Neutron Polarization*

J. FLEEMAN, D. B. NICODEMUS, AND H. H. STAUB**
Stanford University, Stanford, California
(Received September 1, 1949)

Experiments for the determination of the polarization cross section p of iron with monochromatic and non-monochromatic neutrons are described. The absolute value of p as well as its dependence on the neutron velocity is found to be in good agreement with the recent calculations of Steinberger and Wick and also with other experiments. For a complex neutron spectrum originating from a paraffin moderator $\bar{p}$ is found to be $(2.35\pm0.1)\times10^{-24}$ cm² in agreement with previous investigations. It is shown that the larger value of 3.15×10^{-24} cm² found by Hughes, Wallace, and Holtzman must be due to spectral differences. The approach of the magnetization of iron towards saturation is also investigated.

I. INTRODUCTION

THERMAL neutrons which pass through a slab of magnetized ferromagnetic material will undergo partial polarization because the scattering cross section depends, by way of the magnetic interaction of the iron atom and the neutron, on the relative orientation of the neutron spin and the magnetic moment of the iron atom. Bloch[1] first showed that the scattering cross section per nucleus of a magnetically saturated material can be written as

$$\sigma=\sigma_0\pm p, \tag{1}$$

where σ_0 represents the cross section for complete demagnetization (including a quadratic term due to magnetic interaction) and p the polarization cross section. The $\pm$ sign refers to the two possible magnetic quantum numbers $m_s=\pm\frac{1}{2}$ of the incident neutron. Although the magnetic contribution to σ_0 might be different for complete magnetization and complete demagnetization, the difference is so small that usually it can safely be neglected. As a consequence of the difference in the scattering cross section for the two states, $m_s=\pm\frac{1}{2}$, the intensity I of a beam of neutrons passing through a slab of iron will undergo an increase ΔI upon complete magnetization which is given by

$$\eta_\infty=\Delta I/I=\cosh npd-1 \tag{2}$$

$$\approx\tfrac{1}{2}n^2p^2d^2 \quad \text{for} \quad npd\ll1. \tag{2'}$$

d represents the thickness of the iron slab and n the number of iron atoms per unit volume.

If the iron is partially magnetized, not only will the neutrons passing through the material undergo polarization, but also the inhomogeneities of the magnetic field will induce transitions between the two states and thereby will again partly depolarize the beam. Halpern and Holstein[2] have made a detailed theoretical study of this effect and have shown that for partly magnetized iron Eq. (2') has to be replaced by

$$\left.\begin{aligned}&\eta=\tfrac{1}{2}n^2p^2d^2f(\lambda/\epsilon d),\\ \text{where}\quad&f(x)=2x^2[e^{-1/x}+(1/x)-1]\end{aligned}\right\}. \tag{3}$$

The depolarization factor $f(\lambda/\epsilon d)$ depends on the lack of magnetic saturation,

$$\epsilon=\frac{M_\infty-M}{M_\infty}, \tag{4}$$

where M represents the actual magnetization, M_∞ its saturation value, and λ a parameter related to the size of the microcrystals in the iron. The microcrystals are supposed to be magnetically saturated but their deviation from perfect alignment is responsible for the incomplete magnetic saturation and the depolarization of the neutrons. The quantity λ depends not only on the average linear dimension δ of the microcrystals but in general also on a characteristic length $2\pi l$, the distance over which a neutron travels in the material during one full Larmor precession about the internal field B.

$$l=v/B\gamma, \tag{5}$$

where v is the velocity of the neutron and γ its gyromagnetic ratio (1.83×10^4 e.m.u.). For $v=2200$ m/sec. and $B=20{,}000$ gauss, l is about 6×10^{-4} cm. For the two extreme cases of large and small crystalline dimensions λ is given by

$$\left.\begin{aligned}&\lambda\approx\delta/2 \quad \text{for} \quad \delta\gg l\\ \text{and}\quad&\lambda\approx l^2/\delta \quad \text{for} \quad \delta\ll l\end{aligned}\right\}. \tag{6}$$

The polarization cross section p depends strongly on the velocity of the neutrons. In the first place, this dependence is caused by the form factor of the $3d$ electrons in the iron atom, which describes the magnetic moment density within the scattering atom. In the second place, since p represents an interference term between nuclear and magnetic scattering, it will exhibit a similar dependence on the neutron velocity as does the scattering cross section itself. In a crystalline material the scattered intensity is largely concentrated in the well-known Debye-Scherrer rings, whose number decreases rapidly with increasing de Broglie wave-length of the neutrons. Whenever the neutron wave-length becomes too large to satisfy the Bragg condition for a particular set of lattice planes

* Assisted by the Joint Programs of the ONR and the AEC.
** Now at the University of Zurich, Switzerland.
[1] F. Bloch, Phys. Rev. **50**, 259 (1936); **51**, 994 (1937).
[2] O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941).

there will be a discontinuous drop in the scattering intensity. These wave-lengths are given by

$$\lambda_c=\frac{2a}{(h_1^2+h_2^2+h_3^2)^{\frac{1}{2}}},$$

where a is the lattice constant of iron and h_1, h_2, h_3, the Miller indices of the lattice plane. For neutrons with wave-length larger than 4.04A no coherent crystalline scattering can take place and likewise one would expect the polarization effect to be absent. In addition to the coherent scattering, however, a small amount of diffuse incoherent scattering is present. It is caused by

(a) Scattering processes in which the neutron spin changes its orientation. Their effect is, however, negligible in the case of iron, since both predominant isotopes (Fe^{54} and Fe^{56}) have presumably zero spin.

