Experimental Investigations of Magneto-Hydrodynamic Waves

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The importance of magneto-hydrodynamic phenomena to different parts of cosmic physics is becoming more and more evident. This fact and the extreme mathematical difficulties involved in an exact treatment of many problems make it necessary to consider the possibility for experimental investigations. In this paper the fundamental equations are solved for cylindrically limited waves, a case which seems to be best adapted for experiments. It is shown that an "ideal" magneto-hydrodynamic wave in a liquid with finite conductivity can exist only in a certain frequency interval. The upper limit is set by the conductivity and the magnetic field strength. Above this value the waves degenerate into skin-waves. The lowest frequency is determined by the geometrical dimensions and by the conductivity. Waves in different liquids are compared, and liquid sodium is found to be the best medium for an experiment. A brief account is given for a preliminary experiment and other investigations are proposed.

I. INTRODUCTION

THE mutual interaction between electromagnetic and hydrodynamic forces in an electrically conducting fluid which is subject to a magnetic field gives rise to a certain type of waves, called magneto-hydrodynamic waves, which travel in the direction of the outer magnetic field carrying with them an induced magnetic field as well as a velocity field.

This phenomenon was discovered in 1942 by Alfvén.[1] In the following year Alfvén outlined a theory of sunspots.[2] According to this theory, the spots are generated by a disturbance in the central part of the sun and then transported to the surface by means of magneto-hydrodynamic waves.

Walén has made a detailed theoretical study of the waves.[3] He considers especially traveling magneto-hydrodynamic whirl-rings. Such a ring may give rise to a bipolar spot pair when intersecting the surface.

In three papers Alfvén has developed his theory of sunspots further and also made comparisons with observations.[4, 5] He has also given a theory for the granulation and the heating of the solar corona, based on magneto-hydrodynamic waves.[6]

Recently, some theories on the origin of the cosmic radiation have been proposed, in which magneto-hydrodynamic phenomena play an important role.[7–9]

From the above it is seen that these waves are very important in cosmic physics. Many problems connected with the waves are difficult to treat mathematically, even with the computing machines now existing, so it will be necessary to consider the possibility for an experimental investigation. This is done in the following paragraphs. A brief account of a preliminary experiment is also given.

II. THE FUNDAMENTAL EQUATIONS

The motion of an incompressible fluid with the density ρ, the electrical conductivity σ, and the permeability μ, placed in a magnetic field H_0 is described by Maxwell's equations

$$\mathrm{curl}\,\mathbf{H}=4\pi/c\cdot\mathbf{i} \tag{1}$$

$$\mathrm{curl}\,\mathbf{E}=-\frac{\mu}{c}(\partial\mathbf{H}/\partial t) \tag{2}$$

$$\mathrm{div}\,\mu\cdot\mathbf{H}=0 \tag{3}$$

$$\mathbf{i}=\sigma\left(\mathbf{E}+\frac{\mu}{c}\mathbf{v}\times\mathbf{H}\right) \tag{4}$$

and the hydrodynamical equations

$$\frac{d\mathbf{v}}{dt}=\frac{\mu}{\rho c}\mathbf{i}\times\mathbf{H}-\frac{1}{\rho}\,\mathrm{grad}\,p, \tag{5}$$

$$\mathrm{div}\,\mathbf{v}=0. \tag{6}$$

Here $\mathbf{v}$ denotes the material velocity, $\mathbf{H}$ and $\mathbf{E}$ the magnetic and the electric field strength, respectively, $\mathbf{i}$ the current density, and p the pressure. In (1) the displacement current has been neglected. As Walén has shown, this system of equations may be reduced in the following way. Eliminating $\mathbf{i}$ and $\mathbf{E}$, we have

$$-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}=\frac{c}{4\pi\mu\sigma}\,\mathrm{curl\,curl}\,\mathbf{H}-\frac{1}{c}\,\mathrm{curl}\,\mathbf{v}\times\mathbf{H} \tag{7}$$

$$\frac{d\mathbf{v}}{dt}=\frac{\mu}{4\pi}\,\mathrm{curl}\,\mathbf{H}\times\mathbf{H}-\frac{1}{\rho}\,\mathrm{grad}\,p. \tag{8}$$

Putting $\mathbf{H}=\mathbf{H}_0+\mathbf{h}$, where $\mathbf{h}$ is the induced magnetic field, and $a^2=c^2/4\pi\mu\sigma$, we get after some transforma-

