tron arc modulated type. Jones has reported measurements on water over the same energy range covered here.[4] His cross-section results are consistently lower than those of Fig. 17. For neutron energies greater than 0.005 ev this discrepancy amounts to only a few percent and may be attributed to difficulties of making absolute measurements. At lower energies, however, this difference becomes worse and reaches 10 percent at 0.003 ev. The larger cross-section results are probably more reliable because all known difficulties of measurement tend to give too low a value for the observed cross section. Rainwater, Havens, Dunning, and Wu have published spectrometer measurements on paraffin.[2] Between 0.05 and 0.005 ev their results are in agreement with Bethe's curve (with his own choice of vibration frequencies) which is shown in Fig. 13.

Using a crystal spectrometer Fermi and Marshall[18] have investigated H_2 gas at room temperature and at 83°K from 0.75 to 2.5A neutron wave-length. Their results for the gas at room temperature are included in Fig. 8 and indicate good agreement with the present measurements for wave-lengths shorter than 1.2A. At longer wave-lengths, however, there is a marked discrepancy. Again the larger values for the cross section are likely to be more reliable.

Fermi and Marshall[18] have also measured the cross section of O_2, N_2, and H_2 gases for neutrons filtered through BeO. They initially reported the effective wave-length of these neutrons as 5.1A, and their measured cross-section values were about 10 percent larger than the present results as the stated wave-length.[7] In a subsequent report,[19] however, they gave 5.43A as the effective wave-length. This value has been used in plotting their results on Figs. 6–8. The agreement is now much better.

ACKNOWLEDGMENTS

The author wishes to express appreciation to Professor J. R. Dunning who suggested this problem, and to Professors L. J. Rainwater and W. W. Havens, Jr., who took an active interest in the experiments and made numerous invaluable suggestions. Thanks are also also due to Professor T. I. Taylor for the mass-spectrographic analyses of some of the gas samples; Drs. G. Placzek, H. A. Bethe, and O. Halpern for interesting discussions on several points; and Miss Miriam Levin and members of the cyclotron staff for assistance in making some of the measurements. The author also wishes to thank the AEC whose support made these researches possible.

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[18] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).

[19] E. Fermi and L. Marshall, Phys. Rev. **75**, 578 (1949).

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The Stopping Power for Protons in Several Metals*

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The rate of energy loss of low velocity (i.e., from 50 to 400 kev) protons has been measured in five different stopping materials: Be, Al, Cu, Ag, and Au. Agreement with previous measurements at higher velocities (greater than 400 kev) is good where such measurements have been published. The stopping measurements were made on foils prepared by an evaporation technique and are believed to be more accurate than measurements made on the usual (hammered or rolled) foils; the results give a value for stopping about 15 percent higher (in Au) and about 7 percent lower (in Al) than those published by Wilcox. A comparison to existing theory at very low and at high velocities (the critical velocity being defined as $e^2/\hbar$) was made with good agreement shown at the low end. The high end theory is of doubtful validity and gives a stopping power about 10 percent higher than observed. It is argued that this discrepancy cannot be accounted for by a proton charge less than unity due to the electron capture-loss process.

I. INTRODUCTION

SEVERAL recent discussions[1] of the stopping power for charged particles in the low velocity region make an experimental determination interesting from a theoretical point of view. An adequate theory of stopping power, in the region where ionic velocity becomes comparable with the orbital electronic velocity, does not really exist. The measurements that will be reported on in this paper are, therefore, compared to qualitative extensions of a familiar theory that is valid at very high proton energies, but rather less accurate for energy about 500 kev—and quite invalid below this point. A recent argument that should be valid in the region of "adiabatic" collisions, i.e., below about 15 kev, will also be discussed. The particular metals on which

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[1] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947); N. Bohr, Kgl. Danske Vid. Sels. Math.-fys. Medd. **18**, 8 (1948) (this work will be referred to as (N.B.) in the following); see also J. Knipp and E. Teller, Phys. Rev. **59**, 659 (1941).

stopping measurements have been made have been selected to cover a wide range in nuclear charge so that the resulting stopping curves should be of use to workers in the field of nuclear reactions involving thick targets.

Wilcox[2] has previously utilized the advantages of the Allen-type electron multiplier tube to determine the rate of energy loss for several light nuclei in gold and aluminum. However, one of Wilcox's results was a slight mass dependence in the stopping power, and while the failure to verify this result has been discussed elsewhere,[3] it was thought worthwhile to obtain again the stopping curves for protons in both of these metals.