(b) Lattice irregularities due to the presence of isotopes.

(c) Inelastic scattering due to temperature motion of the crystal lattice (Debye-Waller factor) and lattice imperfections.

Halpern[3] has shown that the incoherent scattering processes under (c) are the only ones which will contribute to the polarization cross section p.

Values of p have previously been calculated by Halpern, Hamermesh, and Johnson[4] and by Hamermesh.[5] Their results were very much smaller than the observed values.[6–9] More recently Steinberger and Wick[10] recalculated p for the velocities of the neutrons between 1000 and 2800 m/sec. They found values which were indeed considerably larger than those of Halpern, Hamermesh and Johnson, and Hamermesh. The increase is caused by the following factors: (1) The nuclear scattering cross section as redetermined, for instance, by Fermi and Marshall.[11] (2) The contribution of the diffuse temperature scattering. (3) A revised form factor of the $3d$ electrons in iron. The results of Steinberger and Wick† are shown as the solid curve in Fig. 4.

On the experimental side Powers[12] showed that polarization of the neutrons could be obtained only if the iron was almost completely magnetized, as one would expect from expression (3). For a thickness d of 1 cm, for instance, and $\lambda=\delta/2\sim10^{-3}$ cm, the function f(x), defined in (3) has the value 1/2 for $\epsilon=2.5\times10^{-3}$. In a detailed study Bloch, Hamermesh and Staub[6] and Bloch, Condit and Staub[8] investigated the dependence of the polarization effect η on the thickness d‡ and the degree of saturation ϵ. They showed that the polarization effect for complete saturation increases quadratically with the thickness. By direct magnetic measurements of ϵ at magnetizing fields up to 5000 gauss they found that the polarization effects could be represented by the expression (3) given by Halpern and Holstein. They also determined the value of $\bar{p}$ for an average thermal neutron velocity as about 2.1×10^{-24} cm^2. Subsequently Fryer measured the value of p with monochromatic neutrons at several different velocities. While the relative increase of p with decreasing velocity was in reasonable agreement with the predictions of Halpern, Hamermesh and Johnson, the absolute magnitude of p was much larger than the calaulated value.

More recently Hughes, Wallace, and Holtzman[9] (referred to hereafter as H.W.H.) made a detailed study of the neutron polarization effect. Instead of measuring ϵ directly, they determined the two parameters λ/ϵ and p from measurements of η at a number of values of d for a constant magnetization close to saturation. Utilizing the thermal neutrons from the graphite column of a pile, they determined an average value $\bar{p}$ of 3.15×10^{-24} cm^2 which is considerably larger than the value found in the previous work. H.W.H. also measured p for different neutron velocities, particularly in the region between the last two Bragg discontinuities. At low velocities they found p to be considerably larger than one value given by Fryer at $v=1310$ m/sec., while at higher velocities their results agree well with those of Fryer. A direct metallographic determination, as well as the fact that H.W.H. found $f(x)$ to be independent of the velocity of the neutrons, showed clearly that for the material used in their investigation the crystalline dimensions were large compared to the critical length l.

Since the measurements of p at low velocities of H.W.H. are in very good agreement with the theoretical predictions of Steinberger and Wick, it was felt that more accurate measurements should be made at higher velocities. The values of the atomic form factor F at low velocities are quite certain, since for $v=0$, F approaches unity. At higher velocities, however, the form factor depends very critically on the particular choice of the electron distribution and it is therefore of interest to verify the calculations of Steinberger and Wick in this region. In addition, the present investigation was made in order to determine the reason for the considerable discrepancy in the average value $\bar{p}$ as found at Stanford and by the Argonne Laboratory. H.W.H. suggested that the discrepancy was caused by an overestimate of the quantity ϵ by the Stanford workers. While $1/\epsilon$ is proportional to H^2, the square of the magnetizing field, in a region of moderate fields

[3] O. Halpern, Phys. Rev. **72**, 260 (1947).
[4] Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[5] M. Hamermesh, Phys. Rev. **61**, 17 (1942).
[6] Bloch, Hamermesh, and Staub, Phys. Rev. **64**, 47 (1943).
[7] E. M. Fryer, Phys. Rev. **70**, 235 (1946).
[8] Bloch, Condit, and Staub, Phys. Rev. **70**, 972 (1946).
[9] Hughes, Wallace, and Holtzman, Phys. Rev. **73**, 1277 (1948).
[10] J. Steinberger and G. C. Wick, Phys. Rev. **74**, 1207 (1948).
[11] E. Fermi and L. W. Marshall, Phys. Rev. **71**, 666 (1947).
† We are greatly indebted to Professor Wick for kindly sending us a copy of his curve prior to publication.
[12] P. Powers, Phys. Rev. **54**, 827 (1938).
‡ It should be noted that, contrary to the statement of Hughes, Wallace and Holtzman (see reference 9), both previous measurements (see references 6 and 8), at this laboratory were carried out with two different thicknesses of iron. It was indeed just the particular form of $f(x)$ predicting a linear increase of η at low magnetizing fields and a quadratic dependence near saturation which prompted the use of different thicknesses.