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tions:

$$\frac{\partial \mathbf{h}}{\partial t}-a^2\Delta\mathbf{h}-(\mathbf{H}_0\nabla)\mathbf{v}+(\mathbf{v}\nabla)\mathbf{H}_0=\operatorname{curl}\mathbf{v}\times\mathbf{h} \tag{9}$$

$$\frac{\partial \mathbf{v}}{\partial t}-\frac{\mu}{4\pi\rho}[(\mathbf{H}_0\nabla)\mathbf{h}-(\mathbf{h}\nabla)\mathbf{H}_0]+\frac{1}{\rho}\operatorname{grad}\left(p+\frac{\rho v^2}{2}+\frac{\mu}{4\pi}\mathbf{H}_0\cdot\mathbf{h}\right)=\mathbf{v}\times\operatorname{curl}\mathbf{v}-\frac{\mu}{4\pi}\mathbf{h}\times\operatorname{curl}\mathbf{h}. \tag{10}$$

These equations determine $\mathbf{h}$ and $\mathbf{v}$. They are not linear. Walén, however, has shown that if the conductivity σ is infinite, the equations reduce to a linear system, the right members of (9) and (10) exactly canceling in this case. With some approximation the same is true for a finite but still very good conductivity. In an experiment where the amplitudes are small, we can simply neglect the second-degree terms. For this case and with a homogeneous outer magnetic field, we get:

$$[(\partial/\partial t)-a^2\Delta]\mathbf{h}=(\mathbf{H}_0\nabla)\mathbf{v} \tag{11}$$

$$\frac{\partial \mathbf{v}}{\partial t}=\frac{\mu}{4\pi\rho}(\mathbf{H}_0\nabla)\mathbf{h}-\frac{1}{\rho}\operatorname{grad}\left(p+\frac{\rho v^2}{2}+\frac{\mu}{4\pi}\mathbf{H}_0\cdot\mathbf{h}\right). \tag{12}$$

III. A SPECIAL SOLUTION OF THE EQUATIONS

Equation (3) requires the motion to be closed. The simplest way of satisfying this condition in an experiment is to choose a motion in which the liquid rotates about a fixed axis. This case seems to be of general interest for experimental investigations and therefore we deduce the corresponding solution. We assume the rotation to take place in a plane perpendicular to the vertical magnetic field B_0 and around the z axis of a cylindrical reference system (r, φ, z). The components of $\mathbf{v}$ and $\mathbf{h}$ are:

$$v_r=0,\quad v_\varphi=v,\quad v_z=0$$
$$h_r=0,\quad h_\varphi=h,\quad h_z=0.$$

That $\mathbf{v}$ and $\mathbf{h}$ are parallel is seen by considering the induced currents. Equations (11) and (12) may be written

$$\frac{\partial h}{\partial t}=a^2\left(\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}-\frac{h}{r^2}+\frac{\partial^2 h}{\partial z^2}\right)+H_0\frac{\partial v}{\partial z} \tag{13}$$

$$\frac{\partial v}{\partial t}=\frac{\mu H_0}{4\pi\rho}\frac{\partial h}{\partial z}. \tag{14}$$

Eliminating v, we get

$$\frac{\partial^2 h}{\partial t^2}=a^2\frac{\partial}{\partial t}\left[\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}-\frac{h}{r^2}+\frac{\partial^2 h}{\partial z^2}\right]+V^2\frac{\partial^2 h}{\partial z^2}, \tag{15}$$

where

$$V^2=(\mu H_0^2)/(4\pi\rho). \tag{16}$$

We want to find a wave with the angular frequency ω, which travels along the z axis, i.e., all quantities are to vary as $\exp(j\omega t+\alpha z)$. Accordingly, we put $\partial/\partial t=j\omega$ and $\partial/\partial z=\alpha$.

Then (15) may be written

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}+\left(k^2-\frac{1}{r^2}\right)h=0, \tag{17}$$

where $k^2=\alpha^2+(\omega^2+\alpha^2V^2)/j\omega a^2$. A solution of (17), which is zero for $r=0$, is

$$h=A\cdot J_1(kr)\cdot\exp(j\omega t+\alpha z) \tag{18}$$

with

$$\alpha^2=(j\omega k^2a^2-\omega^2)/(j\omega a^2+V^2), \tag{19}$$

where J_1 is the Bessel function of the first order.

