discussions and assistance, and D. S. Bushnell, who aided in the collection of much of the data.

APPENDIX

Tangen[2] believes his accuracy of the Li(p,γ) resonance to be 0.5 percent, but believes his voltage scale has a much higher relative accuracy. The linearity of his voltmeter scale was tested with protons and diatomic ions, and he got excellent correspondence with two reactions below 450 kev. As noted previously, he obtained 339 kev for the F($p\ \alpha',\gamma$) and 162 for the B(p,γ) resonances.

The author's value of boron was found at 162.8±0.2 kev and assuming a linear scale, we get 340.7 kev for the fluorine resonance when Tangen's value of 339 kev is used. The value actually obtained was 340.4±0.4 kev, adding weight to the belief that Tangen's resistor was truly ohmic in character. A linear extrapolation from the fluorine value, using Tangen's result of 440 kev for the lithium resonance, gives 441.8 kev. A linear extrapolation from the boron value gives 442.2 kev, and the average of the two is 442.0 kev. This is in good agreement with the value 441.4±0.5 kev obtained by Fowler and Lauritsen,[n] and the value 442.4 kev obtained by Hudspeth and Swann,[21] both obtained using the F($p\ \alpha',\gamma$) resonance at 873.5 kev as recently determined by Herb, Snowdon, and Sala[8] in an absolute measurement.

Recently, N. P. Heydenburg has informed me he has made absolute determinations using a new, carefully calibrated resistor in his high resistance voltmeter. He obtained for $B^{11}(p,\gamma)$ 161.7 ±0.2 kev; for F($p\ \alpha',\gamma$) 339.7±0.2 kev; and for $Li^7(p,\gamma)$ 440.8 ±0.5 kev. These values are 1.1 and 0.7 kev lower than the ones made in this laboratory, and 1.2 kev lower than the estimate for lithium. If an average of these values is taken, one obtains the following table, in which extra weight has been given to Heydenburg's $Li^7(p,\gamma)$ value:

	kev
$B^{11}(p,\gamma)$	162.2
$F(p\ \alpha',\gamma)$	340.0
$Li^7(p,\gamma)$	441.2
$F(p\ \alpha',\gamma)$	873.5

These values are quite close on a percentage basis with the results from each laboratory, and one has some confidence they are good to ±0.5 kev, a fairly satisfactory situation.

[21] E. L. Hudspeth and C. P. Swann, Phys. Rev. 75, 1272 (1949).

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The Application of the Tomonaga-Schwinger Theory to the Interaction of Nucleons with Neutral Scalar and Vector Mesons

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The Tomonaga-Schwinger theory is applied to the interaction of neutral scalar and vector mesons with nucleons. The Hamiltonians are derived in the interaction representation and virtual effects are transformed away. The meson and nucleon self-energies are calculated. It is shown that they are invariant and, like the electron self-energy, can be transformed away by formal renormalizations of the meson and nucleon masses. All effects are independent of the direction of the general space-like surface in spite of the occurrence in the Hamiltonian of terms explicitly dependent on this direction.

INTRODUCTION

THE generalized Schrödinger equation in the interaction representation was first given by Tomonaga[1] for a Hamiltonian which commuted with itself at all points on a general space-like surface. When this condition was not immediately satisfied, it was found necessary to add certain terms explicitly dependent on the normal to the space-like surface and then to verify that this new Hamiltonian led to an integrable equation and that the generalized equation reduced to the ordinary formalism for a special choice of the general surface. By this method the Hamiltonians have been obtained by Myamoto[2] for the cases to be considered here, namely, the interaction of nucleons with scalar or vector mesons. It has been shown by the present author[3] that the generalized Schödinger equation in the interaction representation can be derived by a development of the work of Weiss.[4] The Hamiltonians of this equation for the nucleon-meson interactions are here deduced by an application of this theory.

The equations so obtained are then transformed by the methods which Schwinger[5] used for the interaction of the electron with the electrodynamic field. Besides real effects, the transformed Hamiltonians contain terms which give rise to infinite self-energies of both types of particle. These are evaluated and it is shown that they can be transformed away, leaving an equation in terms of field variables which satisfy the free field equations of the separate meson and nucleon fields with renormalized masses. Thus the observed free particle is taken to be the "bare" particle plus the vacuum effects.

The main difference between the electromagnetic case dealt with by Schwinger and those dealt with here is that now interactions which contain derivatives of the

[1] S. Tomonaga, Prog. Theor. Phys. 1, 27 (1946); Koba, Tati, and Tomonaga, Prog. Theor. Phys. 2, 101, 198 (1947); S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. 3, 1, 101 (1948).
[2] Y. Myamoto, Prog. Theor. Phys. 3, 124 (1948).
[3] P. T. Matthews, Phys. Rev. 75, 1270 (1949). See also T. S. Chang, Phys. Rev. 75, 967 (1949).
[4] P. Weiss, Proc. Roy. Soc. A 169, 102 (1938).
[5] J. Schwinger, Phys. Rev. 74, 1492 (1948); 75, 615 (1949).

field variables are being considered. Consequently, even the transformed Hamiltonian contains terms which depend explicitly on the direction of the general space-like surface. The most important result of the present paper is that the transformed Hamiltonian contains a further implicit dependence on the surface direction which to the second order in the coupling constants exactly cancels the explicit term. It follows that all self-energies and cross sections predicted by the theory are independent of the surface direction and are thus relativistically invariant. It will be shown elsewhere that this independence of the surface direction can be proved up to any order for any effect.

The integrals in the self-energy calculations are not regular which has given rise to the different values for the photon self-energy obtained by Schwinger[5] and Wentzel.[6] To resolve this difficulty Pauli[7] has suggested a regularizing procedure. The effect of "regularizing" meson self-energies is discussed.

Only neutral meson fields will be considered. The introduction of charge will not affect the first-order self-energies calculated in this paper since for a proton only the positive, and for a neutron only the negative mesons will be significant.

I. THE SCHRÖDINGER EQUATION IN THE INTERACTION REPRESENTATION

Let x_μ be a general point in space-time, the Greek suffix taking the values from 1 to 4, $(x_4=ix_0)$. $\sigma(x)$ is a general space-like surface through x_μ and n_μ is the normal to the surface at the point x_μ, pointing to the future, $(n_0>0,\ n_\mu{}^2=-1)$. Consider the field whose Lagrangian is $\mathcal{L}(\varphi^\alpha, \varphi_\mu{}^\alpha)$ where $\varphi_\mu{}^\alpha=\partial\varphi^\alpha/\partial x_\mu$. The energy-momentum tensor is defined in the usual way,

$$U_{\mu\nu}=\mathcal{L}\delta_{\mu\nu}-(\partial\mathcal{L}/\partial\varphi_\mu{}^\alpha)\varphi_\mu{}^\alpha. \tag{1.1}$$

Then it has been shown[3] that the canonical conjugate of φ^α can be defined as

$$\pi_\alpha=-(\partial\mathcal{L}/\partial\varphi_\mu{}^\alpha)n_\mu, \tag{1.2}$$

and the commutation relations for points with space-like separation for Bose statistics are

$$[\varphi^\alpha(x),\ \pi_\beta(x')]=0 \quad \text{if} \quad (x-x')^2>0, \tag{1.3}$$

$$\int[\varphi^\alpha(x),\ \pi_\beta(x')]d\sigma'=i\hbar c\delta_{\alpha\beta}, \tag{1.4}$$

and for Fermi statistics are

$$\{\varphi^\alpha(x),\ \pi_\beta(x')\}=0 \quad \text{if} \quad (x-x')^2>0, \tag{1.5}$$

$$\int\{\varphi^\alpha(x),\ \pi_\beta(x')\}d\sigma'=i\hbar c\delta_{\alpha\beta}. \tag{1.6}$$

Further, if two fields interact, the Schrödinger equation in the interaction representation is

$$i\hbar c\delta\Psi/\delta\sigma=\mathcal{H}_{\text{int}}(\phi,\ \psi)\Psi, \tag{1.7}$$

where φ and ψ are the field variables of the two interacting fields, $\mathcal{H}_{\text{int}}$ is the interaction terms of the Hamiltonian defined

$$\mathcal{H}=U_{\mu\nu}n_\mu n_\nu \tag{1.8}$$

$$=-\mathcal{L}-(n_\mu\partial\mathcal{L}/\partial\varphi_\mu{}^\alpha)(n_\nu\varphi_\nu{}^\alpha), \tag{1.9}$$

and ϕ and ψ satisfy the equations of motion of their respective free fields. Thus,

$$\phi=R^{-1}\varphi_s R, \tag{1.10}$$

where

$$i\hbar c\delta R/\delta\sigma=\mathcal{H}_{\text{free}}(\varphi_s)R. \tag{1.11}$$

