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Are Mesons Elementary Particles?

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The hypothesis that π-mesons may be composite particles formed by the association of a nucleon with an anti-nucleon is discussed. From an extremely crude discussion of the model it appears that such a meson would have in most respects properties similar to those of the meson of the Yukawa theory.

I. INTRODUCTION

IN recent years several new particles have been discovered which are currently assumed to be "elementary," that is, essentially, structureless. The probability that all such particles should be really elementary becomes less and less as their number increases.

It is by no means certain that nucleons, mesons, electrons, neutrinos are all elementary particles and it could be that at least some of the failures of the present theories may be due to disregarding the possibility that some of them may have a complex structure. Unfortunately, we have no clue to decide whether this is true, much less to find out what particles are simple and what particles are complex. In what follows we will try to work out in some detail a special example more as an illustration of a possible program of the theory of particles, than in the hope that what we suggest may actually correspond to reality.

We propose to discuss the hypothesis that the π-meson may not be elementary, but may be a composite particle formed by the association of a nucleon and an anti-nucleon. The first assumption will be, therefore, that both an anti-proton and an anti-neutron exist, having the same relationship to the proton and the neutron, as the electron to the positron. Although this is an assumption that goes beyond what is known experimentally, we do not view it as a very revolutionary one. We must assume, further, that between a nucleon and an anti-nucleon strong attractive forces exist, capable of binding the two particles together. We assume that the π-meson is a pair of nucleon and anti-nucleon bound in this way. Since the mass of the π-meson is much smaller than twice the mass of a nucleon, it is necessary to assume that the binding energy is so great that its mass equivalent is equal to the difference between twice the mass of the nucleon and the mass of the meson.

According to this view the positive meson would be the association of a proton and an anti-neutron and the negative meson would be the association of an anti-proton and a neutron. As a model of a neutral meson one could take either a pair of a neutron and an anti-neutron, or of a proton and an anti-proton.

It would be difficult to set up a not too complicated scheme of forces between a nucleon and an anti-nucleon, without about equally strong forces between two ordinary nucleons. These last forces, however, would be quite different from the ordinary nuclear forces, because they would have much greater energy and much shorter range. The reason why no experimental indication of them has been observed for ordinary nucleons may be explained by the assumption that the forces could be attractive between a nucleon and an anti-nucleon and repulsive between two ordinary nucleons. If this is the case, no bound system of two ordinary nucleons would result out of this particular type of interaction. Because of the short range very little would be noticed of such forces even in scattering phenomena.

Ordinary nuclear forces from the point of view of this theory will be discussed below.

Unfortunately we have not succeeded in working out a satisfactory relativistically invariant theory of nucleons among which such attractive forces act. For this reason all the conclusion that will be presented will be

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extremely tentative. It would be undesirable to assume that the attraction is due to a special field of force since in this case the quanta of this new field would be themselves new elementary particles which is just what we hope to be able to avoid. Therefore, only forces of zero range appear compatible with relativistic invariance. In Section II the attempt will be discussed to represent the interaction by a term of the fourth degree in the amplitudes of the nucleon fields. We do not know whether this attempt can be made mathematically self-consistent and we have not succeeded in finding a way to treat it, except by the most crude approximation. The main difficulty is that no stationary state exists with one pair of nucleons only, but only mixed states with one pair, two pairs and many pairs. In our simplified discussion we have neglected this important factor, and treated the problem of a nucleon and an anti-nucleon alone. Assuming hopefully that these mathematical difficulties can be overcome, we have investigated the symmetry properties of the quantum states of the system of a nucleon and an anti-nucleon, in particular for the states of total angular momentum zero, $^1S_0$ and $^3P_0$. The former of these two states corresponds to a pseudoscalar meson and the latter to a scalar meson. If the ground state of the two-nucleon system had a resultant angular momentum 1, one could get in a similar way a model of the vector meson.