By means of the original equations the other variables are found:

$$v=\frac{\mu\alpha H_0}{j4\pi\omega}\cdot h \tag{20}$$

$$i_r=-\alpha c/4\pi\cdot h \tag{21}$$

$$i_z=\frac{kc}{4\pi}\frac{J_0(kr)}{J_1(kr)}\cdot h \tag{22}$$

$$E_r=-\frac{\mu\alpha}{j\omega c}(V^2+j\omega a^2)\cdot h \tag{23}$$

$$E_z=\frac{\mu a^2 k}{c}\cdot\frac{J_0(kr)}{J_1(kr)}\cdot h. \tag{24}$$

Suppose that we want to study the waves under the following simple experimental conditions: (a) The liquid is contained in a cylinder with the radius R and the height L; (b) at the bottom, $z=0$, a motion $v(r,t)=v_0\cdot r/R\cdot e^{j\omega t}$ is excited.

The following boundary conditions determine the solution:

(1) $r=R\quad i_r=0\quad \therefore h=0$
(2) $z=0\quad v=v_0\cdot r/R\cdot e^{j\omega t}$
(3) $z=L\quad i_z=0\quad \therefore h=0.$

The first condition gives the proper values k_ν of k as solutions of the equation $J_1(k_\nu R)=0$. The corresponding α_ν are then given by (19).

Conditions (2) and (3) may be satisfied in this way. We consider the repeated reflections of the waves. A wave traveling up to the surface is reflected without phase difference in v but h is in opposite phase in the upward wave and the downward wave. At the bottom the conditions for h and v are reversed. In a point z we add all the waves traveling up and down.

$$v(z)=v(0)(e^{\alpha z}+e^{\alpha(2L-z)}-e^{\alpha(2L+z)}-e^{\alpha(4L-z)}+\cdots)$$

or

$$v(z)=v_0\cdot\frac{\cosh\alpha(L-z)}{\cosh\alpha L}. \tag{25}$$

The solution is

$$v(r,z,t)=\sum_{\nu=1}^{\infty} A_\nu J_1(k_\nu r)\cdot\frac{\cosh\alpha_\nu(L-z)}{\cosh\alpha_\nu}\cdot e^{j\omega t}. \tag{26}$$

For $z=0$ and $r<R$ we have

$$v_0\cdot\frac{r}{R}=\sum_{\nu=1}^{\infty} A_\nu J_1(k_\nu\cdot r), \tag{27}$$

from which the A_ν may be determined. Finally, we give the expression for the velocity in a point (r,z), valid for $0\leq r<R$ and $0\leq z\leq L$

$$\frac{v(r,z)}{v(r,0)}=\sum_{\nu=1}^{\infty}\frac{2}{k_\nu\cdot rJ_2(k_\nu R)}\cdot J_1(k_\nu r)\frac{\cosh\alpha_\nu(L-z)}{\cosh\alpha_\nu L}. \tag{28}$$

IV. IDEAL WAVES AND DEGENERATED WAVES

Introducing $U=j\omega/\alpha$ we see that the field strength of the waves varies as $\exp[j\omega(t-z/U)]$, where U is the phase velocity. A complex value of U means damping of the waves. Equation (19) may be written

$$\frac{1}{U^2}=\frac{1}{V^2}\frac{1-j(k^2a^2/\omega)}{1+j(\omega a^2/V^2)}. \tag{29}$$

The propagation of the waves depends upon both the frequency, ω, and the wave number, k; i.e., upon the spectrum of the wave in time and space. If the conductivity is infinite ($a=0$), we get $U=\pm V$. In this case all waves travel without damping with the same constant velocity $V=H_0\mu^{\frac{1}{2}}(4\pi\rho)^{-\frac{1}{2}}$, parallel or antiparallel to the magnetic field H_0. Every velocity-state is moving with this speed without any deformation. Equation (20) gives $v/V=h/H_0$, and accordingly, we have $\rho v^2/2=\mu h^2/8\pi$, which means the ideal case in which kinetic and magnetic energy are equal.

In reality we always have $a\neq 0$ but in the cosmical applications k and ω are so small that the waves may be considered as ideal.

