One interesting experiment would be to study the reflection of a magneto-hydrodynamic whirl-ring at a non-conducting surface. Especially important for the theory of sunspots is the behavior of rings parallel to the magnetic field.

The generation of the whirl-rings by convection also ought to be studied.

The experiments with waves in mercury have given the impression that all magneto-hydrodynamic phenomena have a quite different character than the corresponding purely hydrodynamic phenomena. Perhaps many theories on hydrodynamic problems in cosmical physics have to be reconsidered.

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I want to express my gratitude to Professor H. Alfvén, who has suggested this work.

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The Decay Scheme of Hf^{181}

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The radiations of Hf^{181} have been studied with delayed coincidence and absorption techniques. No γ-rays are found to precede the formation of the 20-microsecond metastable state in Ta^{181}, thus confirming the decay scheme of Chu and Wiedenbeck. From the values of the internal conversion coefficients together with delayed coincidence absorption measurements it is concluded that the previously reported 0.134-Mev γ-ray is most probably emitted from another metastable state of about one-second duration in Ta^{181}. A theoretical discussion of the multipole order of the different radiations lead to assignments of probable spin and relative parity values to the excited states of Ta^{181}. Beginning with the ground state, the deduced spin and parity values are: $7/2(\pm)$, $1/2(\pm)$, $5/2(\pm)$, and $1/2(\mp)$.

SOME uncertainty exists as to the succession of the γ-transitions in Ta^{181}, following the β^--decay of Hf^{181}. The metastable state in Ta^{181} of 20-μsec. duration was first observed by DeBenedetti and McGowan,[1] and later studied by Bunyan *et al.*[2] The β-rays, γ-rays, and conversion electrons have been subject to investigations by several authors,[3] some of which claim to have observed a γ-transition preceding the formation of the metastable state.

With the delayed coincidence recorder in Fig. 1 we have remeasured the half-life of the metastable state in Ta^{181} [4] with both the differential and integral procedures, the results being in agreement with earlier determinations. Inserting a 1-mm lead absorber between the source and the Geiger counter in the B channel and then in the A channel, we find a genuine delayed coincidence rate of 1.5 min.^{-1} (corresponding to infinitely long resolving time) and less than 10^{-2} min.^{-1}, respectively. With the same Pb absorbers in front of both Geiger counters, we could detect neither delayed nor instantaneous coincidences exceeding the random coincidences and in the latter case the background coincidence rate due to cosmic rays and other instantaneous coincidence sources. We therefore conclude that γ-rays with an energy larger than 0.25 Mev can scarcely precede the isomeric transition nor can two or more such γ-rays follow each other in cascade. The modified decay scheme of Chu and Wiedenbeck[3] given in Fig. 2 therefore appears to be the most satisfactory at present.

Taking into account window absorption in the Geiger counters, the delayed coincidence absorption measurements[2] are in excellent agreement with the result from the β-spectrograph[3] as to the value of the integrated internal conversion coefficients for the different electron energy groups.

A theoretical interpretation of the decay scheme meets with certain difficulties owing to a lack of exact numerical calculations of the internal conversion coefficients in the relativistic region. However, some numerical calculations have been carried out for the natural radioactive elements ($Z=83$).[5] In the approximate calculations, the ratio of the number of conversion electrons to the number of gamma-rays, the branching ratio N_e/N_γ, is proportional to Z^3,[6] thus making the internal conversion in Ta about 35 percent less than in RaC. We have applied this correction factor to the numerical calculations referred to above in the following comparison of experiment with theory.

[1] S. DeBenedetti and F. K. McGowan, Phys. Rev. **70**, 569 (1946).

[2] Bunyan, Lundby, Ward, and Walker, Proc. Phys. Soc. **61**, 300 (1948).

[3] Cork, Shreffler, and Fowler, Phys. Rev. **72**, 1209 (1947); Beneš, Ghosh, Hedgran and Hole, Nature **162**, 261 (1948); K. Y. Chu and M. L. Wiedenbeck, Phys. Rev. **75**, 226 (1949).

[4] The sources were irradiated in the big pile at the Atomic Research Establishment, Harwell, England.

[5] H. R. Hulme, Proc. Roy. Soc. **A138**, 643 (1932); H. M. Taylor and N. F. Mott, Proc. Roy. Soc. **A138**, 665 (1932).

[6] This is because of the volume concentration factor, the radius of the K-orbit being proportional to $1/Z$.

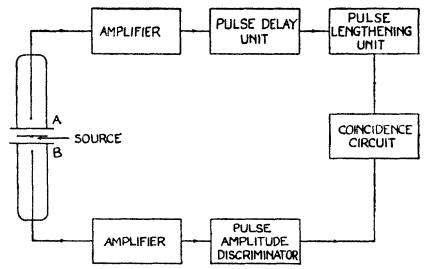

FIG. 1. Schematic diagram of delayed coincidence recorder.