A peculiarity of the wave functions of the meson is that they decrease extremely rapidly with the distance between the two nucleons, so that the dimensions of the meson appear to be of the order of magnitude of the Compton wave-length of the nucleon, which is roughly 1/10 of the classical electron radius. This feature may make the experimental detection of the complex nature of the meson extremely difficult.

In the Yukawa theory of nuclear forces it is postulated that virtual mesons are continuously created and re-absorbed in the vicinity of a nucleon. When two nucleons are close to each other, the process of absorption by one nucleon of the virtual meson originated by the other is responsible for the nuclear forces. According to the present view, the main features of this theory can be kept even when the assumption that the meson is an elementary particle is dropped.

One finds that in the vicinity of an isolated nucleon there is a tendency to pair formation of nucleons and anti-nucleons, which will be predominantly formed in the bound state, that is as π-mesons, because such bound states are energetically much lower. From this point on, the Yukawa theory can be taken over almost unchanged as a description of the mechanism of nuclear forces (see Section III).

If the program that has been outlined could be carried out in a mathematically satisfactory way, one might hope to be able eventually to establish a relationship between the strength of the ordinary nuclear forces and the meson mass. Indeed, the difference between the mass of two nucleons and the mass of the meson is the binding energy of the nucleon and the anti-nucleon system. In a consistent theory, therefore, the strength of the coupling term between a nucleon and an anti-nucleon should be adjusted to give the correct value for this binding energy. On the other hand, it is this same coupling which is responsible for the creation of virtual mesons near a nucleon and determines, therefore, the strength of the ordinary nuclear forces. In Section III an estimate of the nuclear forces, calculated as far as is possible according to this program, is given. Considering the extremely primitive mathematical means used, the agreement is not worse than what might be expected.

II. MESONS AS BOUND STATES OF A NUCLEON AND AN ANTI-NUCLEON

We proceed now to discuss the mathematical formalism needed in order to carry out the outlined program.

For this it is necessary to introduce attractive forces between a nucleon and an anti-nucleon capable of binding the two particles together into what we assume to be a meson.

As long as no requirements of relativistic invariance are introduced, this could be done merely by postulating an interaction potential of suitable depth and range. It is useful for what follows to formulate this in the language of the field theory as follows: Two types of particles, for example, protons and anti-neutrons, are described neglecting spin and relativity by two fields, P and A. It is convenient to use here these letters rather than the more usual ψ_P and ψ_A. The following Hamiltonian can be assumed in order to include the attractive potential:

$$\frac{\hbar^2}{2M}\int \nabla P^* \nabla P d^3\mathbf{r} + \frac{\hbar^2}{2M}\int \nabla A^* \nabla A d^3\mathbf{r} - \int\int P^{*\prime}P^{\prime}A^{*\prime\prime}A^{\prime\prime}V(|\mathbf{r}^{\prime}-\mathbf{r}^{\prime\prime}|)d^3\mathbf{r}^{\prime}d^3\mathbf{r}^{\prime\prime}. \quad (1)$$

The first two terms are the kinetic energy of protons and anti-neutrons and the last term introduces the interaction. In this non-relativistic case, states with one proton and one anti-neutron do not mix with any other states. One can therefore confine one's attention to such states only and it is well known that the Hamiltonian (1) is then completely equivalent to that of a two-particle problem with an interaction $V(|\mathbf{r}^{\prime}-\mathbf{r}^{\prime\prime}|)$.

Unfortunately no such simple situation obtains for relativistic particles in the hole theory. There are two reasons for this. One is that two-particle states mix with states in which additional pairs of particles form. The second is that only zero range forces can be used relativistically without adding an essentially new force field. For zero range forces no bound two-particle solution exists.

Since neutrons and anti-neutrons are symmetrical particles, it is immaterial whether we call the anti-neutrons "holes" in a negative neutron sea or vice versa. Since we are interested primarily in an interaction between protons and anti-neutrons, the second alternative is preferable.