The cross sections for proton energy loss in beryllium, copper, and silver have also been determined here, for energies less than 400 kev.

II. TECHNIQUES

The method for obtaining particles and obtaining data is essentially the same as described by Wilcox,[2] but the type of foils used is different.

A. Procedure

The source of energetic protons was the "kevatron" (400 kev Cockcroft-Walton circuit accelerator) which is described elsewhere.[4] After magnetic analysis, the beam of protons was allowed to strike a beryllium target button onto which was evaporated a very thin coat of gold. The protons which scattered from the gold then had an energy $E_0 = 196/198\ E_B$, where E_B is the accelerator voltage, with a negligible smearing due to the degradation in the thin layer. After 90° scattering and collimation, the protons (*a*) passed through an energy analyzer equipped with an Allen-tube detector, giving the initial energy, E_0, or (*b*) first passed through the thin metal foil mechanically interposed between the collimating slit and the entrance to the analyzer, giving the final energy E_f. The loss of energy in the foil is then $E_0 - E_f$.

One technical difficulty, mentioned by Wilcox, was the progressive carbonization of target and foil by diffusion pump oil. This has been completely eliminated by the installation of a new pumping system, which pulls only through a large liquid-air trap provided with an extensive amount of baffling. It should be remarked here that, generally simultaneously with the appearance of oil on the target, the stopping power of the foils increased considerably; moreover, the carbonization did not necessarily take place at a constant or even a slow rate (an assumption made by Wilcox in correcting for this).

B. The Foils and Their Use

After many non-reproducible measurements using commercial (hammered and rolled) gold and aluminum foils, it was decided that the main difficulty was non-uniformity of the foils. For example, although such foils may sometimes be quite uniform, I am informed[5] that a non-uniformity of ±10 percent, or worse, of the average thickness over several square centimeters may be generally expected in hammered gold foil; rolled aluminum is much worse. It is believed that this was the principal reason for the mass dependence determined by Wilcox.

Therefore the foils used in these determinations were prepared by an evaporation technique described in detail elsewhere.[6] (Briefly, this amounts to evaporating the metal in vacuum onto a plate coated with an organic plastic, stripping the metal with its plastic backing from the glass, and dissolving the plastic.) The measurement of thickness was done interferometrically, i.e., the actual thickness of the foil in centimeters was obtained first, and converted to surface density by use of the bulk density of the metal. A possible objection to this is that the bulk density, because of porosity in the evaporated metal, is not appropriate. However, it is unlikely that the values are off by more than 4 percent and they are conceivably much better. To check this, for present purposes, four rolled and two evaporated foils were used to determine the stopping power in aluminum. The surface densities of the commercial foils were determined by accurately weighing known areas (about 50 cm^2). The resulting curves from the commercial foils deviated as much as 30 percent from the average curve. On the other hand, data from both of the evaporated foils fell on the same curve (within the standard deviation of all points) and furthermore within 8 percent of the average of the curves obtained from the commercial foils.

The two beryllium foils used[7] were prepared by an evaporation somewhat different from the one described above, and a spectro-chemical determination of the mass of beryllium corresponding to an accurately known area of foil was used for the thickness measurement. This procedure was necessary because the foils were small, irregularly shaped, very fragile, curled up pieces. The area measurement consisted of dropping a piece on the surface of still water, which immediately flattened it out, and bringing a clean glass plate up from underneath the floating foil; the metal adhered closely and smoothly to the glass surface. The outline of the piece of foil was then carefully traced several times on a sheet of heavy translucent paper, and a "gravimetric integration" performed. (The internal probable error of such area measurements was about 1.5 percent.) Then

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[2] H. Wilcox, Phys. Rev. **74**, 1747 (1948).

[3] T. A. Hall and S. D. Warshaw, Phys. Rev. **75**, 891 (1949).

[4] L. del Rosario, Phys. Rev. **74**, 304 (1948).

[5] By H. C. Anderson of Hastings and Company, Inc., manufacturers of the gold leaf used.

[6] S. D. Warshaw, Rev. Sci. Inst. **20**, 623 (1949).

