^2P^2(\alpha)$, we use the relation (cf. I, (37))

$$P(z)=(1-2\sigma)^{\frac{1}{2}}+\sigma\int_0^1\frac{s}{s+z}\frac{ds}{P(s)}.\qquad(6.22)$$

If we add $P(-z)$ and put $z=\alpha$, then we obtain, because of $P(-\alpha)=0$, the relation

$$P(\alpha)=2(1-2\sigma)^{\frac{1}{2}}+2\sigma\int_0^1\frac{s^2}{s^2-\alpha^2}\frac{ds}{P(s)}.\qquad(6.23)$$

Now (6.1) gives

$$2\alpha(1-2\sigma)^{\frac{1}{2}}\sim\frac{2}{\sqrt{3}}\left(1+\frac{2}{15\alpha^2}\right),\qquad(6.24)$$

and it then follows from (6.23)

$$\alpha\left[\alpha P(\alpha)-\frac{2}{\sqrt{3}}\right]\sim-\int_0^1\frac{s^2ds}{P(s)}\qquad(6.25)$$

in which the integral has to be taken for $\sigma=\frac{1}{2}$. We introduce a numerical constant m by the relation

$$m=\sqrt{3}\int_0^1\frac{s^2ds}{P(s)}=2\left[1-\frac{1}{\pi}\int_0^{\pi/2}\frac{\log3(1-t\operatorname{ctg}t)/\sin^2t}{\sin^2t}dt\right],\qquad(6.26)$$

m can be found from I to be equal to 1.43. Thus we have

$$\alpha P(\alpha)=(2/\sqrt{3})[1-(m/2\alpha)],\qquad(6.27)$$

and from (6.27), (6.21), and (5.13),

$$\int S(\alpha)=1-(m/\alpha).\qquad(6.28)$$

Equations (5.5) and (5.6), therefore, take on the following form for $\sigma\rightarrow\frac{1}{2}$ and $\alpha\rightarrow\infty$,

$$w(0,z)=\frac{z_0}{2(z_0-z)}\frac{1}{P(-z)}\frac{1}{P(z_0)}-\frac{z_0}{2(m+\lambda)}\frac{1}{P(z_0)P(-z)},\qquad(6.3)$$

$$w(l,z)=\frac{z_0}{2(m+\lambda)}\frac{1}{P(z_0)P(z)}.\qquad(6.4)$$

7. SOME EXPRESSIONS FOR $P_\sigma(z)$ FOR $\sigma\sim\frac{1}{2}$

The new transcendental function $P(z)$, which plays a fundamental role in our theory, has been studied in great detail by one of the authors (R.K.L.). We give here a few results of this mathematical investigation which may be useful for applications. We also add in form of tables certain numerical data which will be of assistance in the use of the theory.

The function $P_\sigma(z)$ can be represented for values of σ in the neighborhood of $\frac{1}{2}$ by the following expression:

$$1/[P_\sigma(z)]=[Q_\sigma(z)]/[P_0(z)].\quad P_0(z)=P(z)\sigma=\tfrac{1}{2}.\qquad(7.1)$$

Here, $Q_\sigma(z)$ is defined by

$$Q_\sigma(z)=\frac{1}{1+(q_1/P_0)(1-2\sigma)^{\frac{1}{2}}+(q_2/P_0)(1-2\sigma)+(q_3/P_0)(1-2\sigma)^{\frac{3}{2}}}.\qquad(7.2)$$

The functions q_i can be determined as polynomials of z multiplied by P_0 and its first derivative. These expansions are valid for $z<1$. We present in Table I numerical values for $P_\sigma^{-1}(z)$, (q_i/P_0) for values of z between 0 and 1. Table II contains values of $P_\sigma^{-1}(z)$ as function of z in the range between 0 and 1 and of σ in the range between 0.5 and 0.4.

We also state the result that the straight line which approximates P_0 with the smallest quadratic deviation is given by

$$1/P_0=1.08+1.84z.\qquad(7.3)$$

8. INVERSION OF THE LAPLACE TRANSFORMATION

While the distribution at the boundaries is, according to (1.61) and (1.62), given by $u(z)$, knowledge of the conditions inside the scatterer requires explicit determination of w. Since, according to (1.3),

$$u(z)=-\frac{B}{z}\int_0^l\bar{w}(x)e^{Ax/z}dx,\qquad(1.3)$$

TABLE I. Values of $P_0^{-1}(z)$ and of q_i/P_0.