(1000 to 5000 gauss), it is to be expected that $1/\epsilon$ increases less rapidly at higher fields. The value of ϵ had been obtained by the Stanford group by direct magnetic measurement and the proportionality of $1/\epsilon$ with H^2 had been well established in the region of low H. The values of ϵ at higher fields were simply obtained by extrapolation of the quadratic law and could therefore indeed be overestimated and thereby cause an underestimate of $\bar{p}$. On the other hand, this explanation fails to account for the rather close agreement of the measurements with monochromatic neutrons and the fact that the saturation values of η were very closely proportional to d^2. It must, therefore, be discarded as a satisfactory explanation of the above-mentioned discrepancy; such an explanation can be found, however, as a consequence of differences in the distribution of the neutron velocities (see Section IV), particularly at low values.

II. EXPERIMENTAL ARRANGEMENT

a. Apparatus

The measurements of $\bar{p}$ were performed essentially with the same arrangement as used by Bloch, Condit, and Staub.[8] A beam of collimated slow neutrons from Be bombarded with 2.7-Mev deuterons and moderated in a block of paraffin surrounding the target of the cyclotron, passes through cadmium channels 6 in. long and with an opening of $1\frac{1}{2}$ in.$\times$2 in. attached to both sides of the iron sample. The samples $1\frac{1}{2}$ in.$\times$2 in. fit snugly into the gap of the magnet. The direction of the

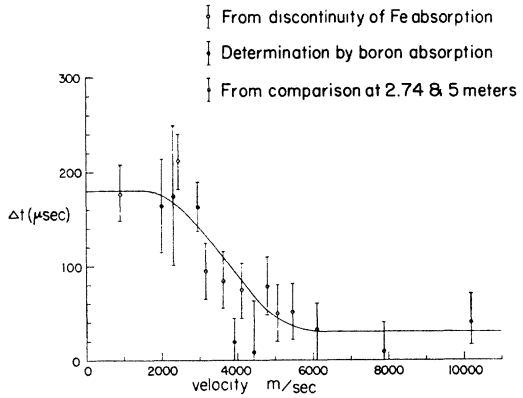

FIG. 1. Delay time Δt as a function of the neutron velocity v.

magnetization is at right angles to the neutron beam. For the present measurements two identical magnets were available and for larger thicknesses of the sample we distributed the iron equally between the two magnets which were operated so that they gave the same magnetizing field. Both fields always pointed in the same direction, and between the magnets there was no region where the field dropped to less than 100 gauss. Careful measurements showed that a piece of 1 in. thickness in one magnet and a $\frac{1}{2}$-in. piece in each magnet gave identical results. By using two magnets we were able to extend our measurements to thicknesses up to 2 in. with a certainty that the pieces were homogeneously magnetized. After passing through the iron blocks the neutrons entered the detector through a circular cadmium channel 30 in. in length and $2\frac{1}{4}$ in. in diameter. In this measurement the filling of the counter of 10-in. effective length consisted of B^{10} enriched boron trifluoride. Except for a $1\frac{1}{2}$ in.$\times$2 in. opening at the front, the chamber was shielded by cadmium, boron carbide, and a thick shield of hydrogenous material. The sensitivity of the detector for fast neutrons is considerably greater than that of the counter used by Bloch, Condit, and Staub. Consequently the present results cannot be compared immediately to the previous ones. The increased sensitivity of the enriched filling for fast neutrons will decrease throughout the polarization effects measured with non-monochromatic neutrons. The counter was placed at various distances, 2.74 m, 4 m, and 5 m, from the neutron source. The measurement of the magnetizing field H was done in the usual manner by placing a small rectangular search coil of $\frac{1}{16}$ in. thickness against the surface of the steel sample. The coil was calibrated against a proton nuclear induction signal. The plates of hot rolled steel were cut from the same material used in the previous investigations.[6-8]

For the measurements with monochromatic neutrons a modulation equipment of the usual design was built. A repetition frequency of 150, 250, or 300 cycles per sec. was derived from an audio frequency oscillator. Its sinusoidal output operated a pulse former which activated one univibrator, to provide the square-wave signal of variable duration τ_A (50 to 100 μsec.) for modulating the ion source of the cyclotron. Simultaneously the pulse former started a second univibrator, whose square pulse determined the variable interval t' (200 to 5000 μsec.) between production of neutrons and their detection. The trailing edge of the latter square pulse in turn operated a third univibrator providing the square pulse of duration τ_B for the activation of the detecting instrument. This latter process was performed by a conventional diode coincidence circuit receiving on one side the amplified pulses from the detector and on its other side the activating square pulse. The various pulses were displayed on a synchronized cathode-ray tube in order to determine their relative positions and duration. Time marks at 333 μsec. intervals served to calibrate the horizontal axis of the oscilloscope. A special circuit was incorporated which permitted the reading of the interval τ_B of the amplifier pulse, by means of a meter, to a high degree of accuracy. This was found necessary, since the system of monitoring the neutron intensity would not account for variations of the quantity τ_B.