[7] I am indebted to H. Bradner at Berkeley for sending several batches of these, and to N. Nachtrieb of the Institute for the Study of Metals for performing the chemical determination. The only reason for the change in technique in the case of beryllium was that the foils were available and measured before the other technique was developed. See Fred, Nachtrieb, and Tomkins, J. Opt. Soc. Am. **37**, 279 (1947) for the chemical method.

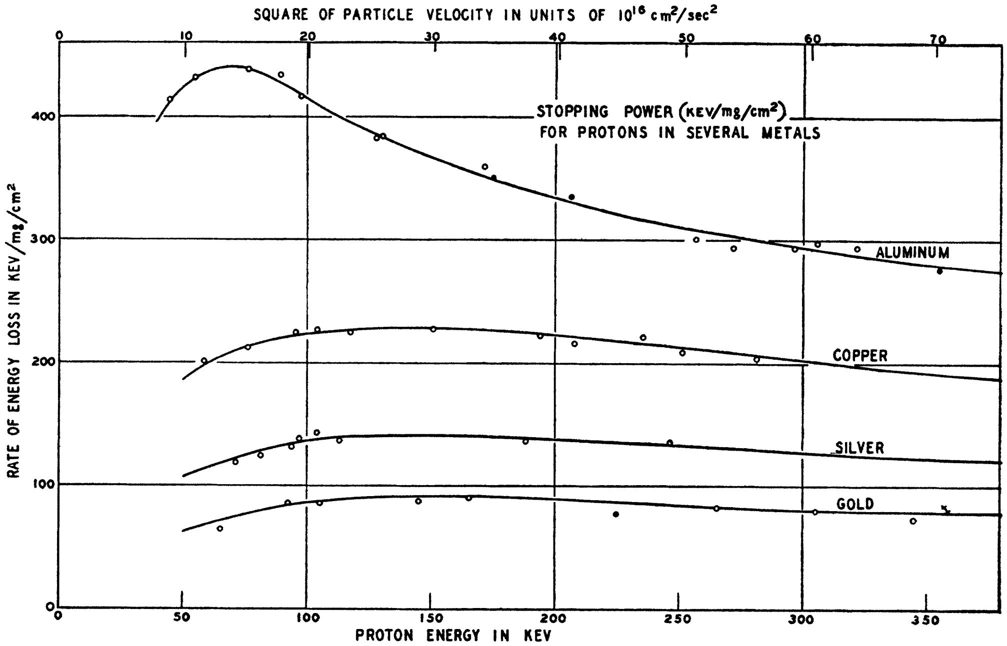

FIG. 1. The stopping power (unit thickness equals 1 mg/cm²) for gold, silver, copper, and aluminum. The few black dots on the gold and aluminum curves are points taken on hammered or rolled foils and normalized to the data from the evaporated foils. The crosses on the gold curve are points measured by Huus and Madsen.

the metal was dissolved off the plate by acid immersion and chemically analyzed. It was assumed here (and also for the interferometer technique) that the pieces of foil used for thickness determination had the same thickness as the piece used for stopping measurements, since they were prepared in the same evaporation.

C. Errors

The quantities measured in this determination, with their probable errors, are:

1. The initial energy E_0 (± 1.0 percent) kev,
2. The final energy E_f (± 1.0 percent) kev,
3. The foil surface density $\tau(\pm 2.5$ percent) mg/cm².

The errors in the energy measurements, as stated, are actually slightly pessimistic. There is, first, a possible (but certainly small) uncertainty in the analyzer constant k,[8] and a possible uncertainty in the "peak height position" due to the finite window curve of the analyzer. The window width is $0.025E$ and the error in obtaining the peak height position is better than 10 percent of the window, so that this uncertainty is better than $0.0025E$. The analyzer voltage was measured by obtaining the current drain through a 50 meg$\pm$0.1 percent resistor stack in parallel with the analyzer plates; the drain was measured with a Weston d.c. milliammeter with a rated accuracy of one-half percent at full scale. Then the maximum uncertainty in the energy measurement is obtained by adding the independent errors without regard to sign; thus, $0.0025+0.001+0.005+0.005=0.013$. The probable error is, of course, 0.2 percent and we take the energy uncertainty as 1 percent for a safe figure. The uncertainty in the foil thickness is discussed above. If porosity is present, the thickness error should more properly be 1.5 percent, -4 percent, but the figure 2.5 percent was chosen because of the internal consistency mentioned in B above. (Actually, each of the two sets of data taken on the two evaporated Al foils differed from the average fitted curve including both sets by a standard deviation of better than 2 percent.)