φ_s is the Schrödinger variable and $\mathcal{H}_{\text{free}}$ is the Hamiltonian of the free field. The commutation relations of ϕ and ψ are given by (1.2)–(1.6) applied to the Lagrangians of their free fields. Also define the energy-momentum vector on the surface as

$$P_\nu=-(1/c)\int U_{\mu\nu}(x')n_\mu{}'d\sigma', \tag{1.12}$$

then

$$(i/\hbar)[\varphi^\alpha(x),\ P_\mu]=(\partial\varphi^\alpha/\partial x_\mu). \tag{1.13}$$

To apply this general theory to the interaction of neutral scalar mesons with nucleons we will use the notation of Schwinger[5] which will not be defined here. The meson potential is $\varphi(x)$ while $\psi_\alpha(x)$ denotes the four-component Dirac spinor of the nucleon field. The differential $\partial\phi/\partial x_\mu$ is written $\varphi_\mu(x)$. Also

$$\kappa_0=m_0c/\hbar, \tag{1.14}$$

and

$$\kappa=mc/\hbar, \tag{1.15}$$

where m_0 and m are the mechanical proper masses of the nucleon and the meson, respectively.

The Lagrangian of the free nucleon field is

$$\mathcal{L}_N=-\tfrac{1}{2}\hbar c\bar{\psi}(\gamma_\mu\partial/\partial x_\mu+\kappa_0)\psi -\tfrac{1}{2}\hbar c\psi(\gamma_\mu{}^T\partial/\partial x_\mu-\kappa_0)\bar{\psi}. \tag{1.16}$$

The Lagrangian of the free scalar meson field is

$$\mathcal{L}_S=-\tfrac{1}{2}\{(\varphi_\mu{}^2)+\kappa^2\varphi^2\}. \tag{1.17}$$

The equations of motion of the free fields are

$$(\gamma_\mu\partial/\partial x_\mu+\kappa_0)\psi=0; \tag{1.18}$$

$$(\gamma_\mu{}^T\partial/\partial x_\mu-\kappa_0)\bar{\psi}=0, \tag{1.19}$$

and

$$(\Box^2-\kappa^2)\varphi=0. \tag{1.20}$$

The Lagrangian of the interacting fields is

$$\mathcal{L}=\mathcal{L}_N+\mathcal{L}_S+\mathcal{L}_{\text{int}}, \tag{1.21}$$

where

$$\mathcal{L}_{\text{int}}=-(1/c)\omega\varphi-(1/c\kappa)j_\mu\varphi_\mu. \tag{1.22}$$

[6] G. Wentzel, Phys. Rev. **74**, 1070 (1948).

[7] W. Pauli, Rouse Ball Lecture, Cambridge University, March, 1949. The "regularizing" process was defined and applied to the derivation of the photon self-energy.

ω and j_μ are covariant expressions in the nucleon variables defined by

$$\omega=\tfrac{1}{2}fc(\bar{\psi}\psi-\psi\bar{\psi}), \quad (1.23)$$

$$j_\mu=\tfrac{1}{2}igc(\bar{\psi}\gamma_\mu\psi-\psi\gamma_\mu{}^T\bar{\psi}). \quad (1.24)$$

The strengths of the interactions are determined by the constants f and g (having the dimensions of charge). Following Pais[8] we will consistently use f for an interaction which does not contain differentials of the field variables and g for one which does. We will speak in this sense of f-interactions and g-interactions.

The coefficients in the definitions of ω and j_μ have been chosen so that ω and $j_i=(-ij_4, j_1, j_2, j_3)$ are Hermitian. It is this condition which gives rise to the factor i in the coefficient for j_μ and ultimately leads to the opposite sign for the nucleon self-energy due to the f-interaction in the scalar and vector meson cases. This is the basis of the Pais f-field theory.[8]

To evaluate $\mathcal{H}_{\text{int}}$ we do not need to consider the Hamiltonian of the free nucleon field because the interactions do not contain the derivatives of the nucleon variables and the canonical conjugates of the nucleon field variables are not altered by the interaction. By (1.17), (1.22), and (1.2) the canonical conjugate of ϕ when there is interaction is

$$\pi=(\varphi_\mu+(1/c\kappa)j_\mu)n_\mu. \quad (1.25)$$

Thus by (9) omitting the free nucleon field

$$\mathcal{H}=\tfrac{1}{2}(\varphi_\mu{}^2+\kappa^2\varphi^2)+(1/c)\omega\varphi+(1/c\kappa)j_\mu\varphi_\mu+(\varphi_\mu+(1/c\kappa)j_\mu)\varphi_\rho n_\mu n_\rho. \quad (1.26)$$

Since n_μ is time-like the tangential part of $\varphi_\mu{}^2$ is

$$\varphi_\mu{}^2+(n_\mu\varphi_\mu)(n_\nu\varphi_\nu). \quad (1.27)$$

This is obvious in the special case when the surface is $x_4=\text{const.}$ since the components of n_μ are then $(0, 0, 0, i)$. The normal derivative must be formally eliminated from (1.26) in favor of the canonical conjugate by (1.25). Thus

$$\begin{aligned}\mathcal{H}=&\tfrac{1}{2}\{\varphi_\mu{}^2+(n_\mu\varphi_\mu)(n_\nu\varphi_\nu)\\&-(\pi-(1/c\kappa)j_\mu n_\mu)(\pi-(1/c\kappa)j_\nu n_\nu)+\kappa^2\varphi^2\}\\&+(1/c)\omega\varphi+(1/c\kappa)\{j_\mu\varphi_\mu+(j_\mu n_\mu)(\varphi_\nu n_\nu)\\&-(j_\mu n_\mu)(\pi-(1/c\kappa)j_\nu n_\nu)\}+\pi(\pi-(1/c\kappa)j_\nu n_\nu).\end{aligned} \quad (1.28)$$

Thus

$$\mathcal{H}=\mathcal{H}_{\text{free}}+\mathcal{H}_{\text{int}}, \quad (1.29)$$

where

$$\mathcal{H}_{\text{free}}=\tfrac{1}{2}\{\varphi_\mu{}^2+(n_\mu\varphi_\mu)(n_\nu\varphi_\nu)+\pi^2+\kappa^2\varphi^2\}, \quad (1.30)$$

and

$$\mathcal{H}_{\text{int}}=(1/c)\omega\varphi+(1/c\kappa)\{j_\mu\varphi_\mu+(j_\mu n_\mu)(\varphi_\mu n_\mu-\pi)\}+(1/2c^2\kappa^2)(j_\nu n_\nu)^2. \quad (1.31)$$

But in the interaction representation the field variables satisfy the free-field equations and thus by (1.2) and (1.17)

$$\pi=\varphi_\mu n_\mu, \quad (1.32)$$

and in the interaction representation

$$\mathcal{H}_{\text{int}}=(1/c)\omega\varphi+(1/c\kappa)j_\mu\varphi_\mu+(1/2c^2\kappa^2)(j_\nu n_\nu)^2, \quad (1.33)$$

which is in agreement with the result of Myamoto.[2]

By (1.2) and (1.16) the canonical conjugates of ψ_α and $\bar{\psi}_\alpha$ are

$$\pi_\alpha=\tfrac{1}{2}\hbar c(\bar{\psi}\gamma_\mu)_\alpha n_\mu,\quad \bar{\pi}_\alpha=\tfrac{1}{2}\hbar c(\gamma_\mu\psi)_\alpha n_\mu. \quad (1.34)$$

By (1.5) and (1.6) the commutation relations are

$$i\int\{\psi_\alpha(x), (\bar{\psi}(x')\gamma_\mu)_\beta\}d\sigma'=\delta_{\alpha\beta}, \quad (1.35)$$

$$i\int\{(\gamma_\mu\psi(x'))_\alpha, \bar{\psi}_\beta(x)\}d\sigma_\mu'=\delta_{\alpha\beta}, \quad (1.36)$$

where

$$d\sigma_\mu'=-n_\mu d\sigma', \quad (1.37)$$

and

$$\begin{aligned}\{\psi_\alpha(x), \psi_\beta(x')\}&=\{\bar{\psi}_\alpha(x), \bar{\psi}_\beta(x')\}\\&=\{\psi_\alpha(x), \bar{\psi}_\beta(x')\}=0 \quad\text{if}\quad (x-x')^2>0.\end{aligned} \quad (1.38)$$