The measurements of the polarization effect with non-monochromatic neutrons were monitored by an integrating boron trifluoride chamber receiving neutrons from the same region of the moderator at which

the main beam originated. For the measurements of the spectral distribution, intervals of equal total number of incident neutrons were determined by recording the total number of counts of the detector through an unmodulated channel. For the measurements of absorption and polarization with monoenergetic neutrons, an additional boron trifluoride counter for monitoring was placed in the vicinity of the main counter.

b. Calibration of Neutron Spectrometer

It is well known that the time interval t', elapsing between the time of the burst of ions in the source of the cyclotron and the time at which the detector becomes active does not give directly the actual flight time t of the neutrons. In fact, t' is always larger than t by an amount Δt,

$$t'=t+\Delta t,$$

The delay Δt is caused by the time required for accelerating the deuterons ($\sim$3 μsec.), slowing down of the neutrons to thermal energies ($\sim$5 μsec.), the effective lifetime of the neutrons in the paraffin (up to 200 μsec.) and delays in the detecting system ($\sim$5 μsec.). The main contribution caused by the finite lifetime of the neutrons in the moderator, due to its exponential character, does not represent simply a time delay but rather a spread with an average delay. Moreover, the average delay will depend on the particular velocity of the neutrons under consideration. In the present experiments we determined the total average delay Δt for the various velocities by three independent measurements. For high velocities ($>$3000 m/sec.) we measured the actual velocity by the transmission of the neutrons through Pyrex glass plates of various thicknesses. The boron content of the plates was determined at one velocity by comparison with the transmission through a plate of fused boron trioxide. Assuming the composition of Pyrex to be boron trioxide and silicon dioxide[13]) it was found that the boron trioxide content was 13.3$\pm$0.4 percent. The velocity of the neutrons was determined from the value of the absorption cross section of boron (743×10^{-24} cm^2 at $v=2200$ m/sec.,[13]) assuming the $1/v$ law and taking into account the attenuation due to silicon and oxygen. Since this additional attenuation is not very large and is due mainly to scattering, a constant cross section was assumed for O and Si. At intermediate velocities from 3000 m/sec. to 1500 m/sec. the delay was determined by comparing similar points in the neutron spectrum measured at two different distances L_1 and L_2. For instance, let t_1' and t_2' be the time interval at which one observes the intensity of the spectrum to have dropped to half its maximum value, measured at distances L_1 and L_2, respectively, between source and detector. Since the actual velocities are the same for both distances, one obtains Δt through the relation

$$v=\frac{L_1}{t_1'-\Delta t}=\frac{L_2}{t_2'-\Delta t}.$$

In this manner Δt was determined from the measurements of the spectrum at distances of 2.74 m and 5 m. For the lowest value of the velocity $v=983$ m/sec. we determined the delay by observing the spectral distribution of the neutron beam after passage through a plate of iron. Since, at a de Broglie wave-length $\lambda=4.04$A, corresponding to a velocity of 983 m/sec., the scattering cross section drops from a value of about 17×10^{-24} cm^2 discontinuously to almost zero, the transmitted spectrum exhibits a sudden increase in intensity at this point. This discontinuity can be very accurately located. Unfortunately, the other discontinuities occurring at velocities of 1390, 1700, 1960, etc., m/sec. are not nearly so pronounced and their location is not so easily determined. From the compound evidence of the various measurements which partially overlapped we were able to construct a plausible curve, shown in Fig. 1, describing the dependence of Δt on the velocity of the neutrons. For velocities at which the neutrons are in a pseudothermal equilibrium with the molecules of the moderator, the delay is of the order of 200 μsec., as should be expected, since this represents the average lifetime of a thermal neutron in paraffin. At higher velocities where many neutrons escape from the moderator without having undergone a large number of collisions, the delay time becomes gradually smaller. For velocities at which a thermal equilibrium distribution would contain hardly any neutrons the delay has dropped to about 20 μsec.

III. MEASUREMENTS

a. Experiments with Non-Monochromatic Neutrons

In a first series of experiments we measured the average polarization effect as a function of the thickness d of the sample with considerable accuracy for thicknesses from 0.635 to 5.08 cm in steps of 0.635 cm at a constant magnetizing field of $H=11{,}200$ gauss. The greater accuracy obtained in the present experiments is mainly due to the increased sensitivity of the detector. For every measurement counts were recorded alternately with the field on and off during short intervals monitored by the integrating boron trifluoride chamber. For every group of measurements the cadmium background ranging from 10 to 40 percent, according to the thickness of iron, was determined and subtracted from the results. No polarization effect could be observed for neutrons passing through cadmium. The net polarization effect η' is given by

$$\eta'=(N'-N)/(N-N_c),$$

where N', N and N_c are the total number of counts for an equal number of monitor intervals with the iron

[13] J. Rainwater and W. W. Havens, Jr., Phys. Rev. **70**, 136 (1946).

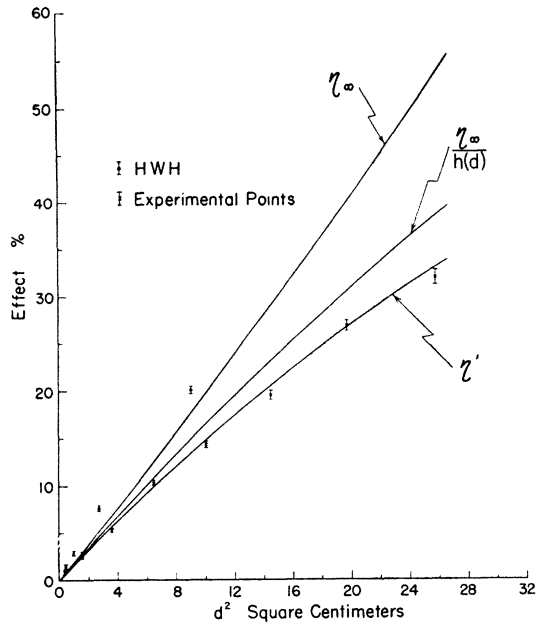

FIG. 2. Transmission effect *versus* d^2. Uppermost curve: $\eta_\infty = (\cosh n\bar{p}d - 1)$ calculated with $\bar{p} = 2.35 \times 10^{-24}$ cm². Middle curve: η_∞/h calculated with the values of h as given in Figure 6. Lowest curve represents η' taking lack of saturation into account using $\lambda/\epsilon = 10$ cm.

