the multiple refraction theory the width of the single Gaussian would be 4530 seconds for the curve shown in Fig. 4.

The theory developed in this paper should be applicable, without serious modification, to the passage of slow neutrons through colloidal materials. Indeed, because of the relatively lesser importance of neutron absorption, higher scattering multiplicities would be experimentally obtainable.

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The Density Effect for Cosmic-Ray Mesons*

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This paper deals with a detailed study of individual pulses produced by fast cosmic-ray mesons in a specimen of silver chloride. A discussion is given of the pulse distributions obtained from the experiment, which was designed to show the increase with energy of the rate of ionization in the relativistic region and the correction to this increase due to the density effect. The measured values are compared with theoretical distributions which take fluctuation and density effect into account. It is concluded that the results agree well with the fluctuation theory developed by Symon and the density effect corrections as given by Halpern and Hall.

1. INTRODUCTION

IN 1945 van Heerden[1] reported the technique of using a crystal of silver chloride as an ionization detector. Such a detector is sufficiently sensitive to measure the ionization produced by a singly charged particle ionizing at the minimum rate. Hofstadter[2] has reported the observation of pulses produced in silver chloride and certain other substances by gamma-rays.

The theoretical treatment of the average ionization produced by fast particles has been given by Bethe and by Bloch. Their results have been summarized by Heitler.[3a] Curves are available in an article by Rossi and Greisen.[3b]

The Bethe-Bloch theory pertains to the case of disperse media. For a condensed medium an additional effect, called the density effect, reduces the rate of ionization for charged particles. The mechanism for the density effect is the polarization of the atoms of the medium with the resulting reduction of the distant electromagnetic interactions. This effect, first suggested by Swann and discussed briefly by Fermi,[4] has been treated in detail by Wick,[5] and Halpern and Hall.[6] The theory of average ionization for an energetic particle ionizing near the minimum rate has been tested experimentally by Corson and Brode[7] and Hazen,[8] who partially verified the rise in the rate of ionization in the relativistic region predicted by the theory. Further tests were performed by Hayward[9] and Hereford[10] to search for the existence of the density effect. Both of these workers report results which agree with the Bethe-Bloch theory of ionization as corrected for the density effect by Halpern and Hall. In each case, however, the experiment is indirect and the results sketchy.

The theory of ionization, including polarization effects, seems to be sound, but has not been tested thoroughly at high energy. In the crystal counter we have an instrument suited for such a study. We cannot directly compare the rate of ionization deduced from the crystal pulse with the *average* rate of ionization given in the above-mentioned theory because the pulses produced in the crystal by monoenergetic particles suffer a large fluctuation due to the fluctuation in energy loss. Landau[11] has investigated this fluctuation in energy lost by a particle in traversing a *thin* thickness of absorber. Similar calculations, carried out in detail by Symon[12] for all thickness of absorber, make it possible to predict the fluctuation in energy loss for mesons. Believing the crystal ionization detector to be ideally suited for the study of ionization at high energy, we have attacked the problem in the following manner.

* Assisted by the ONR and AEC.
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2. PROCEDURE AND EQUIPMENT

The ionization curve for any charged particle, and in particular for mesons, plotted as a function of range, momentum, or energy, exhibits a unique point which is useful as a reference value in carrying out the experiment. This reference point is a minimum in the ionization or rate of energy-loss curve which, for mesons, occurs at about 250 Mev. Mesons having higher, or lower, energies than this ionize at a greater rate, and on the average, produce greater pulses in the crystal than those ionizing at the minimum rate. From the total energy spectrum of cosmic-ray mesons at sea level, those mesons were selected which ionized at the minimum rate. Another group was selected which ionized at a predictably greater rate. The crystal was arranged so as to examine the pulses of first one group and then the other. Because of a statistical fluctuation in the rate of ionization, the pulse heights due to mesons in each group were spread out into a distribution. The plan was to compare the shape of the experimental distributions of pulses in these two groups with the predicted pulse-height distributions and to compare the separation of the maxima of the two experimental distributions with that of the predicted distributions. The shapes of the distributions, especially for the minimum ionization group, constitute a check on the uniformity of response of the crystal specimen. The comparison of the measured separation of the maxima of the two experimental distributions with the predicted separation provides a test of the theory of the density effect.