The simplest relativistically invariant interactions between these two fields are the usual[1] five types:[2]

$$\int A^*\beta A P^*\beta P d^3\mathbf{r} \qquad \text{(Scalar)}$$

$$\int \{A^*AP^*P-A^*\boldsymbol{\alpha}A\cdot P^*\boldsymbol{\alpha}P\}d^3\mathbf{r} \qquad \text{(Vector)}$$

$$\int \{A^*\beta\boldsymbol{\sigma}A\cdot P^*\beta\boldsymbol{\sigma}P+A^*\beta\boldsymbol{\alpha}A\cdot P^*\beta\boldsymbol{\alpha}P\}d^3\mathbf{r} \ \text{(Tensor)} \qquad (2)$$

$$\int \{A^*\boldsymbol{\sigma}A\cdot P^*\boldsymbol{\sigma}P-A^*\gamma_5AP^*\gamma_5P\}d^3\mathbf{r} \qquad \text{(Pseudovector)}$$

$$\int A^*\beta\gamma_5AP^*\beta\gamma_5Pd^3\mathbf{r}. \qquad \text{(Pseudoscalar).}$$

The vector interaction in (2), like the Coulomb forces, has opposite signs for the interaction between a proton and a neutron and between a proton and an anti-neutron. It turns out that the tensor interaction also has this property while the scalar, pseudovector and pseudoscalar interactions have the same sign for a proton-neutron pair and a proton-anti-neutron pair.

As explained in the introduction, one needs an interaction that is attractive for a proton-anti-neutron pair and repulsive for a proton-neutron pair. Thus the vector and tensor interactions in (2) are the possible choices. For definiteness we shall take in what follows the vector interaction and write:

$$H^{\text{int}}=G\int \{A^*AP^*P-A^*\boldsymbol{\alpha}AP^*\boldsymbol{\alpha}P\}d^3\mathbf{r}. \qquad (3)$$

This Hamiltonian represents a δ-function interaction between a proton and an anti-neutron. Indeed, (3) may be written:

$$H^{\text{int}}=G\int\int A^{*\prime}P^{*\prime\prime} \times[\delta(\mathbf{r}'-\mathbf{r}'')(1-\boldsymbol{\alpha}_A\boldsymbol{\alpha}_P)]A'P''d^3\mathbf{r}'d^3\mathbf{r}''. \qquad (4)$$

[1] These are very similar to the interactions used in β-decay theory. See, e.g., H. A. Bethe, Rev. Mod. Phys. **8**, 82 (1936). We use the same notation as Bethe's for the $\boldsymbol{\alpha}$-, β-, and γ-matrices.

[2] In the hole theory to make the vacuum expectation value of these interactions zero one needs actually to subtract from (2) certain terms. For example the correct scalar interaction to take is:

$$\int[N^*\beta N-\langle N^*\beta N\rangle_{\text{vac}}][P^*\beta P-\langle P^*\beta P\rangle_{\text{vac}}]d^3r,$$

where $\langle\ \rangle_{\text{vac}}$ means vacuum expectation value.

It has proved impossible to solve exactly the interaction problem of a proton and an anti-neutron to yield the "meson" bound state. We had to limit ourselves to the extremely crude description in terms of two-particle states only, disregarding thereby the complications due to multiple pair creation.

The following qualitative argument leads us to believe that this approximate description may be fairly good when the two particles are relatively far from each other and may break down when they are close. For a proton-anti-neutron state the unperturbed energy is larger than the actual energy by a little bit less than 2 Mc^2. For a state with an additional pair (two-pairs state), the energy difference[3] is 4 Mc^2, for an N-pairs state, $2N$ Mc^2. One might expect that an N-pair state will last a time of the order of $\hbar/(2N\ \text{Mc}^2)$ during which the particles can move away about $\hbar/(2N\ \text{Mc})$. We expect, therefore, nucleons to be found away from the center up to about this distance. As N increases such configurations will become smaller and smaller. As a confirmation of this qualitative argument we find that actually for the two-nucleon state the wave function depends on the distance approximately as $\exp(-\text{M}cr/\hbar)$.