$\diagdown z$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$P_0^{-1}(z)$	1.000	1.241	1.446	1.640	1.827	2.011	2.193	2.373	2.552	2.730	2.907
q_1/P_0	0.000	0.173	0.346	0.520	0.693	0.866	1.039	1.212	1.386	1.559	1.732
q_2/P_0	0.000	0.113	0.162	0.189	0.214	0.236	0.252	0.260	0.270	0.278	0.286
q_3/P_0	0.000	−0.050	−0.082	−0.109	−0.129	−0.142	−0.154	0.169	−0.180	−0.190	−0.198

or with $A/z=-\gamma$,

$$\frac{1}{\sigma\gamma}u(-A/\gamma)=\int_0 \bar{w}(x)e^{-\gamma x}dx, \tag{8.1}$$

we see that $(1/\sigma\gamma)u(-A/\gamma)$ is the Laplace adjoint of $\bar{w}(x)$. The inversion gives $\bar{w}$ in the form

$$\bar{w}(x)=(1/2\pi i\sigma)\int_L u(-A/\gamma)e^{\gamma x}(d\gamma/\gamma). \tag{8.2}$$

Here L can be the imaginary axis or any line parallel to it; we can also deform the path of integration into any closed curve L_1 which stays in the *left*-half plane and intersects the real axis in the origin with an angle of 90°. This is permissible because $u(z)$ is regular in the left-half plane. If $\bar{w}$ is determined from the inversion theorem, we obtain an expression for w by introducing (8.2) into (1.4) and integrating over x,

$$w(x,\,\xi)=w(0,\,\xi)e^{-(Ax/\xi)}+\frac{1}{2\pi i}\int_{L_1}\frac{u(s)}{s-\xi}(e^{-(Ax/s)}-e^{-(Ax/\xi)})ds. \tag{8.3}$$

Since the curve L_1 can be chosen arbitrarily small, we may construct it so that the point ξ is excluded. Then the second term on the right side of (8.3) becomes simply

$$\frac{1}{2\pi i}\int_{L_1}\frac{u(s)}{s-\xi}e^{-(Ax/s)}ds. \tag{8.31}$$

If $\xi>0$ we obtain w directly from 8.31

$$w(x,\,\xi)=f(\xi)e^{-(Ax/\xi)}+\frac{1}{2\pi i}\int_{L_1}\frac{u(s)e^{-(Ax/s)}}{s-\xi}ds. \tag{8.41}$$

For $\xi<0$ we have $w(0,\,\xi)=u(\xi)$ and, therefore,

$$w(x,\,\xi)=u(\xi)e^{-(Ax/\xi)}+\frac{1}{2\pi i}\int_{L_1}\frac{u(s)e^{-(Ax/s)}}{s-\xi}ds \tag{8.42}$$

or

$$w(x,\,\xi)=\frac{1}{2\pi i}\int_{L_2}\frac{u(s)e^{-(Ax/s)}}{s-\xi}ds,$$

where L_2 *includes* the point $s=\xi$. In both cases the imaginary axis is equivalent to L_1 and L_2. This gives

$\xi<0$:

$$w(x,\,\xi)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{u(s)e^{-(Ax/s)}ds}{s-\xi}. \tag{8.5}$$

$\xi>0$:

$$w(x,\,\xi)=f(\xi)e^{-(Ax/\xi)}+\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{u(s)e^{-(Ax/s)}}{s-\xi}ds. \tag{8.6}$$

9. DISTRIBUTION INSIDE A PLATE OF INFINITE THICKNESS: ASYMPTOTIC FORMULAS

If the thickness of the plate becomes infinite, we find after some simple calculations

$$u(z)=\frac{\sigma z_0}{z_0-z}\frac{1}{P(z_0)}\frac{1}{P(-z)}. \tag{9.1}$$

$\xi>0$:

$$w(x,\,\xi)=\delta(\xi-z_0)e^{-(Ax/\xi)}-\frac{\sigma z_0}{P(z_0)}\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{e^{Ax/s}ds}{(s+\xi)(s+z_0)P(s)}. \tag{9.2}$$

$\xi<0$:

$$w(x,\,\xi)=-\frac{\sigma z_0}{P(z_0)}\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{e^{Ax/s}ds}{(s+\xi)(s+z_0)P(x)}; \tag{9.3}$$

and from (9.1) and (8.2),

$$\bar{w}(x)=-\frac{z_0}{P(z_0)}\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{e^{Ax/s}ds}{s(s+z_0)P(s)}. \tag{9.4}$$

Instead of integrating along the imaginary axis, we can also integrate along a closed curve L_1 which includes all singularities of $1/P$ in the left half of the complex plane; these are the point $s=-\alpha$ and the sector $-1\leqq s\leqq 0$. The curve L_1 can in turn be split into two separate closed curves L_0 and L' including the point $s=-\alpha$ and the section $(-1,0)$ separately. For *large* values of Ax, the main contribution comes from the curve L_0; the integral around it will, therefore, give us

TABLE II. Values of $P_\sigma^{-1}(z)$.