Chu and Wiedenbeck[3] have determined the ratio of the number of conversion electrons emitted in the different transitions to the total number of β-rays preceding the isomeric transition (Table I in their paper). Correcting for the branching between the γ_2- and γ_3-transitions according to Fig. 2, the value of N_e/N_γ is found to be 0.031 for the K-shell in the case of the 0.471-Mev transition. Theoretical calculations[5] give for the same ratio 0.0065 for an electric dipole, 0.018 for an electric quadrupole, and 0.065 for a magnetic dipole transition. The 0.471-Mev γ-radiation is therefore most probably of a mixed electric quadrupole and magnetic dipole character.

For the 0.337-Mev transition the corrected experimental value for the branching ratio in the K-shell is 0.067, while the theoretical calculations give: electric dipole 0.015, electric quadrupole 0.047, and magnetic dipole 0.13. This transition is thus apparently also of a mixed electric quadrupole and magnetic dipole character.

The ratio between internal conversion in the K- and L-shells is for the 0.471-Mev transition 3.0 and for the 0.337-Mev transition 3.6. This is about half the values predicted by theory for an electric quadrupole transition.[7] Numerical calculations have not been carried out for conversion of magnetic multipole radiations in the L-shell. The relative magnitudes of the two K to L ratios are in general agreement with the fact that this ratio should be inversely proportional to the γ-ray energy when other conditions are equal.

As will appear later, the 0.471-Mev and 0.337-Mev transitions are most probably emitted from the same excited state of Ta^{181}. According to the liquid drop model of the nucleus[8] the energy dependence of the transition probability for a multipole γ-radiation should be as E^{2l}, where E is the γ-ray energy and l is the multipole order. For the two radiations in question the ratio of the transition probabilities should be $(0.471/0.337)^4 = 3.8$ according to this theory, while experiments yield a value between 2 and 3. The agreement is reasonable, considering the very rough assumptions made in the liquid drop theory of the nucleus.

The branching ratios for the 0.130-Mev and 0.134-Mev transitions in the K-shell are about 1.3 and 2.8, respectively. Unfortunately, numerical calculations of N_e/N_γ have not been carried out for such low energies. From a rough extrapolation of the data given by Fisk and Taylor[9] it is apparent that electric dipole and quadrupole transitions are ruled out. The extrapolated theoretical value of the branching ratio for magnetic dipole transitions is also rather low, taking into account the fact that the internal conversion in the K-shell decreases in favor of conversion in the L-shell when the energy of the γ-ray approaches the binding energy of the K-electrons. On the other hand, no numerical calculations have been performed for electric octopole transitions. However, comparing the variation of N_e/N_γ with l in the non-relativistic theory[10] and in the exact theory, one can make a rough estimate of the branching ratio which is to be expected for electric octopole radiation in the latter case. The result is about 1.5–2, which is somewhat larger than the experimental value for the 0.130 transition. We can therefore conclude that this γ-radiation is probably of an electric octopole or magnetic quadrupole character or a mixture of both.

A further check on the value for the spin change in this transition can be obtained from the rough energy-lifetime relation which has been deduced from the liquid drop model of the nucleus. This relation being valid for a pure γ-transition, the observed value for the half-life (2.10^{-5} sec.) must be corrected for the probability of ejection of conversion electrons. The corrected half-life comes out as 2.10^{-4} sec. Theoretical calculations give, on the other hand, the following values for τ_γ:

l	2	3	4
τ_γ(sec.)	10^{-10}	10^{-4}	218

For $l=3$ the experimental and theoretical results differ by a factor of 2, which must be considered as a satisfactory agreement. Both the internal conversion coefficient and the energy-lifetime relation therefore lead to the same conclusion, *viz.*, that the 0.130-Mev radiation is electric octopole and/or magnetic quadrupole radiation.