We have attempted therefore to regard the effect of multiple pairs as perturbing the near parts of the single pair wave function as if the δ-function interaction were smeared over a region of dimensions about $\hbar/\text{Mc}$. This procedure is not relativistically invariant and should be substituted by a correct multiple-pairs theory. In lack of this we propose to follow up the two-particle theory assuming instead of the contact interaction one of range $\hbar/\text{Mc}$. The interaction will be modified accordingly by introducing instead of $G\delta(\mathbf{r}'-\mathbf{r}'')$ a finite range attractive potential $-V(|\mathbf{r}'-\mathbf{r}''|)$. With this the interaction term becomes

$$H^{\text{int}}=-\int\int A^{*\prime}P^{*\prime\prime}V(\mathbf{r})(1-\boldsymbol{\alpha}_A\cdot\boldsymbol{\alpha}_P)A'P''d^3\mathbf{r}'d^3\mathbf{r}''. \qquad (5)$$

For simplicity we will take for V a step function

$$\begin{aligned} V(\mathbf{r})&=0 && \text{for} \quad r>\hbar/\text{Mc} \\ V(\mathbf{r})&=V_0=\text{constant} && \text{for} \quad r<\hbar/\text{Mc}, \end{aligned} \qquad (6)$$

where

$$r=|\mathbf{r}'-\mathbf{r}''|, \quad \mathbf{r}=\mathbf{r}''-\mathbf{r}'.$$

We now adopt the two-particle approximation whereby the Schrödinger function will be a function of the spin and positional coordinates of the proton and the anti-neutron. The two spin indices running from 1 to 4 each yield a 16-component wave function. For states of zero total momentum each of the 16 components will depend only on the relative position $\mathbf{r}$.

The Schrödinger equation is:

$$\{-chi(\boldsymbol{\alpha}_P-\boldsymbol{\alpha}_A)\cdot\Delta+\text{Mc}^2\beta_A+\text{Mc}^2\beta_P -V(r)(1-\boldsymbol{\alpha}_A\cdot\boldsymbol{\alpha}_P)\}\psi=E\psi.$$

[3] See, however, Section III, especially footnote 5.

It is convenient to arrange the 16 components of ψ into a 4×4 matrix with the proton spin index vertical and the anti-neutron index horizontal.

For a $^1S_0$ state the rotational invariance specifies the dependence of the 16 components on the angular variables as follows:

$$\psi(^1S_0)=\left|\begin{array}{cccc} 0 & -if_1 & \dfrac{f_2}{r}(-x+iy) & \dfrac{f_2}{r}z \\ if_1 & 0 & \dfrac{f_2}{r}z & \dfrac{f_2}{r}(x+iy) \\ \dfrac{f_3}{r}(-x+iy) & \dfrac{f_3}{r}z & 0 & -if_4 \\ \dfrac{f_3}{r}z & \dfrac{f_3}{r}(x+iy) & if_4 & 0 \end{array}\right|, \tag{7}$$

where f_1, f_2, f_3 and f_4 are functions of the distance r only. The other state of total angular momentum 0, namely, $^3P_0$, has a wave function similar *in form* to (7) in which, however, the first and second rows are interchanged with the third and fourth rows. The $^1S_0$ state yields a particle that behaves as a pseudoscalar meson, whereas the $^3P_0$ state behaves as a scalar meson. This fact surprised us because we had thought that the opposite would be the case. The reason is connected with the different transformation properties under space reflection of the large and small components of the wave functions of a Dirac particle. No such unexpected behavior would have been found if the neutron had been treated in the sense of the hole theory as the particle and the anti-neutron as the anti-particle.