It is well known that at sea level about eighty percent of the cosmic-ray events recorded by a vertical Geiger counter telescope are due to single mesons passing through the system. The remainder of the events is mainly due to electrons which, if of low energy, can be absorbed or, if of high energy, can be made to form an electron shower with high probability. A shower detector can be used to make the equipment relatively insensitive to multiple rays. The apparatus which permitted the study of only those events involving a single high energy meson is shown in Fig. 1. Above the crystal was placed a 1.8-cm slab of lead. Immediately below the crystal was a piece of lead 8.9 cm thick, large enough to cover completely the solid angle (0.1 steradian) defined by the Geiger counters C_1 and C_2. Between the counters C_2 and AC_2 were 23 cm of lead and between the counters AC_2 and C_3 there were 76 cm of lead and 2 cm of iron.

In addition to the desired events, a few, in which high energy mesons did not penetrate the crystal, were selected by the geometrical arrangement shown in Fig. 1. The solid angle was defined by the Geiger counter C_1 and the three counters at C_2. To select only those mesons penetrating the crystal, it would have been necessary to use only one counter in tray C_2. With three counters in tray C_2, the addition to the counting rate due to the particles penetrating the crystal was much larger than the addition due to those missing it. The counting rate was very small; therefore, the few events not involving the crystal were accepted in order to increase by nearly a factor of three the desired number. It seemed unwise to use an arrangement which required too long an observation period because of the uncertainty of maintaining constant operation of the crystal.

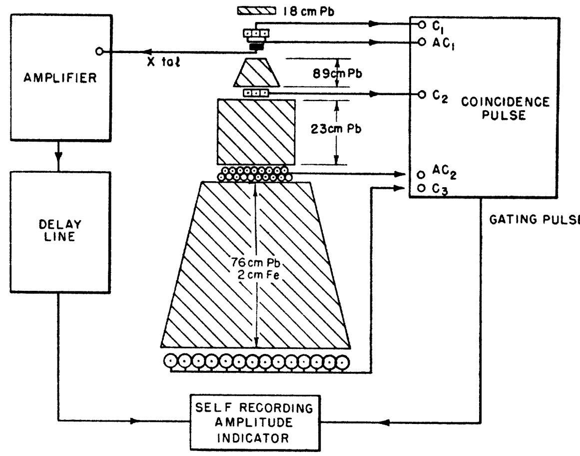

FIG. 1. Arrangement of counters and lead and block diagram of circuits.

As suggested above, it was possible to eliminate a large number of electrons by using a shower detector in anticoincidence. The counters used to detect showers were those designated by AC_1 in Fig. 1. These two counters were connected in parallel and attached to the anticoincidence channel of the coincidence circuit. Thus, whenever a pulse appeared at C_1 simultaneously with one at AC_1, the event was very likely a shower and was automatically not recorded. To cause the largest number of electrons possible to produce showers, the optimum thickness of lead for producing showers was placed above the apparatus.

This entire equipment was placed in a room underground. Over the room there was a six-inch concrete roof covered with two feet of cinders, ashes, and dirt. Above this there was only sky for the entire solid angle subtended by the equipment. It was not essential for this experiment to place the equipment under a thin roof because energetic mesons observed at sea level do not interact strongly with matter. Hence, the absorber above the equipment did not alter appreciably the nature of the meson-component of the radiation used in the experiment.

The Geiger counters in tray AC_2 were 10-inch cylindrical glass counters with copper foil cathodes $\frac{7}{8}$ inch in diameter with an anode of 15-mil tungsten wire (in general the symbol C stands for coincidence, AC for anticoincidence, and the subscript denotes the tray). The counters in tray C_3 were 20-inch glass cylinders with copper cathodes 2 inches in diameter. The counters in the trays marked C_1, AC_1, and C_2 were each constructed from copper in units of three, each individual counter having a sensitive volume of about $1\times1\times1\frac{1}{2}$ inch. All the counters were filled with the standard alcohol-argon mixture (1 part ethyl alcohol, 9 parts argon) to a pressure of about 10 cm of mercury. The operating potential was 990 volts, the plateau of each counter was at least 150 volts, and the mean overvoltage was about 75 volts.

The circuits employed were not unusual for an experiment of this type. The electronic circuit for the Geiger counters was similar to one used by Valley,[13] in that each counter tray like that designated by AC_2 and C_2 was divided into subgroups of 4 counters or less, each group being connected to its own 6AK5 pentode operating at normal plate voltage. The several type 6AK5 pentodes shared a common plate resistor so that all of the Geiger counters in the tray functioned as one tube because of the signal addition property of the common plate resistor. The coincidence circuit, which employed the familiar Rossi-type coincidence elements, produced a long gating pulse of 5000 μsec. duration following the desired chain of events at the Geiger counters.