The 0.134-Mev transition is, however, more exceptional. The branching ratio being in this case even larger than for the 0.130-Mev transition, the radiation is probably of multipole order 3 or possibly 4.[11] We have recorded the delayed coincidence rate for different time delays with three different absorbers (2-, 20-, and 40-mg/cm² aluminum foils) between the source and the Geiger counter in the A channel. The results agreed to within the statistical error in the measurements, which was about 3 percent. The lower metastable state can therefore scarcely have a half-life which lies in the interval from one to some hundred microseconds. If,

[7] J. B. Fisk, Proc. Roy. Soc. **A143**, 674 (1934).
[8] See, e.g., I. S. Lowen, Phys. Rev. **59**, 834 (1941).
[9] J. B. Fisk and H. M. Taylor, Proc. Roy. Soc. **A146**, 178 (1934).
[10] M. H. Hebb and E. Nelson, Phys. Rev. **58**, 486 (1940).
[11] A 0→0 transition is of course ruled out owing to the fact that the ground state of Ta^{181} has half-integer spin.

however, the lifetime of this state exceeds 10^{-2} sec. and is less than the lifetime of Hf^{181}, it would be very difficult to detect. This would certainly be the case if the transition gives rise to a magnetic 2^3-pole or electric 2^4-pole radiation. Correcting the theoretical half-life for $l=4$ for the probability of internal conversion, the result comes out as a few seconds. A probable working hypothesis is therefore to assume that the 0.134-Mev transition is of magnetic 2^3-pole and/or electric 2^4-pole character.

Finally the β-transition from the ground state of Hf^{181} to the higher metastable state in Ta^{181} will be considered. From the values of the disintegration constant and the maximum energy of the β-particles, we find that the position of the point in the Sargent diagram for the β-emitters in the heavier elements[12] corresponds to a once-forbidden transition.

Collecting the results of the above discussion as to the nature of the different transitions, we arrive at the most probable spin and parity assignments to the different nuclear states as indicated in Fig. 2. We have tried a number of alternative models for the decay scheme, but all of them seem to be in disagreement with experiments. It should be remarked that the transition between the two metastable states is very rare owing to the special selection rule operating when the centers of mass and charge of a system coincide. If this transition had not been forbidden it had been necessary to assign a spin 13/2 or larger to the first excited state of Ta^{181}.

[12] N. Feather and E. Bretscher, Proc. Roy. Soc. **A165**, 545 (1938). N. Feather, Nature **161**, 451 (1948).

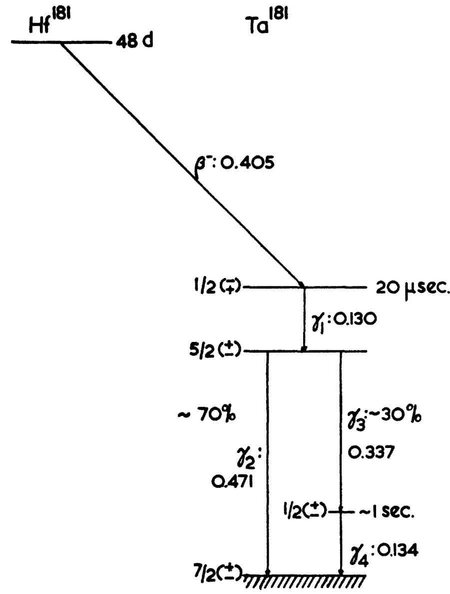

FIG. 2. The decay scheme of Hf [181].

Note added in proof: Comparison with recent discussions of nuclear shell structures (e.g., M. G. Mayer, Phys. Rev. **75**, 1969 (1949)) shows that the energy-levels of Ta^{181} all appear within the same shell. The states in order of increasing excitation energy are then supposed to be formed by the odd proton moving in $g_{7/2}$, $s_{1/2}$, $d_{3/2}$ or $d_{5/2}$ and $h_{11/2}$ orbits. The problem still arises as to why the direct transition from metastable to ground state does not occur. Special selection rules must apparently operate in this case.

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Multiple Scattering of Neutrons. II. Diffusion in a Plate of Finite Thickness

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The diffusion of neutrons in an infinite plate of finite thickness is studied. Analytic expressions are derived for the density and current of the returning and transmitted neutrons at the boundaries. Density and current distribution inside the material at sufficiently large distances from the boundaries are also calculated.

INTRODUCTION

IN I we succeeded[1] in obtaining a rigorous analytical solution for the density and current distributions of neutrons which have been impinging with an arbitrary velocity distribution upon a plate of infinite thickness; the neutrons were assumed to undergo elastic isotropic scattering processes and capture inside the material.

We here extend the treatment under the same physical assumptions to the case of a plate of finite thickness; in the limit of very large thickness the results, of course, will have to agree with those of I. The solutions given here will be *asymptotically* valid if the thickness of the plate is large compared with the mean free path of the neutrons inside the material. They will, therefore, become rigorous for the limiting case of infinitely large thickness.

The treatment is here extended considerably farther than in the first paper. We obtain information not only

[1] This second paper (see Halpern, Luneburg, and Clark, Phys Rev. **53**, 173 (1938), referred to as I) appears belatedly due to reasons beyond the control of the authors; much of its content has been presented orally at an earlier opportunity (Phys. Rev. **56**, 1068 (1939)).