The average energy loss is then taken as

$$\frac{E_0(1\pm 0.01)-E_f(1\pm 0.01)}{\tau(1\pm 0.025)}=\frac{\Delta E}{\tau}\pm 4.5 \text{ percent.}$$

(However, the standard deviation of points on all the curves from the graphically fitted curves was better than 2.5 percent. Hence, although the absolute heights may be in, say, 4 percent error, the shapes are quite accurate.) This average rate can be properly identified with the true, or instantaneous rate of loss, dE/dx, at the arithmetic average energy $\bar{E}=1/2(E_0+E_f)$, only if ΔE is very small. For some portions of the curve, the loss is of the order of one-third to one-half of the initial energy, however, and here a correction is required. This correction will turn out to be actually smaller than the errors inherent in the measurements; however, it is still significant since it can change the qualitative appearance of the curve.

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[8] Defined by the expression $E=kV$, where V is the voltage (in kilovolts) necessary to produce maximum counting at the detector end of the instrument, and E (in kev) is the proton energy. For the analyzer used, $k=19.5$.

For the correction we may use a simple, linear extrapolation to zero foil thickness. Assume that in a small enough interval, the stopping curve can be approximated by a straight line,

$$-dE/dx=\lambda(E-E_0)-L_0, \tag{1}$$

where $-dE/dx$ is the true rate of loss, λ a constant and $-L_0$ the initial rate of loss; at energy E_0, $x=0$; at E_f, $x=\tau$. On rearrangement, (1) becomes

$$dE=L_0e^{-\lambda x}dx,$$

and integrating this over the foil gives, on rearrangement,

$$L_0=\Delta E/\tau(\lambda\tau/e^{\lambda\tau}-1). \tag{2}$$

The loss at $E=\bar{E}$, combined with (2), gives

$$\left.\frac{dE}{dx}\right|_{\bar{E}}=\frac{\Delta E}{\tau}\frac{\lambda\tau}{2}\coth\frac{\lambda\tau}{2}\approx\frac{\Delta E}{\tau}\left(1+\frac{(\lambda\tau)^2}{12}\right) \tag{3}$$

for small $\lambda\tau$. For the correction to be as much as, say, 2 percent, we must have $\lambda\tau\gtrsim 1/2$. Since the effect is, therefore, small ($\lambda\tau\ll 1$ in every case) the procedure was to plot the curve $\Delta E/\tau$ *versus* $\bar{E}$, determine λ from this curve in the neighborhood of each observed point, and correct the ordinates of each point by the factor in (3) to obtain dE/dx. The largest correction that was applied was one-half percent at the low energy end of the silver curve where $\lambda\tau=0.25$ (for $\tau=0.270$ mg/cm^2).

III. RESULTS AND CONCLUSIONS

A. Description of Results

The measured stopping curves for aluminum, copper, silver, and gold are given in Fig. 1, and for beryllium—in a separate graph for convenience in plotting—in Fig. 2; these give the loss in kilovolts per milligram per square centimeter *versus* the proton energy in kilovolts, and a velocity square scale is included at the top of each figure for convenience. The few black dots in the aluminum and gold data represent points from the data taken on commercial foils of essentially arbitrary thickness, and normalized to the data taken on the evaporated foils; the points used for normalization are not shown. There is no distinction made in Fig. 1 between the two evaporated foils used (0.146, 0.193 mg/cm^2) in the aluminum determination.

It should be pointed out that the gold curve (Fig. 1) is somewhat higher than either the "deuteron" or the "proton" curve published by Wilcox.[2] The aluminum curve, well-verified on several foils, is about 8 percent lower, at 300 kev, than the equivalent curve given by Wilcox.

Figure 3 gives the proton stopping power in electron volts per atom per square centimeter, extrapolated to 600 kev; those are constructed from the smooth curves of Figs. 1 and 2, with the aid of some of the conversion factors listed in Table I.