$\sigma\diagdown z$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.500	1.00	1.24	1.45	1.64	1.83	2.01	2.19	2.37	2.55	2.73	2.91
0.499	1.00	1.23	1.42	1.60	1.77	1.94	2.10	2.25	2.40	2.55	2.70
0.498	1.00	1.23	1.41	1.59	1.75	1.90	2.06	2.20	2.35	2.49	2.62
0.496	1.00	1.22	1.40	1.57	1.72	1.86	2.00	2.13	2.27	2.39	2.52
0.494	1.00	1.22	1.39	1.55	1.69	1.84	1.96	2.09	2.21	2.32	2.44
0.492	1.00	1.21	1.38	1.54	1.67	1.81	1.93	2.05	2.17	2.27	2.38
0.490	1.00	1.21	1.37	1.53	1.66	1.79	1.90	2.01	2.13	2.22	2.32
0.485	1.00	1.20	1.36	1.50	1.62	1.74	1.85	1.95	2.05	2.14	2.23
0.480	1.00	1.19	1.35	1.48	1.59	1.70	1.80	1.89	1.98	2.06	2.14
0.470	1.00	1.18	1.32	1.44	1.55	1.64	1.73	1.81	1.88	1.96	2.02
0.460	1.00	1.17	1.30	1.41	1.51	1.60	1.67	1.74	1.81	1.87	1.92
0.450	1.00	1.17	1.29	1.39	1.48	1.55	1.63	1.69	1.75	1.80	1.85
0.440	1.00	1.16	1.27	1.37	1.45	1.52	1.58	1.64	1.70	1.75	1.79
0.430	1.00	1.15	1.26	1.35	1.42	1.49	1.55	1.60	1.65	1.69	1.73
0.420	1.00	1.15	1.25	1.33	1.40	1.46	1.52	1.57	1.61	1.65	1.68
0.410	1.00	1.14	1.24	1.31	1.38	1.44	1.49	1.53	1.58	1.61	1.64
0.400	1.00	1.13	1.23	1.30	1.36	1.41	1.46	1.50	1.54	1.57	1.60

an asymptotic expression for w. Remembering that

$$\frac{1}{P'(\alpha)}=\frac{P(\alpha)}{\rho'(\alpha)},$$

we obtain easily

$$w(x,\xi)=\frac{\sigma z_0}{P(z_0)}\frac{P(\alpha)}{\rho'(\alpha)}\frac{e^{-(Ax/\alpha)}}{(\alpha-\xi)(\alpha-z_0)},\tag{9.5}$$

$$\bar{w}(x)=\frac{z_0}{P(z_0)(\alpha-z_0)}\frac{P(\alpha)}{\alpha\rho'(\alpha)}e^{-(Ax/\alpha)}.\tag{9.6}$$

If we allow α to approach ∞ $(\sigma\rightarrow\frac{1}{2})$, we thus obtain

$$w(x,\xi)=\frac{\sqrt{3}}{2}\frac{z_0}{P(z_0)},\tag{9.7}$$

$$\bar{w}(x)=\sqrt{3}\frac{z_0}{P(z_0)}.\tag{9.8}$$

10. THE DISTRIBUTION INSIDE A PLATE OF FINITE THICKNESS

$u(z)$ is in this general case given by (4.9). Introducing (4.9) into (8.41) and (8.42) and replacing x by xl, we obtain the following rigorous solution for $w(x,\xi)$

$$w(x,\xi)=\frac{\sigma z_0}{2}e^{-(\lambda/2z_0)}[g(z_0)I_1-f(z_0)I_2]+\begin{cases} f(\xi)e^{-(\lambda x/\xi)} & \xi>0\\ 0 & \xi<0.\end{cases}\tag{10.1}$$

In (10.1) the following notation has been used:

$$I_1(x,\xi)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{f(s)e^{(\lambda/2s)-(\lambda x/s)}ds}{(z_0-s)(s-\xi)},\tag{10.21}$$

$$I_2(x,\xi)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{g(s)e^{(\lambda/2s)-(\lambda x/s)}ds}{(z_0-s)(s-\xi)}.\tag{10.22}$$