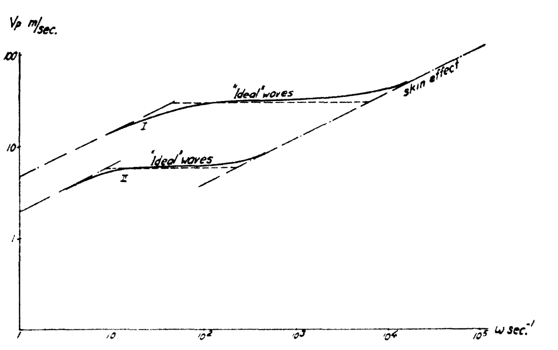

FIG. 1. Phase velocity of magneto-hydrodynamic waves in liquid sodium: —— phase velocity V_p, - - - - ideal wave velocity V_0, —·— degeneration at high frequencies (skin-effect), —··— degeneration at low frequencies. I—Cylinder radius $R=12$ cm. Magnetic field strength $H_0=10{,}000$ gauss. II—Cylinder radius $R=25$ cm. Magnetic field strength $H_0=2000$ gauss.

At high frequencies (or if $V\rightarrow 0$ and $k\rightarrow 0$) we get $U\simeq a\omega^{\frac{1}{2}}e^{j\pi/4}$ and $v\simeq 0$ which means a damped pure electromagnetic wave, independent of H_0 and ρ. The waves have degenerated into skin-effect waves.

If the frequency is very low we get $U\simeq\pm V\cdot\omega^{\frac{1}{2}}(ka)^{-1}e^{j\pi/4}$, which is another type of degenerated waves, where the degeneration is due to the finite value of k. If $k=0$, i.e., $R=\infty$ (or plane waves), we get ideal waves for all frequencies $\omega\ll V^2/a^2$.

V. CONDITIONS FOR AN EXPERIMENT

From (29) we infer that the conditions for getting "ideal" magneto-hydrodynamic waves are

$$k^2a^2/\omega\ll 1 \tag{30}$$

$$\omega a^2/V^2\ll 1 \tag{31}$$

or together

$$k^2a^2\ll\omega\ll V^2/a^2. \tag{32}$$

In order that this shall be possible we must have $V^2\gg k^2a^4$ or

$$H_0R\gg 3.83(4\pi)^{-\frac{1}{2}}c^2\sigma^{-1}\rho^{\frac{1}{2}}\mu^{-\frac{3}{2}}, \tag{33}$$

where k has been replaced by $3.83/R$, corresponding to the lowest wave number, which is possible in a cylinder of the radius R.

In an experiment it is desirable to get a small required magnetic field H_0 and small dimensions. Equation (33) tells us that the required value of $H_0\cdot R$ is small if $\sigma\mu^{\frac{3}{2}}\rho^{-\frac{1}{2}}$ is large. The permeability, μ, being always ≈ 1, we may take $\sigma\rho^{-\frac{1}{2}}$ as a measure of the property of a medium for magneto-hydrodynamic experiments.

The relative values of $\sigma\rho^{-\frac{1}{2}}$ for some possible media are as follows: liquid sodium, 35; liquid lithium, 11; liquid sodium-potassium alloy, 10; mercury, 1; arc discharge in air at 100 mm Hg, 0.1.

Mercury is perhaps most simple to handle, but an experiment with liquid sodium would certainly give much more information.

In Figs. 1 and 2 the phase-velocity and the damping of a magneto-hydrodynamic wave in liquid sodium, corresponding to the lowest possible wave number, k_1,

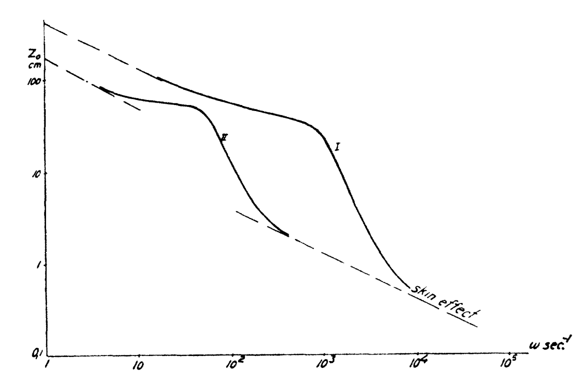

FIG. 2. Damping of magneto-hydrodynamic waves in liquid sodium: —— Z_0 = distance for damping of the wave to e^{-1}, —·— damping at high frequencies (skin-effect), —··— damping at low frequencies. I, II—See text to Fig. 1.