Also

$$\int[\varphi(x), \varphi_\mu(x')]d\sigma_\mu'=-i\hbar c, \quad (1.39)$$

$$[\varphi(x), \varphi(x')]=0 \quad\text{if}\quad (x-x')^2>0. \quad (1.40)$$

In addition the commutation relations between the meson and nucleon field variables are

$$[\varphi(x), \psi(x')]=[\varphi(x), \bar{\psi}(x')]=0. \quad (1.41)$$

The above relations apply only to points with space-like separation. To obtain the general commutation relations for points with any separation introduce the function $\Delta(x)$ such that

$$(\square^2-\kappa^2)\Delta(x)=0,\quad \Delta(x)=0 \quad\text{if}\quad x_\mu{}^2>0, \quad (1.42)$$

and

$$\int(\partial\Delta(x)/\partial x_\mu)d\sigma_\mu=1. \quad (1.43)$$

It then follows by the same argument as that given by Schwinger[5] in the electron-photon case that

$$[\varphi(x), \varphi(x')]=i\hbar c\Delta(x-x'). \quad (1.44)$$

$$\begin{aligned}\{\psi_\alpha(x), \bar{\psi}_\beta(x')&=-i(\gamma_\mu\partial/\partial x_\mu-\kappa_0)_{\alpha\beta}\Delta_0(x-x')\\&=-iS_{\alpha\beta}(x-x').\end{aligned} \quad (1.45)$$

$\Delta_0(x)$ is the same expression as $\Delta(x)$ with κ replaced by κ_0. By (1.5) and (1.33) the Schrödinger equation in the

[8] A. Pais, Kon. Ned. Akad. v. Wet. Verh. D1 19, 1 (1947).

interaction representation is

$$i\hbar c\delta\Psi/\delta\sigma=\{(1/c)\omega\varphi+(1/c\kappa)j_\mu\varphi_\mu+(1/2c^2\kappa^2)(j_\nu n_\nu)^2\}\Psi. \quad (1.46)$$

We now consider the case in which the scalar meson field is replaced by a vector meson field $\varphi_\mu(x)$. Denote the differential $\partial\varphi_\mu(x)/\partial x_\nu$ by $\varphi_{\mu\nu}(x)$. The free field satisfies the equation

$$(\Box^2-\kappa^2)\varphi_\mu(x)=0, \quad (1.47)$$

and in classical theory the relation

$$\varphi_{\mu\mu}(x)=0, \quad (1.48)$$

which ensures that the energy is positive definite. If the Lagrangian of the free field is taken to be

$$\mathcal{L}=-\tfrac{1}{2}\{\tfrac{1}{2}(\varphi_{\mu\nu}-\varphi_{\nu\mu})^2+\kappa^2\varphi_\mu{}^2\}, \quad (1.49)$$

then the variation of $\mathcal{L}$ gives the equation of motion

$$(\partial/\partial x_\nu)(\varphi_{\mu\nu}-\varphi_{\nu\mu})=\kappa^2\varphi_\mu. \quad (1.50)$$

Differentiating gives Eq. (1.48) and substituting back into (1.50) yields Eq. (1.47). It is thus not necessary to introduce (1.48) as a special condition. However the appearance of the differentials of the field variables in the Lagrangian in antisymmetrical combinations leads directly to the equation

$$\pi_\mu n_\mu=0. \quad (1.51)$$

This equation is inconsistent with the commutation relations defined by (1.3), as can immediately be seen by taking the surface to be the plane $x_4=\text{const}$.

The immediate generalization of the scalar meson theory is to take the Lagrangian

$$\mathcal{L}=-\tfrac{1}{2}\{\varphi_{\mu\nu}{}^2+\kappa^2\varphi_\mu{}^2\}, \quad (1.52)$$

with Eq. (1.48) as a supplementary condition on the wave function

$$\varphi_{\mu\mu}\Psi=0. \quad (1.53)$$

The application of the general theory to (1.52) would lead to the quantization of φ_μ as independent scalars and the general commutation relations

$$[\varphi_\mu(x),\ \varphi_\nu(x')]=i\hbar c\delta_{\mu\nu}\Delta(x-x'). \quad (1.54)$$

Thus by (1.54) and (1.47)

$$[\varphi_{\mu\mu}(x),\ \varphi_{\nu\nu}(x')]=i\hbar c\kappa^2\Delta(x-x'), \quad (1.55)$$

which is inconsistent with (1.53) since it is required of a supplementary condition that it commutes with itself at different space-time points.

Both these difficulties are avoided if a form of the theory given by Stueckelberg[9] is adopted. Two subsidiary fields are introduced, a vector field $A_\mu(x)$ and a scalar field $B(x)$ both satisfying Eq. (1.47). Write $\partial A_\mu(x)/\partial x_\nu$ as $A_{\mu\nu}(x)$ and $\partial B(x)/\partial x_\nu$ as $B_\nu(x)$. Define

$$\varphi_\mu=A_\mu+(1/\kappa)B_\mu, \quad (1.56)$$

[9] E. C. G. Stueckelberg, Helv. Phys. Acta **11**, 225 (1938).

and postulate the subsidiary condition in classical theory

$$A_{\mu\mu}+\kappa B=0. \quad (1.57)$$

The Lagrangian of the free field is

$$\mathcal{L}_\nu=-\tfrac{1}{2}\{A_{\mu\nu}{}^2+\kappa^2A_\mu{}^2+B_\nu{}^2+\kappa^2B^2\}, \quad (1.58)$$

from which the correct equations of motion for A_μ and B follow directly. By (1.56) and (1.57) it can be shown that φ_μ so defined satisfies the required Eqs. (1.47) and (1.48).

If the meson field is interacting with a nucleon field the additional terms in the Lagrangian are

$$\mathcal{L}_{\text{int}}=-(1/c)j_\mu(A_\mu+(1/\kappa)B_\mu)-(1/2c\kappa)m_{\mu\nu}(A_{\nu\mu}-A_{\mu\nu}), \quad (1.59)$$

where j_μ is defined by (24) with f replaced by g and

$$m_{\mu\nu}=\tfrac{1}{2}igc(\bar{\psi}\gamma_\mu\gamma_\nu\psi-\psi\gamma_\mu{}^T\gamma_\nu{}^T\bar{\psi}). \quad (1.60)$$

The Lagrangian of the free nucleon field is given by (1.16) and again need not be considered in the calculation of $\mathcal{H}_{\text{int}}$.

By a calculation similar to that given in the scalar case it can easily be shown that the Hamiltonian in the interaction representation is

$$\begin{aligned}\mathcal{H}_{\text{int}}&=(1/c)j_\mu(A_\mu+(1/\kappa)B_\mu)+(1/2c\kappa)m_{\mu\nu}(A_{\nu\mu}-A_{\mu\nu})+(1/2c^2\kappa^2)\{(n_\mu j_\mu)^2+(n_\mu m_{\mu\nu})^2\}\\&=(1/c)j_\mu\varphi_\mu+(1/2c\kappa)m_{\mu\nu}(\varphi_{\nu\mu}-\varphi_{\mu\nu})+(1/2c^2\kappa^2)\{(n_\mu j_\mu)^2+(n_\mu m_{\mu\nu})^2\}. \quad (1.61)\end{aligned}$$

The general commutation relations follow as before

$$[B(x),\ B(x')]=i\hbar c\Delta(x-x'), \quad (1.62)$$

$$[A_\mu(x),\ A_\nu(x')]=i\hbar c\delta_{\mu\nu}\Delta(x-x'). \quad (1.63)$$

Equation (1.56) is replaced in quantum theory by a supplementary condition which in the interaction representation can be written

$$(A_{\mu\mu}+\kappa B)\Psi=\Omega\Psi=0. \quad (1.64)$$

Using the commutation relations (1.62) and (1.63) it can be shown that

$$(\Box^2-\kappa^2)\Omega=0, \quad (1.65)$$

$$[i\hbar c\delta/\delta\sigma(x')-\mathcal{H}(x'),\ \Omega(x)]=0. \quad (1.66)$$

and

$$[\Omega(x),\ \Omega(x')]=0. \quad (1.67)$$

These relations show that the supplementary condition is consistent with the field equations, the equation of motion and the commutation relations. From (1.56) and the commutation relations (1.62) and (1.63) the commutation relations for $\varphi_\mu(x)$ are

$$\begin{aligned}[\varphi_\mu(x),\ \varphi_\nu(x')]&=i\hbar c(\delta_{\mu\nu}-(1/\kappa^2)\partial^2/\partial x_\mu\partial x_\nu)\Delta(x-x')\\&=i\hbar cT_{\mu\nu}(x-x'). \quad (1.68)\end{aligned}$$

By (1.61) the Schrödinger equation in the interaction representation is

$$i\hbar c\delta\Psi/\delta\sigma=[(1/c)j_\nu\varphi_\nu+(1/2c\kappa)m_{\mu\nu}(\varphi_{\nu\mu}-\varphi_{\mu\nu})+(1/2c^2\kappa^2)\{(n_\mu j_\mu)^2+(n_\mu m_{\mu\nu})^2\}]\Psi. \quad (1.69)$$

II. THE VACUUM

To apply Schwinger's methods to the meson-nucleon interactions it is necessary to split the field variables into parts which can be interpreted as annihilation and creation operators and to define the vacuum. It will then be convenient to calculate the expectation values of certain expressions which appear in the later work and to develop explicitly the singular function $\Delta(x)$ and the associated functions $\Delta^{(1)}(x)$ and $\bar{\Delta}(x)$ which will be introduced below. Where this work is the same as that of Schwinger only the results will be quoted.