The polycrystalline specimen of rolled silver chloride was supplied by the Harshaw Chemical Company. The preparation (mainly careful heat treatment and electrode surfacing) and detailed investigation of the crystal will be given in a forthcoming paper. For the purposes of this report it will suffice to say that the crystal was used as an ionization chamber. Full voltage saturation could not be obtained for the maximum voltage used. The degree of saturation (75 percent) obtained with the maximum field available (5240 volt/cm) did not vary during the experiment, which lasted four weeks. The capacity with respect to ground at the input of the amplifier was 72 ± 3 $\mu\mu$f. Most of this was due to the crystal.

The pulse produced in the crystal by a meson was amplified with a Los Alamos Model 100 preamplifier and amplifier with a pulse rise time of about 0.5 μsec. and a voltage gain of about 300,000. The crystal pulse, after amplification, was delayed before being fed into the self-recording amplitude indicator. The design of the indicator was based on a pulse lengthener due to Elmore modified so as to provide a means for blanking out all pulses from the crystal except those accompanied by the gating pulse from the coincidence circuit. The apparatus worked as follows. Depending on whether tray AC_2 or C_3 was used, the meson selected by the counters C_1 and C_2 stopped in the 23-cm block of lead or penetrated all of the lead. The pulse of ionization produced in the crystal by the meson was amplified and fed to the amplitude indicator which made a permanent record. A contribution to each of the two groups of events was made each day. This would eliminate any long term variation of the equipment. It was recognized that a better procedure would have given a simultaneous record of both groups requiring, however, more elaborate recording equipment.

[13] G. E. Valley, Phys. Rev. **72**, 772 (1947).

3. EXPERIMENTAL RESULTS

With the apparatus shown in Fig. 1, the counting rates for the mesons stopping in the 23 cm of lead and for those penetrating 112 cm of lead as well as the counting rate for all mesons penetrating at least 8.9 cm of lead are shown in Table I. In this table we have quoted the standard error determined from the number of counts. The accidental counting rate is, in every case, far smaller and will be neglected.

The present experiment was made uncertain to some extent by scattering. Since scattering is more important for mesons of lower energy, the data most affected are those obtained for mesons stopping in the 23-cm block of lead. A computation has shown that, since the scattering falls off inversely with the energy, the majority of the mesons scattering out of the beam did not have an energy greatly in excess of 10^9 electron volts, and thus would not be expected to ionize at a substantially greater rate than did those which stopped in the lead. Therefore, we feel that the contribution of these pulses to the minimum ionization distribution could not have altered its shape or the position of its maximum by more than the uncertainty already inherent in the distribution.

In Fig. 2, we have exhibited the experimentally obtained differential distributions of mesons possessing the required ranges in lead. The data accumulated during the experiment were analyzed with allowance made for the slight non-linearity of the amplifier. The distributions will be discussed in terms of the voltage input to the main amplifier; but this voltage divided by 82 ± 8, the gain of the preamplifier, will give the voltage pulse at the crystal. The two histograms have essentially the same shape. The main difference is that the histogram obtained for mesons penetrating 112 cm of lead is displaced toward the region of larger pulses by a small amount, as it should be if the rate of ionization increases for high energy mesons.

The separation between the two curves is not large, however. That the separation is real and not due to statistical variation is seen from the fact that, when the two histograms are compared at and near their peaks, the relation of each rectangle of the histogram for mesons penetrating 112 cm of lead bears the relation to the other histogram it should if this histogram is indeed displaced to the region of larger pulses. In particular, the four rectangles to the left of the peak of the histogram for penetrating mesons are less high than the corresponding ones of the histogram for stopped mesons, whereas the rectangle at the peak and the next four are larger than the corresponding ones for the stopped mesons. A definite correlation exists for these nine rectangles. This relationship between the rectangles for the two histograms could occur by chance only once in $(2)^9=512$ times. The small probability that the separation between the histograms occurred by chance is taken as proof of the reality of the separation of the two distributions.

The values of the average and most probable pulses for the histograms will be useful later for the discussion of the results. For reasons to be discussed later, we cut off each histogram at a certain fraction of its peak, both below and above the peak, using the remaining part to calculate the average. For the histogram due to mesons stopping in 23 cm of lead, we used the portion of the data for the pulse heights between 1.60×10^{-2} volt and 3.40×10^{-2} volt. For the other histogram, we used the data for the pulse heights between 1.70 and 3.60×10^{-2} volt. The averages of the pulse heights computed for the case of mesons stopping in 23 cm of lead and those penetrating 112 cm of lead are 2.09 and 2.13×10^{-2} volt respectively. The most probable values for the pulse heights read from the histograms are $1.81(\pm5)\times10^{-2}$ volt and $1.88(\pm5)\times10^{-2}$ volt respectively.