The extrapolation was carefully done by extending each of the curves in Fig. 3 (except that for gold) to a

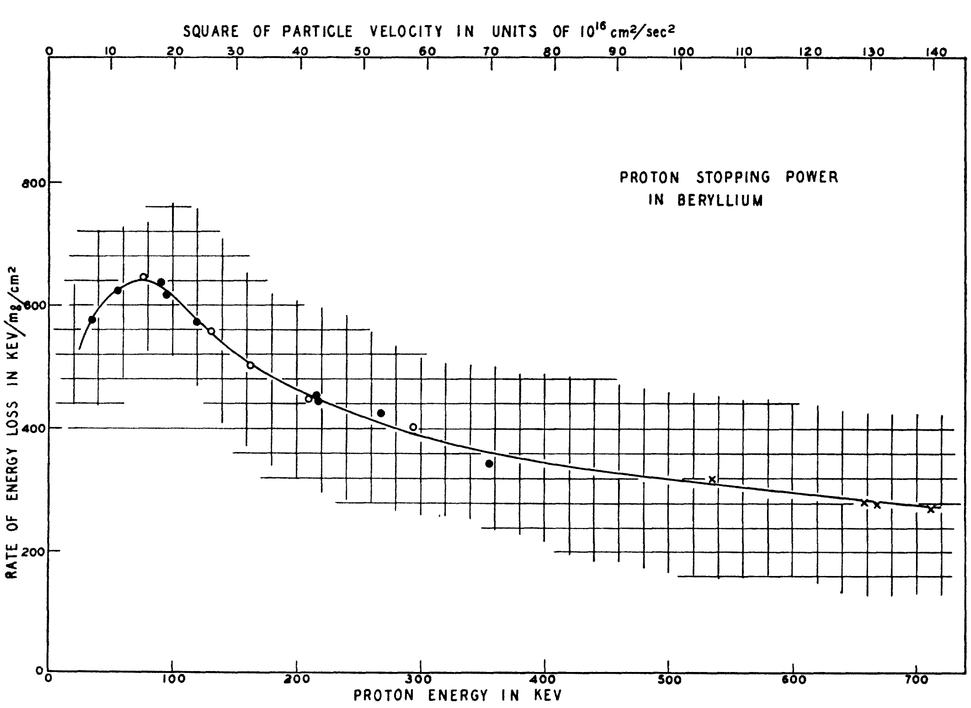

FIG. 2. The stopping power for beryllium (unit thickness equals 1 mg/cm^2). Two foils were used, one with surface density 0.045 mg/cm^2 (circles), the other with essentially arbitrary thickness, with data from this (dots) normalized to the curve drawn through the circles. The crosses are data from the measurements of Madsen and Venkateswarlu.

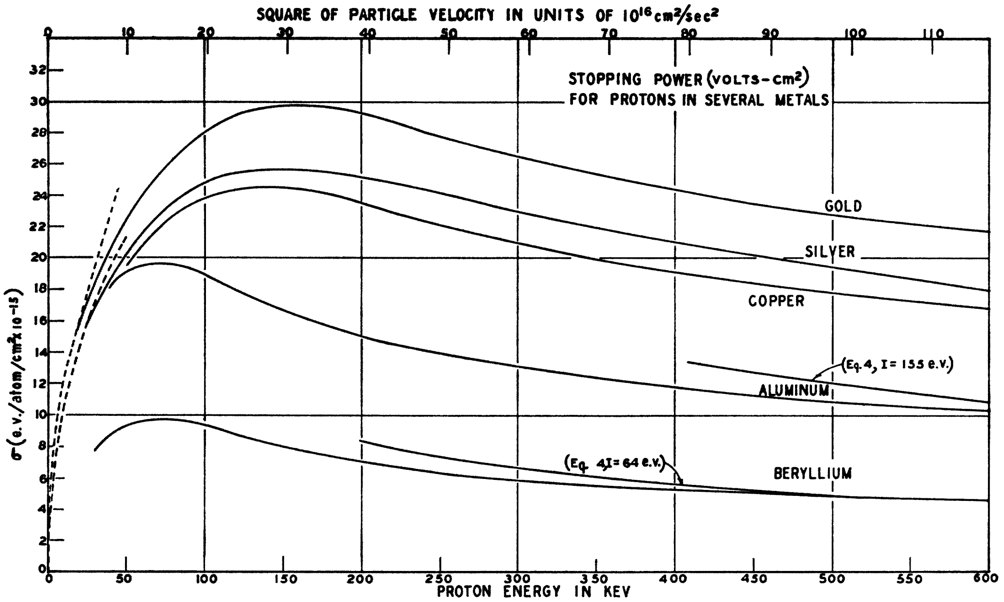

FIG. 3. The stopping power (unit thickness equals 1 atom/cm²) constructed from the smooth curves in Figs. 1 and 2. (Experimental points are not shown.) For comparison with existing theory, Eq. (4) is plotted for aluminum and beryllium using appropriate values for I, and the Fermi-Teller equation (Eq. (5)) is shown dotted for gold and silver.