The scalar meson field can be split into positive and negative frequency parts

$$\varphi(x)=\varphi^{(+)}(x)+\varphi^{(-)}(x). \quad (2.1)$$

Schwinger[5] has shown that this can be done in an invariant manner. Define

$$\varphi^{(1)}(x)=i[\varphi^{(+)}(x)-\varphi^{(-)}(x)]. \quad (2.2)$$

It follows that

$$[\varphi^{(+)}(x),\ \varphi(x')]=i\hbar c\Delta^{(+)}(x-x'), \quad (2.3)$$

$$[\varphi^{(-)}(x),\ \varphi(x')]=i\hbar c\Delta^{(-)}(x-x'), \quad (2.4)$$

$$[\varphi^{(1)}(x),\ \varphi(x')]=i\hbar c\Delta^{(1)}(x-x'), \quad (2.5)$$

where $\Delta^{(+)}(x)$, $\Delta^{(-)}(x)$, and $\Delta^{(1)}(x)$ satisfy the same relations (2.1) and (2.2) as $\varphi^{(+)}(x)$, $\varphi^{(-)}(x)$, and $\varphi^{(1)}(x)$. Then

$$\Delta^{(1)}(x)=\Delta^{(1)}(-x),\quad \Delta(x)=-\Delta(-x). \quad (2.6)$$

$$(\Box^2-\kappa^2)\Delta^{(+)}(x)=(\Box^2-\kappa^2)\Delta^{(-)}(x)=(\Box^2-\kappa^2)\Delta^{(1)}(x)=0. \quad (2.7)$$

The wave function $\Psi(\sigma)$ on any particular surface σ specifies a state of the free field on that surface. The vacuum state must be defined to be that for which the eigenvalue of the energy in any frame is a minimum. Now by (1.12) and (1.13)

$$\varphi^{(+)}(x)P_\nu-P_\nu\varphi^{(+)}(x)=-i\hbar\partial\varphi^{(+)}(x)/\partial x_\nu. \quad (2.8)$$

Applying (2.8) to a Fourier component of $\varphi^{(+)}(x)$ give by

$$\varphi^{(+)}(x)=\int_{-k_\lambda n_\lambda>0}\varphi(k)\delta(k_\lambda^2+\kappa^2)\exp[ik_\mu x_\mu]d^4k,\quad (d^4k=dk_0dk_1dk_2dk_3), \quad (2.9)$$

it follows that

$$\varphi^{(+)}(k)\mathrm{W}-\mathrm{W}\varphi^{(+)}(k)=\hbar\mathrm{w}\varphi^{(+)}(k), \quad (2.10)$$

where $\mathrm{W}=-n_\nu P_\nu c$ and $\mathrm{w}=-n_\nu k_\nu c$ are expressions for the energy and frequency in the arbitrary frame specified by the time-like vector n_μ. Since $-n_\nu k_\nu>0$ by (2.9), ω is positive. Operating with (2.10) on the vacuum state Ψ_0

$$\mathrm{W}\varphi^{(+)}(k)\Psi_0=(\mathrm{W}_0-\hbar\mathrm{w})\varphi^{(+)}(k)\Psi_0. \quad (2.11)$$

Thus $\varphi^{(+)}(k)\Psi_0$ is the state with one less meson in the k energy level and if Ψ_0 is the vacuum

$$\varphi^{(+)}(k)\Psi_0=0. \quad (2.12)$$

This equation is valid for all $\varphi^{(+)}(k)$ and thus

$$\varphi^{(+)}(x)\Psi_0=0. \quad (2.13)$$

In general $\Psi(\sigma)$ specifies occupation numbers of mesons in the various free meson energy states on the surface σ. $\varphi^{(+)}(x)$ and $\varphi^{(-)}(x)$ are annihilation and creation operators, respectively. With this definition the vacuum expectation value of the anticommutator is

$$\langle\{\varphi(x),\ \varphi(x')\}\rangle_0=\hbar c\Delta^{(1)}(x-x'). \quad (2.14)$$

A similar treatment of the free meson field yields the definition of the vacuum

$$\psi^{(+)}(x)\Psi_0=0. \quad (2.15)$$

And the expectation value of the commutator in the vacuum is

$$\langle[\psi_\alpha(x),\ \bar{\psi}_\beta(x')]\rangle_0=-S_{\alpha\beta}^{(1)}(x-x'). \quad (2.16)$$

The above theory can immediately be extended to the vector meson field. In place of (2.3)–(2.5) we have

$$[\varphi_\mu^{(+)}(x),\ \varphi_\nu(x')]=i\hbar cT_{\mu\nu}^{(+)}(x-x'), \quad (2.17)$$

$$[\varphi_\mu^{(-)}(x),\ \varphi_\nu(x')]=i\hbar cT_{\mu\nu}^{(-)}(x-x'), \quad (2.18)$$

$$[\varphi_\mu^{(1)}(x),\ \varphi_\nu(x')]=i\hbar cT_{\mu\nu}^{(1)}(x-x'), \quad (2.19)$$

where

$$T_{\mu\nu}^{(+)}(x-x')=\{\delta_{\mu\nu}-(1/\kappa^2)(\partial^2/\partial x_\mu\partial x_\nu)\}\Delta^{(+)}(x-x'), \quad (2.20)$$

and similar equations hold for $T_{\mu\nu}^{(-)}(x-x')$ and $T_{\mu\nu}^{(1)}(x-x')$. The vacuum is defined by

$$\varphi_\mu^{(+)}(x)\Psi_0=0. \quad (2.21)$$

Difficulties of the false vacuum do not arise as in the electromagnetic case because the supplementary condition does not exclude longitudinal mesons when $\kappa\neq 0$. The vacuum expectation value of the anticommutator is

$$\langle\{\varphi_\mu(x),\ \varphi_\nu(x')\}\rangle_0=\hbar cT_{\mu\nu}^{(1)}(x-x'). \quad (2.22)$$

We proceed to develop the expectation values of various expressions in the invariants ω, j_μ, and $m_{\mu\nu}$. It follows from the definitions that

$$\omega(x)=-\tfrac{1}{2}ifc[\psi_\alpha(x),\ \bar{\psi}_\beta(x)](-i\delta_{\beta\alpha})=M^{(0)}(x), \quad (2.23)$$

$$j_\mu(x)=-\tfrac{1}{2}ifc[\psi_\alpha(x),\ \bar{\psi}_\beta(x)](\gamma_\mu)_{\beta\alpha}=M^{(1)}(x), \quad (2.24)$$

and

$$m_{\mu\nu}(x)=-\tfrac{1}{2}igc[\psi_\alpha(x),\ \bar{\psi}_\beta(x)](\gamma_\mu\gamma_\nu)_{\beta\alpha}=M^{(2)}(x). \quad (2.25)$$

Let r, s be suffixes which take the values 0, 1, 2 and define $L^r(\gamma)$ by

$$(L^{(0)})_{\alpha\beta}=-i\delta_{\alpha\beta}, \tag{2.26}$$

$$(L^{(1)})_{\alpha\beta}=(\gamma_\mu)_{\alpha\beta}, \tag{2.27}$$

$$(L^{(2)})_{\alpha\beta}=(\gamma_\mu\gamma_\nu)_{\alpha\beta}, \tag{2.28}$$

so that

$$M^{(r)}(x)=-\tfrac{1}{2}igc[\psi_\alpha(x),\bar{\psi}_\beta(x)](L^r)_{\beta\alpha}. \tag{2.29}$$