4. DISCUSSION OF RESULTS

We would now like to derive as well as possible the distribution of pulses that one should expect from the experiment and to compare these with those obtained experimentally to see whether there is agreement with theory. We first assume that the crystal is a perfect ionization detector; later we take account, to a certain extent, of imperfections in the crystal, indicating experimental evidence of the divergence of the observations from those expected with an ideal instrument. The starting point for the calculation of the pulse height distributions are two sets of data, the first (theoretical) giving the average rate of ionization of mesons in iron as a function of their remaining range in air, and the second (experimental) giving the differential distribution of mesons as a function of their range in air. The collision loss was taken for iron rather than for silver chloride because calculations of the collision loss for silver chloride were not available and the ones for iron are not very different. This caused a small error in the predicted pulse heights which is of no consequence since the curve to be derived for the distribution as a function of average ionization will be used only illustratively. Actually two curves were considered for the collision loss, one taken from the usual formulas, and a second including the density effect, use being made of the results given by Halpern and Hall.[6] The curve giving the differential distribution of mesons at sea level as a function of the range in air represents the sum of the best experimental measurements to date.[14] From these combined data one can eliminate the range and compute in a standard but tedious manner curves

TABLE I. Counting rates of mesons of various penetrating powers.

	Mesons with range >8.9 cm of Pb	Mesons with 8.9 <range <32 cm Pb	Mesons with range >112 cm Pb
Counting rate	18.5±0.4/hr.	4.0±0.2/hr.	7.5±0.4/hr.
Percentage of mesons	100 percent	22±1 percent	41±2 percent

[14] B. Rossi, Rev. Mod. Phys. **20**, 545 (1948).

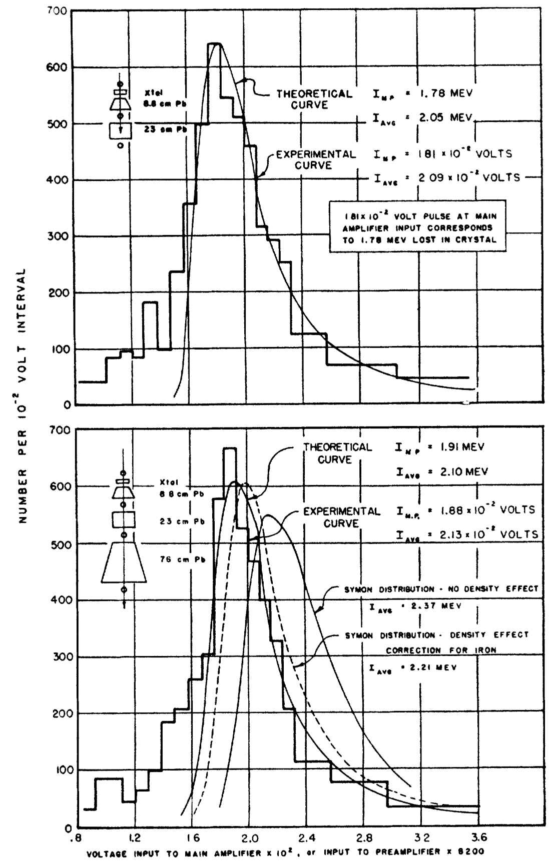

FIG. 2. Pulse height distributions due to cosmic-ray mesons selected as indicated. Histograms are experimental data.

giving the differential distribution of mesons at sea level as a function of the average rate of ionization in the crystal. There will be two such curves, depending upon whether or not the density effect is taken into account. The two curves are exhibited in Fig. 3, where the distribution is plotted on an arbitrary scale as a function of the average rate of ionization. The sharp, narrow peak, which is the same whether the density effect is included or not, gives the distribution for those mesons which penetrate 9 cm of lead and then stop in an additional 23 cm of lead. The other two solid portions of the curve are the distributions of those mesons which penetrate at least 112 cm of lead.

Assuming no fluctuation in ionization, the curve in Fig. 3, taking the density effect into account, would represent the expected distribution in pulse sizes, since the crystal is relatively thin for high energy mesons, and the energy of the particle would change but little in passing through it.