high enough energy so that the familiar theoretical formula[9]

$$\sigma=\frac{4\pi e^4 Z}{mv^2}\left(\log\frac{2mv^2}{I}-\frac{C_K}{Z}\right)(\text{erg/atom/cm}^2), \qquad (4)$$

would be valid, and smoothly joining the extrapolated experimental curves to Eq. (4) calculated using values of the average excitation potential I given in Table II.[10]

TABLE I. Conversion factors for stopping power.

Given \ Result	kev/mg/cm²	ev/atom/cm²	erg/cm
kev/mg/cm²	Au 1	0.327×10^{-15}	3.09×10^{-5}
	Ag 1	0.178	1.69
	Cu 1	0.105	1.43
	Al 1	0.045	0.44
	Be 1	0.015	0.30
ev/atom/cm²	3.06×10^{15}	Au 1	9.48×10^{10}
	5.35	Ag 1	9.48
	9.58	Cu 1	13.53
	22.32	Al 1	9.67
	66.76	Be 1	19.79
erg/cm	3.24×10^{4}	1.06×10^{-11}	Au 1
	6.30	1.06	Ag 1
	7.01	0.74	Cu 1
	23.10	1.03	Al 1
	33.74	0.51	Be 1

TABLE II. Excitation potentials for the elements used.

Metal	*Be	Al	Cu	Ag	Au
I(kev)	0.064	0.155	0.320	0.490	0.780

*I_{Be} is taken from Madsen's data.

Pieces of beryllium foils from two different evaporations were used; the result of the chemical determination gave for one foil (circles in Fig. 2) a surface density of 0.045±3.5 percent mg/cm², with the error obtained from measurement of three separate pieces, and four chemical runs on each piece. Unfortunately, only a few very small pieces were available for determining the thickness of the other foil used, and the deviation from average of the two pieces used was about 15 percent; since the average itself (0.054 mg/cm²) was about 20 percent too low to fit either the curve of the other foil, or previous measurements at higher energies, the data from this foil were normalized to the other (these points shown as black dots in Fig. 2). The resulting curve can be seen to fit the higher energy data of Madsen and Venkateswarlu[11] (crosses) quite well, thus verifying their measurement of the mean excitation potential and corroborating a theoretical[12] estimate ($I_{\text{theory}}=60$ ev) which was supposed to account for lattice effects.

Since the theoretical curve (4) can be expected to be valid only well above its position of maximum loss, given by 1250 I kev, the extrapolation for gold means an extension from 400 kev to several million volts! Fortunately, however, data taken by other workers[13]

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[9] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 264 (1937). C_K is a correction for strong binding in the K-shell and given graphically by these authors. Z is the atomic number of the stopping material, v the proton velocity, m the electron mass, and e the charge on the electron.

[10] M. G. Mano, Comptes rendus 197, 319 (1933).

[11] C. Madsen and P. Venkateswarlu, Phys. Rev. 74, 648 (1948).

[12] A. Bohr, Kgl. Danske Vid. Sels. Math.-fys. Medd. 24, 14 (1948).

[13] T. Huus and C. B. Madsen, Phys. Rev. 76, 323 (1949).

has recently been published for gold (crosses in the gold curve of Fig. 1) which, in view of the good agreement (within 2 percent) with these measurements, establishes the stopping power for gold very well.

B. Comparison with the Theory

As pointed out in the introduction, there exists no adequate theory that can account for all the features of the curves in Fig. 3. The best that can be done would be to compare the high, and low, energy region extrapolations of these curves with existing theory. This procedure is useful, first to verify a previously unverified low energy equation for stopping, second to verify a rough treatment of stopping valid in this high (i.e., 300 to 600 kev) energy region but only for the heavy elements, and third, to provide a convenient check on the experimental consistency since Eq. (4), with the values of I in Table II, represents a survey of all previous high energy experimental data. The third point has already been commented on in part A above.