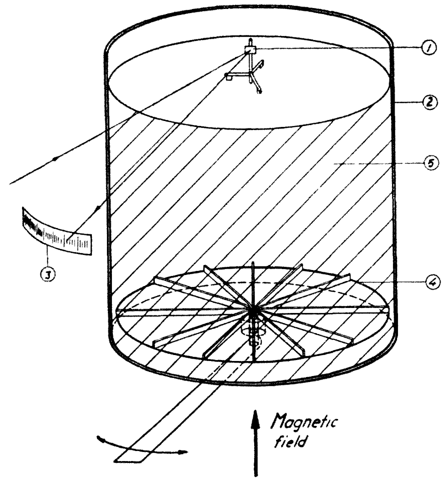

FIG. 3. Arrangement of the experiment on magneto-hydrodynamic waves in mercury: (1) floating mirror, (2) stainless steel cylinder, (3) scale, (4) vibrating disk, (5) mercury.

are given as a function of the frequency for two different cylinder radii.

VI. INFLUENCE OF THE VISCOSITY

In a viscous fluid we get, instead of Eqs. (13) and (14),

$$\frac{\partial h}{\partial t}=a^2\Delta h+H_0\frac{\partial v}{\partial z} \tag{34}$$

$$\frac{\partial v}{\partial t}=\frac{\mu H_0}{4\pi\rho}\frac{\partial h}{\partial z}+\nu\Delta v, \tag{35}$$

where ν denotes the kinematic viscosity.

If $\nu=0$, Eq. (20) gives $v=(\alpha\mu H_0/j4\pi\rho\omega)\cdot h$. We put this value into the last term of (35) and eliminate v between (34) and (35). With $\partial/\partial t=j\omega$ and $\partial/\partial z=\alpha$ we get

$$-\omega^2 h=\Delta h\left(j\omega a^2+\frac{\nu V^2\alpha^2}{j\omega}\right)+\alpha^2V^2h. \tag{36}$$

If $\nu\ll(\omega^2a^2/\alpha^2V^2)=(U^2/V^2)\cdot a^2$, the viscosity may be neglected. If it may not be neglected the influence of the viscosity is to increase the damping of the waves. For an "ideal" wave a^2 should be replaced by $a^2+\nu$, and then we see that the viscosity is unimportant if $\nu\ll a^2$. For liquid sodium ν/a^2 is about 10^{-5} and so the viscosity certainly may be neglected.

VII. A PRELIMINARY EXPERIMENT

An experiment on magneto-hydrodynamic waves in mercury has been carried out at this laboratory. The mercury was contained in a stainless steel cylinder with the diameter 15 cm and the height 15 cm (see Fig. 3). The waves were excited at the bottom by means of a disk with radial strips and the measurements were made by means of a mirror floating on the surface.

The apparatus was placed in a vertical magnetic field with a maximal value of 13,000 gauss. The disk was mechanically driven to perform torsional vibrations with a small amplitude. The damping and the phase difference between surface and bottom were measured directly by means of a stopwatch. The frequency was varied between 0.1 and 1 cm/sec. for different values of the magnetic field. In Fig. 4 the results are compared with the theoretical values.* The discrepancy between theory and experiment may be due to the experimental difficulties to keep the surface of the mercury clean. A layer rapidly formed on the surface, making further measurements impossible. We hope to find the reason for the discrepancy and intend also to repeat the experiment in liquid sodium, if technically possible.

VIII. SOME PROBLEMS SUITED FOR EXPERIMENTAL INVESTIGATIONS

In cosmical physics the conductivity may often be considered as infinite and the waves are correctly described by Eqs. (9) and (10), the right members being equal to zero. These equations are formally linear, but do not permit the adding of solutions to get a new solution. Suppose, in particular, that we have two waves traveling into each other. In each wave alone we have a perfect balance between magnetic forces and centrifugal forces. By adding the waves it may happen that we get a wave which is not balanced, e.g., if the induced fields are antiparallel but the material velocities parallel. A detailed discussion is given in the paper by Walén. This problem is very difficult to treat mathematically. An experimental investigation may be able to give at least a qualitative picture of the situation if sufficiently large amplitudes may be excited that the second-degree terms are important. The condition for this is $h\approx H_0$, or for an ideal wave $v\approx V$.