magnetized, unmagnetized, and with a cadmium shield, respectively. The results of these measurements with their usual standard deviations are represented in Fig. 2. For comparison we have included in the figure the values given by H.W.H., measured at a field of 12,000 gauss, close to that at which our data were taken.

As the neutrons pass through increasing thicknesses of iron a non-monochromatic beam will change its spectral composition, and one can only determine values of $\bar{p}$ and λ/ϵ if proper corrections are applied for this change. If for every neutron velocity $npd \ll 1$ and assuming that the depolarization function $f(\lambda/\epsilon d)$ does not depend on the velocity, the observed transmission effect η' is given by

$$\eta' = \tfrac{1}{2}n^2d^2 \frac{\int_0^\infty p^2(t)F(t)e^{-n\sigma(t)d}dt}{\int_0^\infty F(t)e^{-n\sigma(t)d}dt} f\left(\frac{\lambda}{\epsilon d}\right), \qquad (7)$$

where $F(t)$ represents the spectral distribution of the unattenuated neutron beam in terms of its time of flight t, and $p(t)$ the polarization cross section. If $\eta = h\eta'$ represents the transmission effect at the same thickness without spectral modification, the "hardening factor" h is given by

$$h(d) = \frac{\int p^2Fdt \int Fe^{-n\sigma d}dt}{\int Fdt \int p^2Fe^{-n\sigma d}dt}. \qquad (8)$$

The values of the hardening factor were computed on the basis of the measurements described below. In order to have a rough check on the calculation we measured the transmission effect $\eta''(d)$ for a thickness $d = 1''$ with a continuously unmagnetized piece of iron of thickness $d = 1''$ placed in the path of the beam. From Eqs. (7) and (8) it follows with the above-mentioned assumption that

$$[\eta'(d)]/[\eta''(d)] = [h(2d)]/[h(d)]. \qquad (9)$$

Relations Eqs. (7) and (9) hold, of course, only as long as $npd \ll 1$ for every value of v. For large thicknesses the relations become quite involved, particularly since

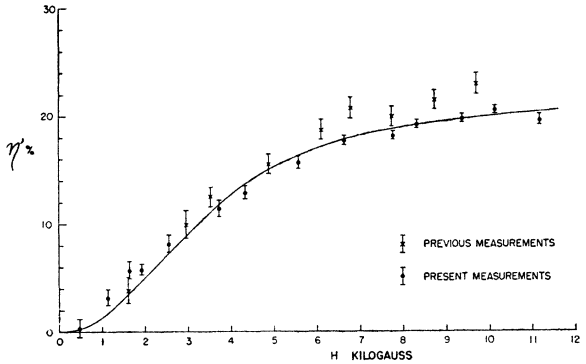

FIG. 3. Dependence of η' on magnetizing field H. Curve calculated with $\bar{p} = 2.35 \times 10^{-24}$ cm² and using data from Fig. 6. ($d = 3.81$ cm.)

lack of saturation can no longer be described by the simple expression Eq. (3). For $d = 1''$, however, the approximation is still justifiable. The result of the measurement of η'' for $d = 2.54$ cm and with prefiltering by 2.54 cm of iron gave a ratio

$$[h(5.08)]/[h(2.54)] = 1.14 \pm .07.$$

Since, as mentioned before, the results of the previous investigation[8] were obtained with a detector of different spectral sensitivity, a direct comparison with the present results is not possible. It can be seen that for a thickness of $1\frac{1}{2}$ in. in the previous work η' was about 23 percent, as compared to 21 percent in the present investigation. It was therefore necessary to repeat the measurements of the dependence of η on the magnitude of the magnetizing field H. This was done for a thickness $d = 3.81$ cm in one of the magnets in the same manner as in the other measurements. The measured uncorrected values of η' are presented in Fig. 3, together with the values found by Bloch, Condit, and Staub.[8] The effect of the different spectral sensitivity is quite apparent. Values of the former work are somewhat higher than the present ones. This is to be expected, as the more shallow chamber used in the earlier work favors slower neutrons showing a higher polarization.

b. Experiments with Monochromatic Neutrons

As indicated in Section I, detailed measurements of the transmission effect with monochromatic neutrons

were made in the velocity region between 1400 m/sec. and 2800 m/sec. Data were taken at distances between source and detector of 2.74 m and 4 m with iron having a thickness of 2.54 cm and distributed equally between the two magnets and magnetized to 11,200 gauss. At a distance of 2.74 m the resolution was not very good, and even though the two time intervals τ_A and τ_B were reduced to 100 μsec. each, the exponential decay of the neutrons in the moderator reduces the resolution considerably. This is particularly true if the intensity of the neutrons decreases rapidly with increasing flight time. The data at 4 m are therefore much more reliable. As a check we also measured the transmission effect at a low velocity $v=1120$ m/sec. The observed values η of the transmission effect were divided by a constant value $f(x)=0.89$ as given in Section IV. The polarization cross section p follows then from Eq. (3) or the corresponding exact expression of Halpern and Holstein. The values of p determined in this manner are shown in Fig. 4. The horizontal bars represent the resolution due to the finite magnitude of τ_A and τ_B without taking into account the exponential decay of the neutron intensity.