A simple argument based on the Thomas-Fermi model has been given by Fermi and Teller[1] which should account for the electronic stopping at proton velocities much less than the electron orbital velocity, i.e., from zero energy to about 20 kev. Their expression, in slightly different form than originally given, is

$$-\frac{dE}{dx}=\frac{4}{3\pi}\frac{E}{vt_0}\log\frac{137v_m}{c}\text{(erg/cm)},\tag{5}$$

where t_0 is a collection of constants (including the proton mass; $t_0=44.16\times10^{-15}$ sec.), v_m is the Fermi velocity, and c the velocity of light. Notice that the entire effect of the character of the stopping material is in the logarithmic factor (and in the density factor from Table I to convert to volt-square centimeters).

Superimposed on the electronic stopping is a nuclear stopping term. For the elements involved, and in this region of energy, direct calculation of an expression for this effect given by Bohr (N.B. Eqs. (2), (3), (9)) shows it to be negligibly small.

The result of calculating (5) is shown dotted in Fig. 3. (For clarity of the figure, copper is not shown.) The experimental data have been extrapolated downward and can be seen to join smoothly onto the dotted curve at about the energy that represents the approximate limit of validity of Eq. (5). While the agreement with the theory therefore happens to be quite good, it should be emphasized that a small change in the fit of the experimental curves for energy in the neighborhood of 70 kev can change this agreement so that it is only fair.

At higher energies we might expect that Eq. (4) would be valid, at least for aluminum and beryllium. Indeed, it is argued (N.B. Chapter 3) that (4) should be quite accurate whenever the parameter,

$$\kappa=\frac{2v_0}{v},\quad v_0\equiv e^2/\hbar,$$

is much smaller than unity, while the argument of the logarithmic factor in (4) remains large. For 500 kev protons, $\kappa=0.46$, while in aluminum $2mv^2/I=7$. On the other hand, (4) fits the data quite well at 500 kev in the case of beryllium, where $2mv^2/I=17$. The second criterion would thus seem to be stronger.

However, a possible discrepancy between the calculated and observed curves in this region might be due to an effective proton charge less than unity, due to an equilibrium between electron capture and loss by the moving proton. This effect can be estimated simply as follows. Let σ_c, σ_l be the cross sections for capture and subsequent loss, respectively, of an electron by the proton. Then, if σ_0 is the stopping power for neutral hydrogen and σ_1, for a unit-charged proton, the net stopping power is

$$\sigma_{\text{net}}=\frac{y}{1+y}\sigma_0+\frac{1}{1+y}\sigma_1,$$

where $y=\sigma_c/\sigma_l$.

Now, in the derivation[14] of Eq. (4) the final expression prior to (4) itself (ignoring C_K) is

$$\sigma=Z\frac{4\pi e^4}{mv^2}\int_{K_{\min}}^{K_{\max}}\frac{dK}{K},\tag{4$'$}$$

where K is the difference between the initial and final wave-number vectors of the incident particle, $K_{\min}=\Delta E_{0n}/\hbar v$, $K_{\max}=2mv/\hbar$, and $\Delta E_{0n}=E_0-E_n$ is the difference in binding energies of the electron in the stopping material, excited by the moving incident particle from the ground level to the nth level. Equation (4′) should, of course, be properly summed over all n, and all electrons; to a reasonable approximation, this is equivalent to replacing ΔE_{0n} by I. The significant point in (4′) is that the integrand contains, implicitly, an excitation cross section, and therefore the square of the matrix element between initial and final states; in (4′) the potential used was a Coulomb interaction between the proton and stopping electron. For the case of neutral hydrogen, the potential is of the form

$$V(R)=(1/R)e^{-2\alpha R}\cdot(1+\alpha R),$$

which is the average potential at a field point R measured from the proton at the origin with an electron in a $(1s)$-state moving at the Bohr radius $a_0=1/\alpha$. If this potential is used in place of the Coulomb potential, then the only change in the result (4′) will be the introduction of a factor

$$\left(\frac{K^2}{4\alpha^2+K^2}+\frac{2\alpha^2K^2}{(4\alpha^2+K^2)^2}\right)^2$$

in the integrand. The integration is straightforward but tedious and the result is that in beryllium, the ratio σ_0/σ_1 is very nearly constant in the range 250 to 700 kev, and equal to 0.46. This means, that, very nearly,

$$\sigma_{\text{net}}=\frac{0.46y+1}{y+1}\sigma_1$$

with σ_1 given by Eq. (4).