Substituting in (6) one finds for f_1, f_2, f_3, f_4 the equations:

$$\begin{aligned} 2\left[r\frac{d}{dr}\left(\frac{f_2}{r}\right)+3\frac{f_2}{r}\right] &= \frac{-2\,\mathrm{Mc}^2+E+V}{c\hbar}f_1+\frac{3V}{c\hbar}f_4 \\ &= \frac{2\,\mathrm{Mc}^2+E+V}{c\hbar}f_4+\frac{3V}{c\hbar}f_1, \\ \frac{d}{dr}(f_1+f_4) &= -\frac{E}{c\hbar}f_2, \\ f_2 &= f_3. \end{aligned} \tag{8}$$

The lowest eigenvalue must be $E=\mu c^2$, the rest energy of the meson. This condition determines[4] the depth V_0 of the potential (6). Assuming the ratio 6.46 between the proton and meson masses one finds:[4]

$$V_0=26.4\ \mathrm{Mc}^2=24.6\ \mathrm{Bev}. \tag{9}$$

The corresponding normalized solution in a large volume Ω is:

$$r>r_0=\frac{\hbar}{\mathrm{Mc}}\begin{cases} f_1=-\dfrac{0.236}{(r_0{}^3\Omega)^{\frac{1}{2}}}\dfrac{1}{u}e^{-u} \\ f_2=f_3=-\dfrac{0.218}{(r_0{}^3\Omega)^{\frac{1}{2}}}e^{-u}\left[\dfrac{1}{u}+\dfrac{1}{u^2}\right] \\ f_4=\dfrac{0.202}{(r_0{}^3\Omega)^{\frac{1}{2}}}\dfrac{1}{u}e^{-u}, \end{cases} \tag{10}$$

$$r<r_0\begin{cases} f_1=-\dfrac{0.0136}{(r_0{}^3\Omega)^{\frac{1}{2}}}\dfrac{\sin v}{v} \\ f_2=f_3=\dfrac{0.370}{(r_0{}^3\Omega)^{\frac{1}{2}}}\left[\dfrac{\cos v}{v}-\dfrac{\sin v}{v^2}\right] \\ f_4=-\dfrac{0.0147}{(r_0{}^3\Omega)^{\frac{1}{2}}}\dfrac{\sin v}{v}, \end{cases} \tag{11}$$

where

$$u=rc/\hbar[M^2-(\mu^2/4)]^{\frac{1}{2}},\quad v=2.03(r/r_0).$$

Notice that the wave function at large distances decreases like $\exp[-cr/\hbar(M^2-\mu^2/4)^{\frac{1}{2}}]$; thus the geometrical size of the meson is of the order of $\hbar/\mathrm{Mc}$ which is the Compton wave-length of the nucleon.

The inconsistencies of this representation should be emphasized. In particular we have given arguments to prove that the two-particle description breaks down at distances $\hbar/\mathrm{Mc}$ and this very distance turns out to be the size of the meson. One could, therefore, state that the wave function becomes reliable only where it vanishes. Our only excuse in adopting it is that we have been unable to do better.

III. RELATIONSHIPS WITH THE YUKAWA THEORY

In spite of the differences between the Yukawa elementary particle model of the meson and the present model, most features of the Yukawa theory can be

[4] There are some undesirable solutions of (8) with energy values E that go to zero when $V_0\rightarrow0$. These solutions are discarded because they do not adiabatically approach the state of two free particles when $V_0\rightarrow0$. Also they would not appear at all if we had taken the neutron and the proton to be of different masses.

taken over even when the meson is pictured as a proton-anti-neutron bound pair denoted briefly as $(P+A)$.

The fundamental process of Yukawa's theory

$$P \rightarrow N+\pi^{+} \tag{12}$$

now becomes

$$P \rightarrow N+(P+A). \tag{13}$$

This last process essentially is the addition to a proton P of a neutron-anti-neutron pair: N, A. Such pair formation will be induced by the postulated interaction (5). Since the energy of the bound $(P+A)$-system is much lower than that of the free particles the state (13) will be formed rather than a three free-particles state.[5] The matrix element is obtained from (5) by substituting for P the wave function of the proton that disappears, for A the wave function of the anti-neutron that disappears (neutron that appears), for $A'^{*}P''^{*}$ the complex conjugate of the wave function (7) of the bound proton-anti-neutron that appears. In order to express the wave function of the disappearing anti-neutron in terms of that of the neutron that is created, one uses the charge conjugation transformation

$$A=\gamma_2\tilde{N}^{*},$$

where $\sim$ means transposed and * transposed and complex conjugate.