Note that in (2.29) g must be replaced by f if $M^{(r)}$ occurs as a factor in an f-interaction. It then follows after some manipulation of the operators that

$$\begin{aligned}[M^{(r)}(x),M^{(s)}(x')]&=\tfrac{1}{2}ig^2c^2([\bar{\psi}_\beta(x),\psi_\alpha(x')](L^rS(x-x')L^s)_{\beta\alpha}\\&\quad-[\bar{\psi}_\beta(x'),\psi_\alpha(x)](L^sS(x'-x)L^r)_{\beta\alpha}), \qquad (2.30)\\&=\tfrac{1}{2}ig^2c^2([\psi(x),\bar{\psi}(x')L^sS(x'-x)L^r]\\&\quad-[L^rS(x-x')L^s\psi(x'),\bar{\psi}(x)]). \qquad (2.31)\end{aligned}$$

This is a general formula for any commutator of the invariants ω, j_μ, and $m_{\mu\nu}$. By (2.16) and (2.30),

$$\begin{aligned}\langle[M^{(r)}(x),M^{(s)}(x)]\rangle_0&=\tfrac{1}{2}ig^2c^2(\mathrm{Tr}[S^{(1)}(x'-x)L^rS(x-x')L^s]\\&\quad-\mathrm{Tr}[S^{(1)}(x-x')L^sS(x'-x)L^r]), \qquad (2.32)\end{aligned}$$

where $\mathrm{Tr}[\cdots]$ denotes the trace. Introduce the notation

$$\xi=x-x', \tag{2.33}$$

and

$$(\partial/\partial\xi_\mu)\Delta_0(\xi)=\Delta_{0\mu}(\xi). \tag{2.34}$$

By evaluating the traces it follows from (2.32) that

$$\langle[\omega(x),\omega(x')]\rangle_0=4if^2c^2[\Delta_{0\lambda}{}^{(1)}(\xi)\Delta_{0\lambda}(\xi)-\kappa_0{}^2\Delta_0{}^{(1)}(\xi)\Delta_0(\xi)], \tag{2.35}$$

$$\begin{aligned}\langle[j_\mu(x),j_\nu(x')]\rangle_0&=-4ig^2c^2[\Delta_{0\mu}{}^{(1)}(\xi)\Delta_{0\nu}(\xi)+\Delta_{0\nu}{}^{(1)}(\xi)\Delta_{0\mu}(\xi)\\&\quad-\delta_{\mu\nu}(\Delta_{0\lambda}{}^{(1)}(\xi)\Delta_{0\lambda}(\xi)+\kappa_0{}^2\Delta_0{}^{(1)}(\xi)\Delta_0(\xi)), \qquad (2.36)\end{aligned}$$

$$\begin{aligned}\langle[m_{\mu\nu}(x),m_{\pi\rho}(x')]\rangle_0&=-4ig^2c^2[\delta_{\rho\nu}(\Delta_{0\pi}{}^{(1)}(\xi)\Delta_{0\mu}(\xi)+\Delta_{0\mu}{}^{(1)}(\xi)\Delta_{0\pi}(\xi))\\&\quad+\delta_{\pi\mu}(\Delta_{0\rho}{}^{(1)}(\xi)\Delta_{0\nu}(\xi)+\Delta_{0\nu}{}^{(1)}(\xi)\Delta_{0\rho}(\xi))\\&\quad-\delta_{\rho\mu}(\Delta_{0\pi}{}^{(1)}(\xi)\Delta_{0\nu}(\xi)+\Delta_{0\nu}{}^{(1)}(\xi)\Delta_{0\pi}(\xi))\\&\quad-\delta_{\pi\nu}(\Delta_{0\rho}{}^{(1)}(\xi)\Delta_{0\mu}(\xi)+\Delta_{0\mu}{}^{(1)}(\xi)\Delta_{0\rho}(\xi))\\&\quad+(\delta_{\rho\mu}\delta_{\pi\nu}-\delta_{\pi\mu}\delta_{\rho\nu})(\Delta_{0\lambda}{}^{(1)}(\xi)\Delta_{0\lambda}(\xi)\\&\quad-\kappa_0{}^2\Delta_0{}^{(1)}(\xi)\Delta_0(\xi))], \qquad (2.37)\end{aligned}$$

$$\langle[j_\mu(x),\omega(x')]\rangle_0=0, \tag{2.38}$$

$$\langle[j_\mu(x),m_{\pi\rho}(x')]\rangle_0=4ifgc^2(\delta_{\rho\nu}\delta_{\pi\mu}-\delta_{\pi\nu}\delta_{\rho\mu})\times(\Delta_0{}^{(1)}(\xi)\Delta_{0\nu}(\xi)+\Delta_{0\nu}{}^{(1)}(\xi)\Delta_0(\xi)). \tag{2.39}$$

To select out the one nucleon part from an operator of the type $\{M^{(r)}(x),M^{(s)}(x)\}$ the spinors ψ_α must be split into annihilation and creation operators. It is convenient to consider the nucleons in positive energy states to be carrying a charge e. Then the antinucleons, corresponding to the positrons in electron hole theory, will carry a charge $-e$. The charge-current vector is

$$j_\mu'(x)=-\tfrac{1}{2}iec(\bar{\psi}\gamma_\mu\psi-\psi\gamma_\mu{}^T\bar{\psi}). \tag{2.40}$$

The total charge on the surface σ is the integral of the normal component

$$\begin{aligned}Q&=-\frac{1}{c}\int j_\mu(x)n_\mu d\sigma\\&=-\tfrac{1}{2}ie\int(\bar{\psi}(x)\gamma_\mu\psi(x)-\psi(x)\gamma_\mu{}^T\bar{\psi}(x))d\sigma_\mu. \qquad (2.41)\end{aligned}$$

Hence

$$[\psi(x),Q]=-e\psi(x). \tag{2.42}$$

When applied to an eigenstate of the total charge,

$$Q\psi(x)\Psi(Q')=(Q'+e)\psi(x)\Psi(Q'). \tag{2.43}$$

Thus ψ has the effect of increasing the total charge. Therefore, ψ either creates a nucleon or annihilates an anti-nucleon. Similarly $\bar{\psi}$ either creates an anti-nucleon or annihilates a nucleon.

Now in the expression

$$\{M^{(r)}(x),M^{(s)}(x')\}=-\tfrac{1}{4}g^2c^2(L^r)_{\alpha\beta}(L^s)_{\gamma\delta}\times\{[\bar{\psi}_\alpha(x),\psi_\beta(x')],[\bar{\psi}_\gamma(x'),\psi_\delta(x')]\}, \tag{2.44}$$

the terms in ψ and $\bar{\psi}$ are the same as those in $\{j_\mu(x),j_\nu(x')\}$ which has been evaluated by Schwinger.[5] The selection of the one-particle part of the operator can be taken over unchanged and leads to the general result

$$\{M^r(x),M^s(x')\}=\tfrac{1}{2}g^2c^2\{[\bar{\psi}(x),L^rS^{(1)}(x-x')L^s\psi(x')]+[\bar{\psi}(x')L^sS^{(1)}(x'-x)L^r,\psi(x)]\}. \tag{2.45}$$

The one-particle part of any anticommutator of the invariants ω, j_μ, and $m_{\mu\nu}$ can be obtained from (2.45) by substituting for L^r from the definitions (2.26)–2.28).