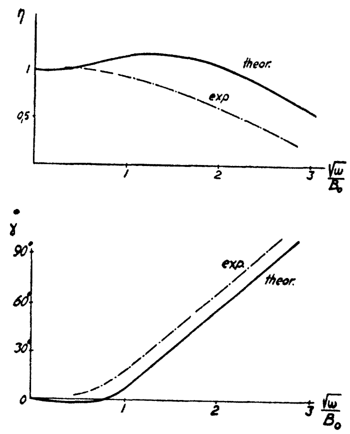

FIG. 4. Measurements on magneto-hydrodynamic waves in mercury. η=amplitude at surface/amplitude at bottom. γ=phase difference between bottom and surface.

* These values have been calculated in the same way as in Section III, only with the exception that the excited velocity at the bottom has been assumed to be $v_0\cdot r/R$ when $0\leq r\leq R_1$, and $v_0\cdot R_1/R\cdot(R-r)/(R-R_1)$ when $R_1\leq r\leq R(R_1/R=0.96)$, which ought to be closer to the real velocity distribution. The difference between these values and Eq. (28) does not exceed 10 percent.

One interesting experiment would be to study the reflection of a magneto-hydrodynamic whirl-ring at a non-conducting surface. Especially important for the theory of sunspots is the behavior of rings parallel to the magnetic field.

The generation of the whirl-rings by convection also ought to be studied.

The experiments with waves in mercury have given the impression that all magneto-hydrodynamic phenomena have a quite different character than the corresponding purely hydrodynamic phenomena. Perhaps many theories on hydrodynamic problems in cosmical physics have to be reconsidered.

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The Decay Scheme of Hf^{181}

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The radiations of Hf^{181} have been studied with delayed coincidence and absorption techniques. No γ-rays are found to precede the formation of the 20-microsecond metastable state in Ta^{181}, thus confirming the decay scheme of Chu and Wiedenbeck. From the values of the internal conversion coefficients together with delayed coincidence absorption measurements it is concluded that the previously reported 0.134-Mev γ-ray is most probably emitted from another metastable state of about one-second duration in Ta^{181}. A theoretical discussion of the multipole order of the different radiations lead to assignments of probable spin and relative parity values to the excited states of Ta^{181}. Beginning with the ground state, the deduced spin and parity values are: $7/2(\pm)$, $1/2(\pm)$, $5/2(\pm)$, and $1/2(\mp)$.

SOME uncertainty exists as to the succession of the γ-transitions in Ta^{181}, following the β^--decay of Hf^{181}. The metastable state in Ta^{181} of 20-μsec. duration was first observed by DeBenedetti and McGowan,[1] and later studied by Bunyan *et al.*[2] The β-rays, γ-rays, and conversion electrons have been subject to investigations by several authors,[3] some of which claim to have observed a γ-transition preceding the formation of the metastable state.

With the delayed coincidence recorder in Fig. 1 we have remeasured the half-life of the metastable state in Ta^{181} [4] with both the differential and integral procedures, the results being in agreement with earlier determinations. Inserting a 1-mm lead absorber between the source and the Geiger counter in the B channel and then in the A channel, we find a genuine delayed coincidence rate of 1.5 min.$^{-1}$ (corresponding to infinitely long resolving time) and less than 10^{-2} min.$^{-1}$, respectively. With the same Pb absorbers in front of both Geiger counters, we could detect neither delayed nor instantaneous coincidences exceeding the random coincidences and in the latter case the background coincidence rate due to cosmic rays and other instantaneous coincidence sources. We therefore conclude that γ-rays with an energy larger than 0.25 Mev can scarcely precede the isomeric transition nor can two or more such γ-rays follow each other in cascade. The modified decay scheme of Chu and Wiedenbeck[3] given in Fig. 2 therefore appears to be the most satisfactory at present.

Taking into account window absorption in the Geiger counters, the delayed coincidence absorption measurements[2] are in excellent agreement with the result from the β-spectrograph[3] as to the value of the integrated internal conversion coefficients for the different electron energy groups.

A theoretical interpretation of the decay scheme meets with certain difficulties owing to a lack of exact numerical calculations of the internal conversion coefficients in the relativistic region. However, some numerical calculations have been carried out for the natural radioactive elements ($Z=83$).[5] In the approximate calculations, the ratio of the number of conversion electrons to the number of gamma-rays, the branching ratio N_e/N_γ, is proportional to Z^3,[6] thus making the internal conversion in Ta about 35 percent less than in RaC. We have applied this correction factor to the numerical calculations referred to above in the following comparison of experiment with theory.

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