If we now use the expressions for f and g as given by (4.85) and (4.86), we have

$$2\pi iI_1(x,\xi)=\int_{-i\infty}^{+i\infty}ds\left[\frac{a(-s)e^{\lambda(1-x)/s}}{P(s)(z_0-s)(s-\xi)}+\frac{a(s)e^{\lambda x/s}}{P(-s)(z_0-s)(s-\xi)}\right],\tag{10.31}$$

$$2\pi iI_2(x,\xi)=\int_{-i\infty}^{+i\infty}ds\left[\frac{b(-s)e^{\lambda(1-x)/s}}{P(s)(z_0-s)(s-\xi)}-\frac{b(s)e^{-(\lambda x/s)}}{P(-s)(s-\xi)(z_0-s)}\right].\tag{10.32}$$

The path of integration can now be replaced by new paths, namely the curves L_1 in the first term and L_2 in the second term of the right side. The curves $L_1(L_2)$ lies in the left- (right-) half plane only and consist of two parts enclosing the sector $-1, 0(0, 1)$ and the point $-\alpha(+\alpha)$. We now assume λ to be so large that

$$e^{-(\lambda x)/\alpha}\ll e^{-\lambda x}.$$

Then the contributions in $L_1(L_2)$ coming from the sections $-1, 0(0, 1)$ will be of the order of $e^{-\lambda x}(e^{-\lambda(1-x)})$ and can be neglected compared with the contributions from the integration around $-\alpha(+\alpha)$. Under these assumptions we have

$$I_1(x,\xi)=\frac{a(\alpha)P(\alpha)}{\rho'(\alpha)}\times\left[\frac{e^{-\lambda(1-x)/\alpha}}{(\alpha+z_0)(\alpha+\xi)}+\frac{e^{-(\lambda x/\alpha)}}{(\alpha-z_0)(\alpha-\xi)}\right],\tag{10.41}$$

$$I_2(x,\xi)=\frac{b(\alpha)P(\alpha)}{\rho'(\alpha)}\times\left[\frac{e^{-\lambda(1-x)/\alpha}}{(\alpha+z_0)(\alpha+\xi)}-\frac{e^{-(\lambda x/\alpha)}}{(\alpha-z_0)(\alpha-\xi)}\right].\tag{10.42}$$

Since the term $\delta(z_0-\xi)e^{-(\lambda x)/\xi}$ can also be neglected, we find from (10.41), (10.42), and (10.1)

$$w(x,\xi)=\frac{\sigma z_0}{2}\frac{P(\alpha)}{\rho'(\alpha)}e^{-(\lambda/2z_0)}\times\left[\frac{g(z_0)a(\alpha)-f(z_0)b(\alpha)}{(\alpha+z_0)(\alpha+\xi)}e^{-\lambda(1-x)/\alpha}+\frac{g(z_0)a(\alpha)+f(z_0)b(\alpha)}{(\alpha-z_0)(\alpha-\xi)}e^{-(\lambda x/\alpha)}\right].\tag{10.5}$$

In order to determine the expressions within the brackets, we use the previously derived asymptotic expressions for a, b, f, g. These formulas were obtained under the assumption that $e^{-\lambda}\ll e^{-\lambda/\alpha}$; terms of the order $e^{-\lambda/\alpha}$ are retained in them. This procedure is only consistent if $e^{-\lambda/\alpha}\gg e^{-\lambda x}$, since in (10.5) terms of the order of $e^{-\lambda x}$ have been neglected. We, therefore, have to require

$$\lambda/\alpha<\lambda x,\quad x>1/\alpha.$$

Similarly,

$$\lambda/\alpha<\lambda(1-x),\quad (1-x)>1/\alpha.$$

This leaves as conditions for the validity of the asymptotic approximations

$$1/\alpha<x<1-1/\alpha\tag{10.51}$$

or $\alpha>2$. The formulas will, therefore, be valid only sufficiently far *inside* the scatterer but not near the boundaries $x=0$ and $x=1$. For these cases we have already obtained expressions before.