We calculate the matrix element for a transition from a slow proton to a slow neutron and a meson at rest. The calculation is straightforward and gives the following result:

$$\int\int V(r)N^{*\prime}Q(\mathbf{r})P''d^3\mathbf{r}'d^3\mathbf{r}'', \tag{14}$$

where Q is the matrix

$$Q=2i(f_1+f_4)\gamma_1\gamma_2\gamma_3+i(f_1-f_2)\gamma_1\gamma_2\gamma_3\gamma_4. \tag{15}$$

If the wave-length of the proton is long compared to $\hbar/\mathrm{Mc}$ (14) can be approximated by

$$\int N^{*}RPd^3\mathbf{r}, \tag{16}$$

where

$$R=\int V(r)Q(\mathbf{r})d^3\mathbf{r}. \tag{17}$$

[5] The contribution to the forces of the virtual creation of free-particle pairs has been discussed in Section II. It was interpreted there as modifying the interaction only at extremely short distances (Order $\hbar/\mathrm{Mc}$). Creation of bound pairs yields inter-nucleon forces of range $\hbar/\mu c$.

Using (10), (11), and (15), and carrying out the integration one finds:

$$R=i\left(\frac{2\pi\hbar^3c^3}{\Omega\mu c^2}\right)^{\frac{1}{2}}(5.3\gamma_1\gamma_2\gamma_3+0.11\gamma_1\gamma_2\gamma_3\gamma_4). \tag{18}$$

This expression can be compared with the conventional interaction of a pseudoscalar meson with nucleons in the Yukawa theory.[6] There are two essentially independent coupling constants: f, the so-called pseudoscalar interaction, and g, the pseudovector interaction. The nucleon-meson interaction Hamiltonian is:

$$i\int N^{*}\left\{f\gamma_1\gamma_2\gamma_3\phi+\sum_{\nu}\frac{\hbar}{\mu c}g\gamma_1\gamma_2\gamma_3\gamma_\nu\frac{\partial\varphi}{\partial x_\nu}\right\}Pd^3\mathbf{r} \tag{19}$$

where ϕ is the pseudoscalar meson field.

The corresponding matrix element for the production of a meson at rest is

$$i\frac{\hbar c}{(2\Omega\mu c^2)^{\frac{1}{2}}}\int N^{*}(f\gamma_1\gamma_2\gamma_3+g\gamma_1\gamma_2\gamma_3\gamma_4)Pd^3\mathbf{r}.$$

Comparison with (18) gives

$$f=(4\pi\hbar c)^{\frac{1}{2}}\times 5.3,\quad g=(4\pi\hbar c)^{\frac{1}{2}}\times 0.11. \tag{20}$$

It has been proved by Case[7] that the terms f and g produce up to the second approximation nuclear forces of the same type. Indeed, their joint contribution is the same as would be obtained by putting $f=0$ and substituting g by

$$g'=g+f(\mu/2M). \tag{21}$$

We find, therefore,

$$g'=(4\pi\hbar c)^{\frac{1}{2}}\times 0.52$$

yielding for $g'^2/4\pi\hbar c$, that is for the analog of the fine structure constant, the value 0.27, which appears quite reasonable.

Naturally the similarity between the present point of view and the Yukawa theory can be carried on only up to a limited extent. The similarity breaks down on the one hand because of the finite size of the meson which introduces naturally a cut-off at short distances. On the other hand it breaks down for phenomena in which sufficiently high energies are involved to break up the meson.

[6] See for example: G. Wentzel, Rev. Mod. Phys. **19**, 1 (1947).
[7] K. M. Case, Phys. Rev. **76**, 14 (1949).