For the determination of the hardening factor h, it is also necessary to know the spectral distribution quite accurately. As mentioned before, it was measured at two distances, 2.74 m and 5.0 m. The intensity at each value of t', measured for equal interval width τ_B, is

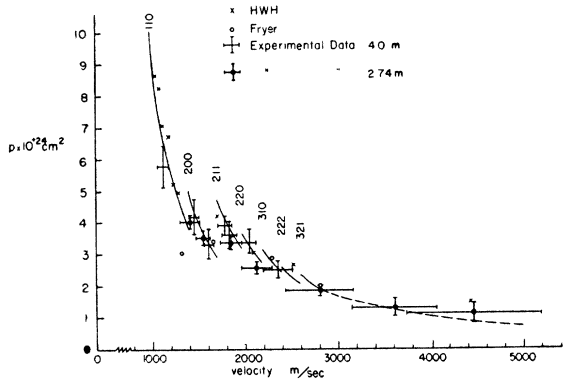

FIG. 4. Dependence of p on neutron velocity v. Curve according to Steinberger and Wick. Horizontal bars indicate resolution. Data of Fryer and H.W.H. are also included.

subject to various corrections. In the first place a small number (≈ 1 percent) of background counts has to be subtracted. In addition, at high neutron velocities there are always a number of low velocity neutrons present from the previous modulation cycle. Their number can be determined directly from the spectral distribution itself. Applying these two corrections one obtains the distribution $F'(t')$ per unit interval of time t'. Finally, since one wants to know the intensity in units of the inverse velocity, $t_0=t/L$, the variation of the delay Δt with flight time has to be considered. Since $F'(t')dt'$ $=F'(t+\Delta t)[1+(d\Delta t/dt)]dt$, one has $F(t_0)=\text{const.}\times F'(t_0L+\Delta t)[1+(d\Delta t/dt_0)/L]$. The values obtained in this manner for both distances are presented in Fig. 5. For comparison the figure also shows the distribution which one would expect if the neutrons were in equilibrium with the moderator. This function is a Maxwell distribution, multiplied with the velocity v to represent the flux and with the sensitivity function $[1-e^{-n_0\sigma(v)}]$ of the detector. n_0 represents the number of boron atoms per unit area of the detector, and $\sigma(v)=a/v$ the cross section of boron. In terms of t_0 the function is therefore

$$F_M(t_0)=\frac{\text{const.}}{t_0^5}\exp\left(-\frac{m}{2kTt_0^2}\right)[1-e^{-n_0 a t_0}].$$

This function was matched to the experimental points for equal maximum value. Figure 5 shows that the positions of the maxima of the actual distribution and the Maxwellian are very close. For large values of t_0 there is a marked deficiency and for high velocities a rather large surplus of neutrons as compared to the Maxwellian; this is to be expected since the neutrons will not reach perfect thermal equilibrium in the moderator.

Finally, the determination of h also requires the knowledge of the total transmission cross section σ of iron as a function of the neutron velocity. Since H.W.H. made a very detailed investigation of this quantity, we determined σ as a check at three velocities, 1770, 3390, and 4750 m/sec. The values of σ were found to be 13.0 ± 0.1, 12.6 ± 0.1, and $12.5\pm 0.1\times 10^{-24}$ cm^2, respectively, in good agreement with H.W.H.

In all our calculations we have assumed that, contrary to our previous belief, the size of the microcrystals in iron is large compared to the critical length l, as it was found by H.W.H. In order to ascertain this fact, the grain size in the iron samples was investigated

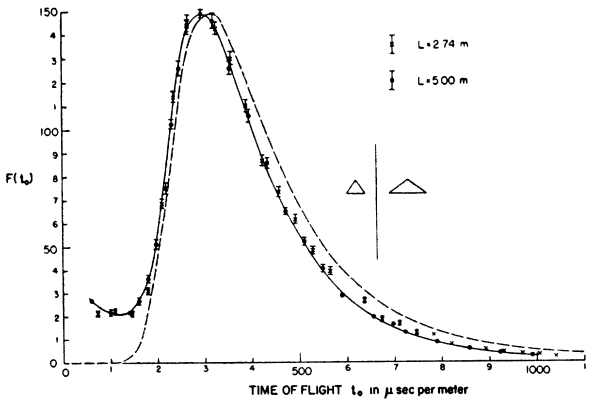

FIG. 5. Spectral distribution of neutrons per unit flight time interval. Dotted curve calculated for Maxwellian distribution at room temperature. The points on the right and on the left side of the vertical line were taken with different time intervals; the detection sensitivity in the two cases is indicated respectively by the two triangles.

metallographically.†† It was found that in two planes, oriented differently relative to the rolling direction of the steel, the average grain size was identical and equal to 35×10^{-4} cm. This is very much larger than the critical length and consequently the first alternative of Eq. (6) is well justified.