The discrepancy at 350 kev, between the calculated and observed curves for beryllium, is about 10 percent; in order to account for this by the capture-loss process, we must have $y\doteq0.2$. But reasonable estimates by Bohr for σ_c and σ_l give[15]

$$y=\frac{Z}{Z^2+Z}(v_0/v)^4=(v_0/2v)^4.$$

Direct calculation of σ_{net} with this expression shows that the

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[14] H. A. Bethe, Ann. d. Physik **5**, 325 (1930).

[15] N.B. Eqs. (4.2.4), (4.3.5). σ_c is really valid only for heavier elements than beryllium, but the estimate of σ_l is good. The exponent is probably smaller than four but greater than three. In the case of aluminum, Bohr estimates a more rapid variation: $y=(4/Z^{\frac{1}{3}})(v_0/v)^5=(v_0/0.9v)^5$, somewhat more accurately than the corresponding result for beryllium.

change of charge effect can account for only a small part (i.e., less than one percent) of the stopping discrepancy. However, work now in progress on measurements of σ_c and σ_l, to be published by T. A. Hall, may modify this argument.

In the heavier elements Bohr (N.B. Chapter 3) attempts qualitative explanation for the somewhat different velocity dependence apparent in the region above the maximum (Fig. 3). The argument is based on the statistical model, i.e., it accounts for the stopping power due to electrons which actually are moving slower than the incident particle by introducing an appropriate electron density function into an expression like (4). However, since (4) itself cannot be very accurate in this energy region, even for the slowest electrons, this procedure should give only qualitative agreement, at best, with experiment. Using the statistical model for the number of electrons in the stopping atom with velocities less than u_s, Bohr puts

$$n(u_s)=Z^{\frac{1}{3}}(u_s/v_0).$$

For $u_s=v$, this means that roughly 20 percent of the electrons in gold, 30 percent in silver, and 45 percent in copper are moving more slowly than the incident proton at about 350 kev. The very firmly bound inner electrons will not contribute much to the stopping but there will still be a large group of intermediate electrons which probably have excitation cross sections that are significant compared to the cross sections for the loosely bound electrons, and may, therefore, be effective in stopping the moving proton. If, however, all these are ignored, and $n(u_s)$ is introduced into (4) with $I=\frac{1}{2}mu_s^2$, and the result integrated over u_s, then the stopping power becomes (N.B. Eq. (3.5.7))

$$\sigma=\text{const. } Z^{\frac{1}{3}}/v. \tag{6}$$

It might be expected that if the inner electrons do contribute significantly, then (6) would be least accurate in gold, except, of course, that the statistical model itself is least accurate, for these three elements, in copper. However, both the physical argument and the very crude application of the statistical model are of a nature that there can be no insistence on other than qualitative agreement. Therefore, to verify Eq. (6), a tabulation was made of the quantity $b=\sigma v/Z^{\frac{1}{3}}$, which, according to (6), should be constant. The result is given in Table III, with b (in arbitrary units) shown for four energies.

TABLE III. Verification of Eq. (6).

	σ_{obs}(ev-cm$^2\times10^{-15}$)					$b=\sigma v/Z^{\frac{1}{3}}$				
	Energy (kev)				$\frac{\Delta\sigma}{\sigma}$	Energy (kev)				$\frac{\Delta b}{b}$
Z	350	400	450	550		350	400	450	550	
29	19.9	19.2	18.3	17.3	13%	100	104	103	107	7%
47	21.9	21.0	20.7	18.7	15%	100	101	105	106	7%
79	25.4	24.4	23.2	22.0	15%	98	100	100	104	7%

For all three elements, b is significantly more constant than the observed stopping power, as indicated in the fractional change columns. However, the power of v is probably smaller than given in Eq. (6). (In fact a plot of log σ *versus* log v is not a very straight line, so that a power function is a poor approximation.) The power of Z is probably rather accurate since the fractional change in σ at a constant velocity is 25 percent compared to a corresponding change in b of only 3 percent. The indication is that a more careful application of the statistical model as outlined by Bohr might be fruitful.

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Note added in proof: Approximate preliminary results, privately communicated by T. Hall, disagree only slightly with Bohr's estimates; Hall finds, e.g., a somewhat less marked dependence of y on the Z of the stopping material, but his results are still in substantial agreement with the rough theory.