We next consider the singular functions $\Delta(x)$ and $\Delta^{(1)}(x)$. First consider the function which for a scalar variable a is defined,

$$\epsilon(a)=a/|a|. \tag{2.46}$$

For a vector variable,

$$\epsilon(x)=x_0/|x_0|. \tag{2.47}$$

If σ and σ' are any two space-like surfaces through x

and x', respectively, define

$$\epsilon(\sigma, \sigma') = \pm 1, \tag{2.48}$$

according to whether σ' lies before or after σ. Then $\epsilon(\sigma, \sigma')$ can be replaced by $\epsilon(x-x')$ in an integral over x' if the integrand is zero outside the light cone of x. By considering the integral

$$\int_\alpha^\beta f(a)(d\epsilon(a)/da)da$$

in the two cases when the origin does occur and does not occur in the domain of integration, it can easily be shown that

$$d\epsilon(a)/da = 2\delta(a), \tag{2.49}$$

and hence

$$\partial\epsilon(x)/\partial x_4 = -2i\delta(x_0),\ \partial\epsilon(x)/\partial x_p = 0,\ p = 1, 2, 3. \tag{2.50}$$

Define

$$\bar{\Delta}(x) = -\tfrac{1}{2}\Delta(x)\epsilon(x). \tag{2.51}$$

Then

$$\begin{aligned}\Box^2\bar{\Delta}(x) &= -\tfrac{1}{2}\epsilon(x)\Box^2\Delta(x) - (\partial\Delta(x)/\partial x_\mu)(\partial\epsilon(x)/\partial x_\mu)\\ &\qquad -\tfrac{1}{2}\Delta(x)\Box^2\epsilon(x),\\ &= -\tfrac{1}{2}\epsilon(x)\Box^2\Delta(x) + 2i\delta(x_0)(\partial\Delta(x)/\partial x_4)\\ &\qquad + i\Delta(x)(\partial\delta(x_0)/\partial x_4),\\ &= -\tfrac{1}{2}\kappa^2\epsilon(x)\Delta(x) + i\delta(x_0)(\partial\Delta(x)/\partial x_4),\end{aligned} \tag{2.52}$$

where the last term has been integrated by parts. By (1.43) in the "natural" coordinate system (with time axis in the direction n_μ),

$$(\partial\Delta(x)/\partial x_4) = i\delta(x_1)\delta(x_2)\delta(x_3). \tag{2.53}$$

Thus

$$(\Box^2 - \kappa^2)\bar{\Delta}(x) = -i\delta_4(x). \tag{2.54}$$

Now

$$\delta_4(x) = \left(\frac{1}{2\pi}\right)^4 \int_{-\infty}^{\infty} \exp[ik_\mu x_\mu]d^4k. \tag{2.55}$$

Therefore

$$\bar{\Delta}(x) = \left(\frac{1}{2\pi}\right)^4 P\int_{-\infty}^{\infty} \exp[ik_\mu x_\mu]/(k_\mu^2 + \kappa^2)d^4k, \tag{2.56}$$

where the principal value is taken at $k_\mu^2 + \kappa^2 = 0$. Performing the integration over

$$\Delta(x) = -\left(\frac{1}{2\pi}\right)^3 \times \int_{-\infty}^{\infty} \frac{\exp[i(\mathbf{k}\cdot\mathbf{x})]\sin[x_0(\mathbf{k}^2+\kappa^2)^{\frac{1}{2}}]}{(\mathbf{k}^2+\kappa^2)^{\frac{1}{2}}}d^3k. \tag{2.57}$$

Hence

$$\Delta(x) = -i\left(\frac{1}{2\pi}\right)^3 \times \int_{-\infty}^{\infty} \epsilon(k)\exp(ik_\mu x_\mu)\delta(k_\beta^2 + k^2)d^4k, \tag{2.58}$$

and

$$\Delta^{(1)}(x) = \left(\frac{1}{2\pi}\right)^3 \int_{-\infty}^{\infty} \exp(ik_\mu x_\mu)\delta(k_\mu^2 + \kappa^2)d^4k. \tag{2.59}$$

It follows from (2.57) that $\Delta(x) = 0$ if $x_0 = 0$. It is then easy to prove that

$$(\partial\bar{\Delta}(x)/\partial x_\mu) = -\tfrac{1}{2}\epsilon(x)(\partial\Delta(x)/\partial x_\mu). \tag{2.60}$$

III. THE SCHWINGER TRANSFORMATION

It follows from the above development of the field variables that the Hamiltonian in the interaction representation contains terms of the first order in the coupling constants which have matrix elements for the absorption or emission of mesons by free nucleons. These processes cannot occur with conservation of energy and momentum which suggests that they should be removed by a transformation. Following Schwinger,[5] make the unitary transformation

$$\Psi(\sigma) \rightarrow e^{iS(\sigma)}\Psi(\sigma), \tag{3.1}$$

where

$$S(\sigma) = (1/2\hbar c^2)\int_{-\infty}^{\infty} \{\omega(x')\varphi(x') + (1/\kappa)j_\mu(x')\varphi_\mu(x')\}\epsilon(\sigma, \sigma')d\omega'. \tag{3.2}$$

This leads to an equation of motion for the new $\Psi(\sigma)$ from which all first-order transitions have been eliminated.

$$\begin{aligned} i\hbar c\delta\Psi(\sigma)/\delta\sigma(x) = \Big\{&(i/4\hbar c^3)\int_{-\infty}^{\infty}[\omega(x')\varphi(x')\\ &+ (1/\kappa)j_\mu(x')\varphi_\mu(x'),\ \omega(x)\varphi(x)\\ &+ (1/\kappa)j_\nu(x)\varphi_\nu(x)]\epsilon(\sigma, \sigma')d\omega'\\ &+ (1/2c^2\kappa^2)(j_\mu(x)n_\mu)^2\Big\}\Psi(\sigma), \end{aligned} \tag{3.3}$$

$$= \{\mathcal{H}_1{}^s(x) + \mathcal{H}_2{}^s(x) + \mathcal{H}_3{}^s(x) + \mathcal{H}_4{}^s(x)\}\Psi(\sigma), \tag{3.4}$$

where

$$\mathcal{H}_1{}^s(x) = (-i/4\hbar c^3)\int_{-\infty}^{\infty} [\omega(x)\varphi(x),\ \omega(x')\varphi(x')] \times \epsilon(\sigma, \sigma')d\omega', \tag{3.5}$$

$$\mathcal{H}_2{}^s(x)=(-i/4\hbar c^3\kappa^2)\int_{-\infty}^{\infty}[j_\mu(x)\varphi_\mu(x),\, j_\nu(x')\varphi_\nu(x')]\times\epsilon(\sigma,\sigma')d\omega', \quad (3.6)$$

$$\mathcal{H}_3{}^s(x)=(-i/4\hbar c^3\kappa)\int_{-\infty}^{\infty}([j_\mu(x)\varphi_\mu(x),\, \omega(x')\varphi(x')]-[j_\mu(x')\varphi_\mu(x'),\, \omega(x)\varphi(x)])\epsilon(\sigma,\sigma')d\omega', \quad (3.7)$$

and

$$\mathcal{H}_4{}^s(x)=\left(\frac{1}{2c^2\kappa^2}\right)(j_\mu(x)n_\mu)^2. \quad (3.8)$$

Consider the terms arising from $\mathcal{H}_2{}^s(x)$.

$$\begin{aligned}\mathcal{H}_2{}^s(x)&=(-i/8\hbar c^3\kappa^2)\int([j_\mu(x),\, j_\nu(x')]\{\varphi_\mu(x),\, \varphi_\nu(x')\}\\&\quad+\{j_\mu(x),\, j_\nu(x')\}[\varphi_\mu(x),\, \varphi_\nu(x')])\epsilon(\sigma,\sigma')d\omega',\\&=(-i/8\hbar c^3\kappa^2)\int([j_\mu(x),\, j_\nu(x')](\{\varphi_\mu(x),\, \varphi_\nu(x')\}\\&\quad-\langle\{\varphi_\mu(x),\, \varphi_\nu(x')\}\rangle)\\&\quad+\hbar c[j_\mu(x),\, j_\nu(x')](\partial^2/\partial x_\mu\partial x_\nu')\Delta^{(1)}(x-x')\\&\quad+i\hbar c\{j_\mu(x),\, {}_{\mu\nu}(x')\}(\partial^2/\partial x_\mu\partial x_\nu')\Delta(x-x'))\\&\quad\times\epsilon(\sigma,\sigma')d\omega', \quad (3.9)\\&=(-i/8\hbar c^3\kappa^2)\int(([j_\mu(x),j_\nu(x')]\\&\quad-\langle[j_\mu(x),\, j_\nu(x')]\rangle_0)(\{\varphi_\mu(x),\, \varphi_\nu(x')\}\\&\quad-\langle\{\varphi_\mu(x),\, \varphi_\nu(x')\}\rangle_0)\\&\quad+\hbar c[j_\mu(x),j_\nu(x')](\partial^2/\partial x_\mu\partial x_\nu')\Delta^{(1)}(x-x')\\&\quad+i\hbar c\{j_\mu(x),\, j_\nu(x')\}(\partial^2/\partial x_\mu\partial x_\nu')\Delta(x-x')\\&\quad+\langle[j_\mu(x),\, j_\nu(x')]\rangle_0(\{\varphi_\mu(x),\, \varphi_\nu(x')\}\\&\quad-\langle\{\varphi_\mu(x),\, \varphi_\nu(x')\}\rangle_0))\epsilon(\sigma,\sigma')d\omega', \quad (3.10)\end{aligned}$$

where (1.44) and (2.14) have been used. The first three lines of (3.10) which will be denoted by $\mathcal{H}_{2,1}{}^s(x)$ are zero unless some of both types of particle are present and have matrix elements for real effects of meson scattering. The next two lines, $\mathcal{H}_{22}{}^s(x)$, are non-zero when no mesons are present and give part of the nucleon g self-energy and part of the g-interaction Yukawa potential. The last two lines, $\mathcal{H}_{23}{}^s(x)$, are non-zero when no nucleons are present and account for the meson f self-energy. The other parts of the Hamiltonian $\mathcal{H}_1{}^s(x)$ and $\mathcal{H}_3{}^s(x)$ can be split up in the same way and the notation introduced above can obviously be extended. Thus, for example, the scalar meson f self-energy will arise from the last two lines of $\mathcal{H}_1{}^s(x)$ to be denoted by $\mathcal{H}_{13}{}^s(x)$.