IV. RESULTS AND DISCUSSION

Figure 4 shows that the present experimental data are in good agreement with the recent calculations of Steinberger and Wick. Actually, these authors have computed p for the two alternatives, assuming the electronic wave function of the iron atom to have either zero slope or zero value at the atomic radius. The values of p computed for zero slope are somewhat smaller throughout and seem to fit the present data and those of H.W.H. somewhat better. These latter values were computed from the observed transmission effects η as given by these authors, using a constant value of $f(x)=0.81$. Four values of Fryer[7] were also recalculated on the basis of a constant $f(x)=0.71$. Our present data show rather clearly that the magnitude of the discontinuities of p at $v=1390$ and 1700 m/sec. are in good agreement with the calculations of Steinberger and Wick. There arises, of course, the question whether this agreement is not somewhat fortuitous. While the inelastic scattering due to thermal motion has been taken into account in the calculations, other sources of incoherent scattering have been disregarded. The only additional type of incoherent scattering which can contribute to the polarization are lattice imperfections, while spin-dependent scattering and isotopic-disorder scattering should not contribute to p. A preliminary study in this laboratory by Mr. M. Nielsen of the transmission of monochromatic neutrons through a single crystal of iron as suggested by Halpern[3] seems to indicate that the scattering cross section of a single crystal is considerably larger than one would expect from inelastic lattice scattering alone. However, the observed polarization effects in the single crystal seem to be very small, indicating, as is to be expected, that only the inelastic lattice scattering contributes to the polarization effect.

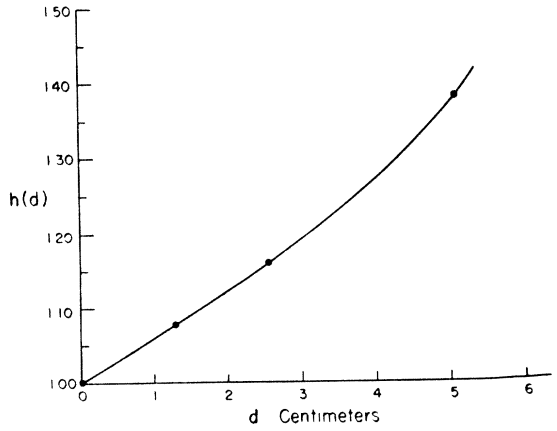

FIG. 6. Hardening factor h *versus* thickness d.

From the polarization cross section, the total transmission cross section and the spectral distribution, the values of the hardening factor h were calculated by numerical integration for three thicknesses of $d=1.27$, 2.54, and 5.08 cm, according to Eq. (8). The values of h for other thicknesses were obtained by interpolation according to the curve shown in Fig. 6. Since the theoretical values of p seem to be consistently somewhat higher than our observed values, we have lowered the theoretical values by 5 percent for our calculations. At velocities larger than 2800 m/sec. we have extrapolated Steinberger and Wick's curve, using the form factor given by Hamermesh.[5] The values of the transmission cross section σ were obtained from a curve fitted to the observed values of H.W.H. in such a manner that the ratio of the discontinuities agreed with that of the curve computed by Halpern, Hamermesh, and Johnson.[4] The results of the calculations of h for 2.54 and 5.08 cm are 1.163 and 1.39, respectively, giving a ratio of 1.19 as compared to the measured ratio 1.14 ± 0.07. Similarly, one can obtain a crude check of the calculations from the data of Bloch, Condit, and Staub. The difference in the observed values of $\bar{p}$ at $d=1.90$ and 3.8 cm indicates a ratio of the hardening factors of 1.15 ± 0.05 as compared to the calculated values of $1.256/1.116=1.12$. After correcting the values of η' with the calculated hardening factors, the average value $\bar{p}$ and the parameter λ/ϵ can be found by fitting the η-values to a law of the form,

$$\eta=(\cosh n\bar{p}d-1)f(\lambda/\epsilon d).$$

This law is not correct, of course, for $npd\approx1$, but should at least be a better approximation than a simple proportionality with d^2. The best fit is obtained with $\bar{p}=2.35\pm.1\times10^{-24}$ cm^2 and λ/ϵ lying between 5 and 35 cm with a most probable value of 10 cm. The reason for this large uncertainty is obvious. Since all $f(x)$ values deviate only little from unity, they are very insensitive to large variations in x, or in other words, the iron samples are indeed quite well saturated magnetically. The resultant value of $\bar{p}$ is in excellent agreement with the value given by Bloch, Condit, and Staub. If one applies the same corrections for hardening of the beam to their result (2.1 and 2.2×10^{-24} cm^2, respectively) one obtains the same value for both thicknesses of about 2.45×10^{-24} cm^2. The slightly larger value of $\bar{p}$ is of course caused by the use of a detector of different sensitivity. The value of 2.35×10^{-24} cm^2 is again very much lower than 3.15×10^{-24} cm^2 as given by H.W.H., but the reason for the difference seems clearly to lie in the different spectral composition of the neutrons and possibly a different sensitivity of the detector. The values of η' at different thicknesses d

†† We are greatly indebted to Professor O. C. Shepard of the Department of Metallurgical Engineering at Stanford for carrying out this investigation.