There is, however, a substantial fluctuation in the rate of ionization of a high energy meson in penetrating a thin absorber. This fluctuation in ionization is contributed to a large degree by the higher energy transfers involved in the collision process. The very complete tables given in K. Symon's thesis[13] make it a simple matter to calculate the effect of fluctuation on the distribution of pulse heights given in Fig. 3. The calculations give directly the expected distributions of energy losses in the 0.3-cm specimen of silver chloride used in the experiment. These presumably correspond to the expected pulse height distributions modified by a suitable proportionality constant.

In the calculation of the fluctuation, it is sufficient to consider directly the effect of the fluctuation spread on the single peak due to mesons that stopped in 23 cm of lead. However, the distribution of mesons penetrating

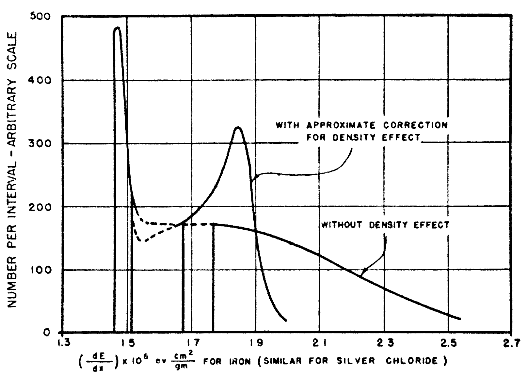

FIG. 3. Calculated distribution of mesons at sea level as a function of the average rate of ionization.

112 cm of lead must be treated as consisting of at least five separate bands, each modified by fluctuations. The reason for this is that the unmodified distribution in this case is about as broad as the spread due to fluctuation. The sum of the modified bands must be taken as the expected distribution. These calculations were made using the Symon results which do not take the density effect into account and the resulting curves are shown in Fig. 4.

It was not possible to calculate the exact distribution when the density effect is taken into account because, as yet, no numerical data for this effect in silver chloride are available. The nearest that one can come, *a priori*, to the proper correction is to apply the density-effect correction for iron. This, at best, is only an approximation to the proper correction. The correction, which depends on the energy of the incident particle, was determined from the curve for iron in Fig. 5, taken from the work of Halpern and Hall.[6] The curve for mesons stopping in 23 cm of lead was not altered because the density-effect correction for iron for mesons of this energy amounted to 0.5 percent at most. The correction for the other distribution was considerable. Each of the five bands, into which the distribution of average rate of ionization for mesons penetrating 112 cm of lead was split, corresponds to a different high energy. For each of these energies, the density-effect correction was determined from Fig. 5 for the case of iron. Considering the distributions in Fig. 4, we see that the effect of fluctuations in energy loss and the density effect are both very important in modifying the distribution in average ionization shown in Fig. 3.

Several additional effects might cause a broadening of the observed distributions. These are the fluctuation in identical pulses caused by imperfections in recorder, amplifier noise, and the difference in pulse sizes due to the variation in path lengths in the crystal. Careful analysis of each source reveals that the additional broadening due to these effects is small compared to the large fluctuation predicted by Symon.

To make comparison with the experimental results, we have added to Fig. 2 the various theoretical curves. Because the density effect is small for the mesons which stopped in 23 cm of lead, we have neglected this correction for these mesons and fitted the theoretical distribution of stopped mesons to the experimental histogram so that both distributions have the same average abscissa between the values of 1.60 and 3.40×10^{-2} volt (upper graph). The fitting process called essentially for a determination of the correlation between the energy lost in the crystal and the resulting voltage peak. The result of the fitting is that an average energy loss of 2.05 Mev corresponded to an average pulse of 2.09×10^{-2} volt at the input of the main amplifier. The theoretical most probable energy loss, 1.78 Mev, corresponded to a pulse height of 1.81 $(\pm5)\times10^{-2}$ volt at the input.** The histogram for stopped mesons has the same shape as the Symon distribution except in its initial part which exhibits an excess number of small pulses. The fluctuation theory predicts that 5 percent of the pulses should saturate the amplifier; indeed, 5 percent of the pulses did. The widths at half-value of the theoretical and experimental curves agree very well. There is a slight difference between the peaks of the two distributions, but not more than would be expected statistically.