There is an additional contribution to the nucleon g self-energy and Yukawa potential from $\mathcal{H}_4{}^s(x)$. It will be shown how this combines with $\mathcal{H}_{22}{}^s(x)$.

$$\begin{aligned}\mathcal{H}_{22}{}^s(x)&=(i/8c^2\kappa^2)\int([j_\mu(x),\, j_\nu(x')]\\&\quad\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta^{(1)}(x-x')+i\{j_\mu(x),\, j_\nu(x')\}\\&\quad\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta(x-x'))\epsilon(x-x')d\omega'. \quad (3.11)\end{aligned}$$

$\epsilon(\sigma,\sigma')$ has been replaced by $\epsilon(x-x')$ since the integrand is zero outside the light cone. In the second term the ϵ factor can be taken under the differential and the additional terms subtracted explicitly. These can be evaluated in the "natural" frame by (2.50) and (2.53) and it follows that

$$\begin{aligned}-\tfrac{1}{2}\{j_\mu(x),\, j_\nu(x')\}\epsilon(x-x')(\partial^2/\partial x_\mu\partial x_\nu)\Delta(x-x')\\=\{j_\mu(x),\, j_\nu(x')\}(\partial^2/\partial x_\mu\partial x_\nu)\bar{\Delta}(x-x')\\-2(j_\mu(x)n_\mu)(j_\nu(x')n_\nu')\delta_4(x-x'). \quad (3.12)\end{aligned}$$

The second term on the right-hand side is just that required to cancel $\mathcal{H}_4{}^s(x)$. Thus

$$\begin{aligned}\mathcal{H}_{22}{}^s(x)+\mathcal{H}_4{}^s(x)&=(i/8c^2\kappa^2)\int[j_\mu(x),\, j_\nu(x')]\\&\quad\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta^{(1)}(x-x')\cdot\epsilon(x-x')d\omega'\\&\quad+(1/4c^2\kappa^2)\int\{j_\mu(x),\, j_\nu(x')\}\\&\quad\times(\partial^2/\partial x_\mu\partial x_\nu)\bar{\Delta}(x-x')d\omega'. \quad (3.13)\end{aligned}$$

By (2.45) the anticommutator in the second term is a function of $S^{(1)}(x-x')$. By (2.31) the commutator in the first term is a function of $S(x-x')$ which by (2.51) and (2.60) combines with $\epsilon(x-x')$ to give a function of $\bar{\Delta}(x-x')$. Thus the whole integrand can be expressed in terms of $\bar{\Delta}(x-x')$ and $\Delta^{(1)}(x-x')$ and the integral is an invariant independent of the particular choice of n_μ. This is the extremely satisfactory result discussed in the introduction. It will now be shown that there are two similar effects in the vector case.

The reader is reminded that in the above treatment of the scalar meson interaction a suffix μ or ν attached to the field variable $\varphi(x)$ denotes differentiation. In the vector meson case only the second suffix ν in $\varphi_{\mu\nu}(x)$ or $A_{\mu\nu}(x)$ denotes differentiation. To avoid confusion, in any equation in which this notation is used a suffix s (scalar) or v (vector) will also occur to distinguish the two cases.

A transformation similar to (3.1) on the wave equation for the vector interaction (1.69) leads to

$$i\hbar c\delta\Psi/\delta\sigma(x)=\{\mathcal{H}_1{}^v(x)+\mathcal{H}_2{}^v(x)+\mathcal{H}_3{}^v(x)+\mathcal{H}_4{}^v(x)\}\Psi(\sigma), \quad (3.14)$$

where

$$\mathcal{H}_1^v(x)=(-i/4\hbar c^3)\int[j_\mu(x)\varphi_\mu(x),j_\nu(x')\varphi_\nu(x')]\times\epsilon(\sigma\cdot\sigma')d\omega', \quad (3.15)$$

$$\mathcal{H}_2^v(x)=(-i/16\kappa^2\hbar c^3)\times\int[m_{\mu\nu}(x)(\varphi_{\nu\mu}(x)-\varphi_{\mu\nu}(x)),\, m_{\pi\rho}(x')\times(\varphi_{\rho\pi}(x')-\varphi_{\pi\rho}(x'))]\epsilon(\sigma,\sigma')d\omega', \quad (3.16)$$

$$\mathcal{H}_3^v(x)=(-i/8\kappa\hbar c^3)\times\int([m_{\mu\nu}(x)(\varphi_{\nu\mu}(x)-\varphi_{\mu\nu}(x)),\, j_\rho(x')\varphi_\rho(x')]-[m_{\mu\nu}(x')(\varphi_{\nu\mu}(x')-\varphi_{\mu\nu}(x')),\, j_\rho(x)\varphi_\rho(x)])\times\epsilon(\sigma,\sigma')d\omega', \quad (3.17)$$

$$\mathcal{H}_4^v(x)=(1/2c^2\kappa^2)\{(j_\mu(x)n_\mu)^2+(m_{\mu\nu}(x)n_\mu)^2\}. \quad (3.18)$$

The various parts of the Hamiltonian can be split up as above. Consider $\mathcal{H}_{12}^v(x)$ obtained from $\mathcal{H}_1^v(x)$ as $\mathcal{H}_{22}^s(x)$ was from $\mathcal{H}_2^s(x)$.

$$\mathcal{H}_{12}^v(x)=(-i/8c^2)\int([j_\mu(x),\, j_\nu(x')]T_{\mu\nu}^{(1)}(x-x')+i\{j_\mu(x),\, j_\nu(x')\}T_{\mu\nu}(x-x'))\epsilon(\sigma,\sigma')d\omega', \quad (3.19)$$

$$=(-i/8c^2)\int([j_\mu(x),\, j_\nu(x')]\Delta^{(1)}(x-x')+i\{j_\mu(x),\, j_\nu(x')\}\Delta(x-x'))\epsilon(x-x')d\omega'+(i/8c^2)\int([j_\mu(x),\, j_\nu(x')]\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta^{(1)}(x-x')+i\{j_\mu(x),\, j_\nu(x')\}\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta(x-x'))\epsilon(x-x')d\omega'. \quad (3.20)$$

The second term of (3.20) can be expressed in terms of $\bar{\Delta}(x-x')$. The third and fourth terms are identical with $\mathcal{H}_{22}^s(x)$ and combine with the first term, $\mathcal{H}_{41}^v(x)$, of $\mathcal{H}_4^v(x)$. Thus

$$\mathcal{H}_{12}^v(x)+\mathcal{H}_{41}^v(x)=(-i/8c^2)\int[j_\mu(x),\, j_\mu(x')]\times\Delta^{(1)}(x-x')\epsilon(x-x')d\omega'-(1/4c^2)\times\int\{j_\mu(x),\, j_\mu(x')\}\bar{\Delta}(x-x')d\omega'+(i/8c^2\kappa^2)\int[j_\mu(x),\, j_\nu(x')]\times(\partial^2/\partial x_\mu\partial x_\nu)\Delta^{(1)}(x-x')\cdot\epsilon(x-x')d\omega'+(1/4c^2)\int\{j_\mu(x),\, j_\nu(x')\}\times(\partial^2/\partial x_\mu\partial x_\nu)\bar{\Delta}(x-x')d\omega'. \quad (3.21)$$

Virtual meson effects in the g-interaction in the vector meson case can be expressed in terms of the subsidiary field $A_\mu(x)$. Thus

$$m_{\mu\nu}(x)(\varphi_{\nu\mu}(x)-\varphi_{\mu\nu}(x))=m_{\mu\nu}(x)(A_{\nu\mu}(x)-A_{\mu\nu}(x))=2m_{\mu\nu}(x)A_{\nu\mu}(x). \quad (3.22)$$