Assuming (10.51) to be valid, we now collect the expressions:

$$a(\alpha)=1/(1-Se^{-\lambda/\alpha}),\quad b(\alpha)=1/(1+Se^{-\lambda/\alpha}),$$

$$f(z_0)e^{-(\lambda/2z_0)}=\frac{1}{P(z_0)}\left[1+\frac{2\alpha Se^{-(\lambda/\alpha)}}{(1-Se^{-(\lambda/\alpha)})(\alpha-z)}\right],\quad (10.52)$$

$$g(z_0)e^{-(\lambda/2z_0)}=\frac{1}{P(z_0)}\left[1-\frac{2\alpha Se^{-(\lambda/\alpha)}}{(1+Se^{-(\lambda/\alpha)})(\alpha-z)}\right].\quad (10.53)$$

Introducing these expressions into (10.5), we have

$$e^{-(\lambda/2z_0)}[g(z_0)a(\alpha)-f(z_0)b(\alpha)]=\frac{-2Se^{-(\lambda/\alpha)}(\alpha+z_0)}{(1-S^2e^{-(2\lambda/\alpha)})(\alpha-z_0)}\frac{1}{P(z_0)},\quad (10.54)$$

$$e^{-(\lambda/z_0)}[g(z_0)a(\alpha)+f(z_0)b(\alpha)]=\frac{2}{P(z_0)(1-S^2e^{-(2\lambda/\alpha)})},\quad (10.55)$$

and thus finally for w

$$w(x,\,\xi)=\frac{z_0}{P(z_0)(\alpha-z_0)}\frac{2\sigma\alpha S}{P(\alpha)(1-S^2e^{-(2\lambda/\alpha)})}\times\left[\frac{e^{-(\lambda x/\alpha)}}{\alpha-\xi}-Se^{-\lambda/\alpha}\frac{e^{-\lambda(1-x)/\alpha}}{\alpha+\xi}\right]\quad (10.6)$$

or by integrating over ξ,

$$\bar{w}(x)=\frac{z_0}{P(z_0)(\alpha-z_0)}\frac{2S(\alpha)}{P(\alpha)(1-S^2e^{-2\lambda/\alpha})}\times[e^{-(\lambda x/\alpha)}-Se^{-\lambda/\alpha}e^{-\lambda(1-x)/\alpha}].\quad (10.61)$$

In the special case of $x=\frac{1}{2}$, we have

$$\bar{w}(\tfrac{1}{2})=\frac{z_0}{(\alpha-z_0)P(z_0)}\frac{2Se^{-(\lambda/2\alpha)}}{P(\alpha)(1+Se^{-(\lambda/\alpha)})}.\quad (10.62)$$

Let us now compare this density, valid in the middle of a plate of thickness l, with the density at the far boundary of the plate of thickness $l/2$. This last quantity is given by (5.6) after integration over ξ:

$$\bar{w}(l/2)=\frac{z_0}{(\alpha-z_0)P(z_0)}\frac{2\alpha\sigma Se^{-(\lambda/2\alpha)}}{1-S^2e^{-(\lambda/\alpha)}}\times\int_0^1\frac{dz}{(\alpha-z)P(z)}.\quad (10.63)$$

To evaluate the definite integral in (10.63), we remember that

$$P(z)=(1-2\sigma)^{\frac{1}{2}}+\sigma\int_0^1\frac{sds}{(s+z)P(s)}=1-\sigma z\int_0^1\frac{ds}{(s+z)P(s)},\quad (10.64)$$

or for $z=-\alpha$,

$$\int_0^1\frac{ds}{(\alpha-s)P(s)}=\frac{1}{\sigma\alpha}.\quad (10.65)$$

Thus,

$$\bar{w}(l/2)=\frac{z_0}{(\alpha-z_0)P(z_0)}\frac{2Se^{-(\lambda/2\alpha)}}{1-S^2e^{-\lambda/\alpha}}.\quad (10.66)$$

The ratio $\bar{w}(\frac{1}{2})/\bar{w}(l/2)$ is given by

$$\frac{\bar{w}(\frac{1}{2})}{\bar{w}(l/2)}=\frac{1-S^2e^{-\lambda/\alpha}}{P(\alpha)(1+Se^{-\lambda/\alpha})}.\quad (10.67)$$

For large α, but $e^{-\lambda/2\alpha}\ll 1$, we then obtain

$$\bar{w}(1/2)/\bar{w}(l/2)\sim(\alpha/2)\sqrt{3}.\quad (10.68)$$

11. APPLICATIONS OF THE THEORY

The theory here developed obviously lends itself to the evaluation of a large number of experiments dealing with diffusion of neutrons. So far as we know, no investigations are reported in the literature which have been made with sufficient care of the geometrical conditions so as to allow immediate evaluation. It is to be expected that the presently available intensities of neutron beams will soon permit a number of investigations of interest in this field.

One of the authors (O.H.) has extended the present theory to include the treatment of the diffusion of polarized neutrons under the influence of spin dependent forces.[2] More detailed calculations will be reported shortly in a following paper.

[2] Otto Halpern, Phys. Rev. 75, 1633A (1949).