The essential dissimilarity between the distributions appears for small pulses. The histogram has a tail for small pulses, constituting about 16 percent of the total area, whereas the theoretical curve falls abruptly to a very small fraction of its maximum value on this part of the curve. The excess number of small pulses observed in the experiment is most likely due to poor regions in the

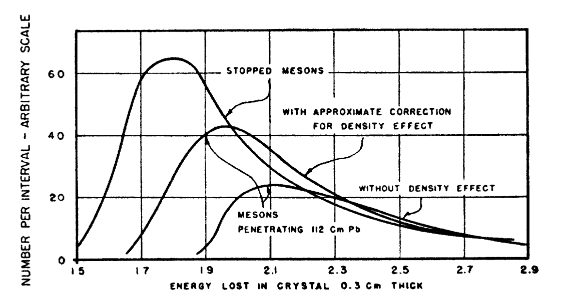

FIG. 4. Expected pulse distributions modified by allowance for fluctuation in energy loss.

** Using these values and the fact that 75 percent voltage saturation was obtained, we calculate that, on the average, 6.6±0.7 electron volts were required to release one conduction electron in the crystal.

crystal. It is not clear whether the failure of the crystal was due to spotty variations throughout the volume, or to poor regions near the edges. Attempts to obtain larger crystals of good quality have not been successful. Detailed area tests of a crystal using cosmic rays are too slow to be practical. Since the histogram agrees well with the theoretical distribution over most of the range of pulses, we conclude that the major portion of the crystal behaved as an efficient detector of ionization.

In Fig. 2 (lower graph), we have plotted the theoretical distribution curves for mesons penetrating 112 cm of lead. In fitting these curves to the amplitude scale, use was made of the result cited above that 1.81×10^{-2} volt corresponds to 1.78 Mev lost in the crystal. The distribution with no correction for the density effect is obviously a poor approximation to the experimental results. The width at half-value of the theoretical curve is somewhat large and the average of this curve is far too great. The theoretical distribution, corrected for the density effect by an amount appropriate for iron as described above, is shown in the same figure. This curve is a fair approximation to the experimental distribution, but the agreement is still not good because the average energy loss is 2.21 Mev, whereas the average of the experimental curve is 2.10 Mev, when the tail due to small pulses is neglected.

Lacking a theoretical density-effect correction derived especially for silver chloride, we have evaluated the correction needed to get good agreement between the experimental and theoretical distributions. It is possible to make a reasonable guess at the correction because the shape of the curve (as a function of p/μ) for the correction is known from the work of Halpern and Hall.[6] All that remains is to determine one parameter. Over a considerable range, the correction is represented by a straight line when plotted as a function of p/μ on semilog paper (see Fig. 5). The dashed curve of Fig. 5 gives the correction, which, when applied to the theoretical distribution, best describes the results of the experiment. The resulting calculated pulse distribution fits the experimental data well. The widths at half-value agree very well. The theory predicts that about 9 percent of the pulses should saturate the amplifier, whereas actually 6.5 percent did. For this distribution, as for the one due to mesons stopping in 23 cm of lead, a relatively large number of pulses were smaller than the theory predicts. The explanation here, as before, is probably that these pulses are due to regions of low sensitivity.

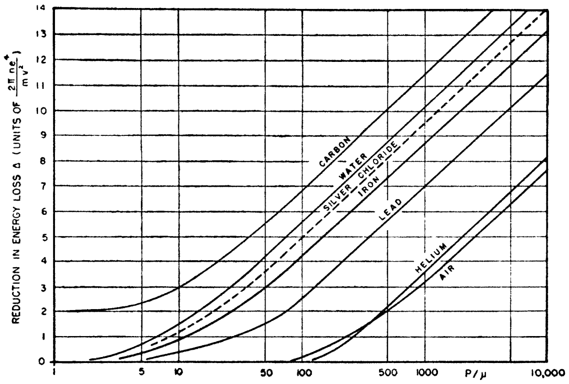

FIG. 5. Reduction in ionization loss due to density effect for various materials (after Halpern and Hall). $2\pi ne^4/mc^2$ for silver chloride is 0.0685 Mev/(g/cm^2).

In the light of the material presented in this section, we draw the following conclusions. In the first place, the fluctuation theory of Symon seems to be well verified experimentally for the two groups of high energy mesons selected for this experiment. In the second place, the density effect for high energy mesons provides a satisfactory explanation of the fact that we obtained a smaller increase in the rate of ionization than the theory of Bethe and Bloch predicts.

We wish to express our indebtedness to Dr. M. L. Sands for many helpful suggestions on circuit design, to Dr. H. G. Voorhies who aided in taking some of the data, and to Mr. Carl Johnson and Mr. Stanley Gardner who were responsible for the mechanical construction of the crystal holder and certain other parts of the equipment.