were measured by us at a field strength very close to that of H.W.H. Assuming that the degree of magnetic saturation is roughly the same, it appears from the fact that the values of H.W.H. at small d's are about 1.7 times larger than ours. The effective cross section should be $(1.7)^{\frac{1}{2}}$ times larger than in our case, or about 3.05×10^{-24} cm^2. Actually, the drop in the ratio of the values of η' of H.W.H. and the present measurements at larger thicknesses indicates that the degree of magnetic saturation in our case is slightly higher. In Fig. 2 the curve $\eta'=\eta_\infty f(x)/h$ is computed with the values of $\bar{p}$ and λ/ϵ as given above. We have also calculated by numerical integration over the measured spectrum the polarization cross section $\bar{p}$ which one would expect for non-monochromatic neutrons, again using the values of $p(v)$ of Steinberger and Wick, lowered by 5 percent. The calculations give $\bar{p}=2.42$, 2.45, 2.61×10^{-24} cm^2 for $d=1.27$, 2.54, and 5.08 cm, respectively, as compared to the measured value of 2.35×10^{-24} cm^2. For the purpose of evaluating $\bar{p}$ from the measured values of η we assumed a relation of the form: $\eta=[\cosh npd-1]f(x)$ to hold. While for $npd\ll1$ this is correct even for the case of a complex neutron spectrum, the relation certainly does not hold for $npd\sim1$ and $f(x)\neq1$ even for monochromatic neutrons. The exact law for monochromatic neutrons given by Halpern and Holstein is quite involved and for a complex neutron spectrum it would predict a rather complicated dependence of η on d. While the simple relation $\eta=\frac{1}{2}n^2\bar{p}^2d^2f(x)$ for large values of npd leads to an underestimate of η, a relation of the form $\eta=(\cosh npd-1)f(x)$ results for values of $f(x)$ close to unity in a slight overestimate of η. The agreement between the values of $\bar{p}$ calculated from the observed values of η and from the spectral distribution is satisfactory for thicknesses $d=1.27$ and 2.54 cm. For $d=5.08$ cm the difference of 12 percent between the value of $\bar{p}$ inferred from the observed values of η and that calculated from the spectral distribution is therefore not surprising in view of the fact that in this case, for certain values of the velocity, npd becomes as large as 4. For a true Maxwellian distribution and a thick detector, the calculation for $d=2.54$ cm gives a value of $\bar{p}=2.75\times10^{-24}$ cm^2, showing clearly the rather sensitive dependence of the transmission effect on the spectral distribution.

The evidence that the difference in the observed values of $\bar{p}$ is not caused by an overestimate of $1/\epsilon$ by Bloch, Condit, and Staub, as suggested by H.W.H., is further corroborated by the evaluation of the measurements of the dependence of η on H. With the value $\bar{p}=2.35\times10^{-24}$ cm^2 the quantity x is determined for each value of the field H, as represented in Fig. 7. In order to find the proportionality constant λ/d we used the direct magnetic measurements of Bloch, Hamermesh, and Staub in the region between 1600 gauss and

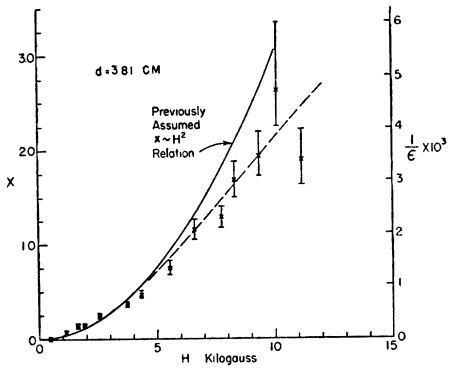

FIG. 7. Dependence of $1/\epsilon$ the degree of magnetic saturation on the magnetizing field H. Solid curve according to the previously assumed quadratic law.

4500 gauss. The observed values of x indeed fit the law $1/\epsilon=bH^2$ quite well in this region and the parameter λ is found to be $\lambda=(21.5\pm2)\times10^{-4}$ cm, and therefore the size of the microcrystals: $\delta=2\lambda=(43\pm4)\times10^{-4}$ cm. in close agreement with the metallurgical determination. Actually, not too much significance should be attributed to this agreement. The microcrystalline size δ as determined by neutron depolarization is certainly only to the order of magnitude equal to that of the average microscopic determination.

Finally, Fig. 7 shows, at the right-hand ordinate, how $1/\epsilon$ depends on H. It can indeed be seen that $1/\epsilon$ does not increase as rapidly with H as the quadratic law predicts for fields in excess of 5000 gauss. It is difficult to establish the exact form of the law since again the measured quantity $f(x)$ becomes increasingly less sensitive to changes in x. Assuming, for instance, a linear dependence as indicated in Fig. 7 by the dotted line, one can, with the reduced values of x and $f(x)$ replot the data of Bloch, Condit, and Staub. With the values $\bar{p}=2.45\times10^{-24}$ cm^2 and $\lambda=21.5\times10^{-4}$ cm their data fit the calculated curve exceedingly well. The value of $\bar{p}$ thus determined is slightly higher than either of the two reported previously. It is to be remembered, however, that the previous figures were not corrected for hardening. Although the present lower values of $1/\epsilon$ shift the points at the highest magnetic fields towards appreciably lower values of the abscissa, this does not noticeably affect the resultant value of $\bar{p}$ since η values at these points are close to the saturation value η_∞ as shown by their almost complete independence of $1/\epsilon$.

V. ACKNOWLEDGMENT

The authors wish to thank Professors F. Bloch and G. C. Wick for many helpful discussions and Mr. Melvin Nielsen for assistance in the construction of some of the equipment and the taking of data.