Hence

$$\mathcal{H}_{22}^v(x)=(-i/8\hbar c^3\kappa^2)\int([m_{\mu\nu}(x),\, m_{\pi\rho}(x')]\times\langle\{A_{\nu\mu}(x),\, A_{\rho\pi}(x')\}\rangle_0+i\{m_{\mu\nu}(x),\, m_{\pi\rho}(x')\}\times[A_{\nu\mu}(x),\, A_{\rho\pi}(x')])\epsilon(\sigma,\sigma')d\omega'$$
$$=(i/8c^2\kappa^2)\int([m_{\mu\nu}(x),\, m_{\pi\nu}(x')](\partial^2/\partial x_\mu\partial x_\pi)\times\Delta^{(1)}(x-x')+i\{m_{\mu\nu}(x),\, m_{\pi\nu}(x')\}\times(\partial^2/\partial x_\mu\partial x_\pi)\Delta(x-x'))\epsilon(x-x')d\omega'. \quad (3.23)$$

The final term combines with the second term, $\mathcal{H}_{42}^v(x)$, of $\mathcal{H}_4^v(x)$ to give an integral of the required form

$$\mathcal{H}_{22}^v(x)+\mathcal{H}_{42}^v(x)=(i/8c^2\kappa^2)\int[m_{\mu\nu}(x),\, m_{\pi\nu}(x')]\times(\partial^2/\partial x_\mu\partial x_\pi)\Delta^{(1)}(x-x')\cdot\epsilon(x-x')d\omega+(1/4c^2\kappa^2)\int\{m_{\mu\nu}(x),\, m_{\pi\nu}(x')\}(\partial^2/\partial x_\mu\partial x_\pi)\times\bar{\Delta}(x-x')d\omega'. \quad (3.24)$$

IV. THE MESON SELF-ENERGIES

The f self-energy of the scalar meson is determined by

$$\mathcal{H}_{13}^s(x)=(-i/8\hbar c^3)\int\langle[\omega(x),\,\omega(x')]\rangle_0\times\{\varphi(x),\,\varphi(x')\}_1\epsilon(\sigma,\sigma')d\omega', \quad (4.1)$$

$$=\{\varphi(x),\,\eta(x)\}_1, \quad (4.2)$$

where

$$\eta(x)=(-i/8\hbar c^3)\int\langle[\omega(x),\,\omega(x')]\rangle_0\times\epsilon(x-x')\varphi(x')d\omega'. \quad (4.3)$$

It must be shown that $\eta(x)$ is a multiple of $\varphi(x)$. By (2.35) and (2.51)

$$\eta(x)=(-f^2/\hbar c)\int\{\Delta_{0\lambda}^{(1)}(x-x')\bar{\Delta}_{0\lambda}(x-x')-\kappa_0^2\Delta_0^{(1)}(x-x')\bar{\Delta}_0(x-x')\}\varphi(x')d\omega', \quad (4.4)$$

$$=(-f^2/\hbar c)\int \mathrm{K}(x-x')\varphi(x')d\omega', \tag{4.5}$$

$$=(-f^2/\hbar c)(1/2\pi)^4\int\int \mathrm{K}(k)\times\exp[ik_\mu\cdot x_\mu-x_\mu']\varphi(x')d\omega' d^4k, \tag{4.6}$$

where

$$\mathrm{K}(x)=(1/2\pi)^4\int K(k)\exp[ik_\mu x_\mu]d^4k. \tag{4.7}$$

If $\eta(x)$ is evaluated for a particular Fourier component $\varphi(p)\exp[ip_\mu x_\mu]$ of $\varphi(x)$ then with ξ defined, by (2.33),

$$\begin{aligned}\eta(x)&=(-f^2/\hbar c)(1/2\pi)^4\int\int \mathrm{K}(k)\exp[ik_\mu\xi_\mu]\varphi(p)\times\exp[ip_\mu\cdot x_\mu-\xi_\mu]d\omega_\xi d^4k,\\&=(-f^2/\hbar c)\varphi(p)\exp[ip_\mu x_\mu]\times\int \mathrm{K}(k)\delta_4(k_\mu-p_\mu)d^4k,\\&=(-f^2/\hbar c)\varphi(x)\mathrm{K}(p).\end{aligned} \tag{4.8}$$

Since $\varphi(p)\exp[ip_\mu x_\mu]$ is a Fourier component of $\varphi(x)$, by (1.20),

$$p_\mu^2=-\kappa^2. \tag{4.9}$$

Now

$$\mathrm{K}(x)=\Delta_{0\lambda}{}^{(1)}(x)\bar{\Delta}_{0\lambda}(x)-\kappa_0^2\Delta_0{}^{(1)}(x)\bar{\Delta}_0(x). \tag{4.10}$$

And by (2.56) and (2.59)

$$\Delta_0{}^{(1)}(x)=\left(\frac{1}{2\pi}\right)^3\int \exp[iq_\nu x_\nu]\delta(q_\nu^2+\kappa_0^2)d^4q, \tag{4.11}$$

$$\bar{\Delta}_0(x)=\left(\frac{1}{2\pi}\right)^4 P\int \frac{\exp[i(p_\mu-q_\mu\cdot x_\mu)]}{(p_\lambda-q_\lambda)^2+\kappa_0^2}d^4p. \tag{4.12}$$

Thus

$$\mathrm{K}(x)=\left(\frac{1}{2\pi}\right)^7\int \frac{\exp[ip_\mu x_\mu]}{(p_\lambda-q_\lambda)^2+\kappa_0^2}\delta(q_\lambda^2+\kappa_0^2)\times[-q_\lambda(p_\lambda-q_\lambda)-\kappa_0^2]d^4pd^4q, \tag{4.13}$$

and

$$\mathrm{K}(p)=\left(\frac{1}{2\pi}\right)^3\int \frac{\delta(q_\lambda^2+\kappa_0^2)}{(p_\lambda-q_\lambda)^2+\kappa_0^2}\times[-q_\lambda(p_\lambda-q_\lambda)-\kappa_0^2]d^4q. \tag{4.14}$$

Now

$$\delta(q_\lambda^2+\kappa_0^2)=(1/2\pi)\int_{-\infty}^{\infty}\exp[i(q_\lambda^2+\kappa_0^2)a]da, \tag{4.15}$$

and

$$(1/(p_\lambda-q_\lambda)^2+\kappa_0^2)=-\tfrac{1}{2}i\int_{-\infty}^{\infty}\exp[ib\{(p_\lambda-q_\lambda)^2+\kappa_0^2\}]\epsilon(b)db. \tag{4.16}$$

Substituting in (4.14),

$$\mathrm{K}(p)=\left(\frac{1}{2\pi}\right)^4\frac{i}{2}\int d^4q\int da\int db\exp[ia(q_\lambda^2+\kappa_0^2)+ib\{(p_\lambda-q_\lambda)^2+\kappa_0^2\}]\epsilon(b)[q_\lambda(p_\lambda-q_\lambda)+\kappa_0^2]. \tag{4.17}$$

Define

$$q_\nu=Q_\nu+(b/a+b)p_\nu. \tag{4.18}$$

Then

$$\mathrm{K}(p)=\left(\frac{1}{2\pi}\right)^4\frac{i}{2}\int d^4Q\int da\int db\times\exp\left[i(a+b)Q_\lambda^2+i\frac{ab}{a+b}p_\lambda^2+i(a+b)\kappa_0^2\right]\times\epsilon(b)\left[\frac{ab}{(a+b)^2}p_\lambda^2+Q_\lambda p_\lambda\frac{a-b}{a+b}-Q_\lambda^2+\kappa_0^2\right]. \tag{4.19}$$

Now

$$\begin{aligned}&\int_{-\infty}^{\infty}\exp[iaQ_\lambda^2]d^4Q=(i\pi^2/a^2)\epsilon(a),\\&\int_{-\infty}^{\infty}Q_\mu\exp[iaQ_\lambda^2]d^4Q=0,\\&\int_{-\infty}^{\infty}Q_\mu Q_\nu\exp[iaQ_\lambda^2]d^4Q=-\delta_{\mu\nu}(\pi^2/2a^3)\epsilon(a),\\&\int_{-\infty}^{\infty}Q_\mu^2\exp[iaQ_\lambda^2]d^4Q=-(2\pi^2/a^3)\epsilon(a).\end{aligned} \tag{4.20}$$

Thus integrating with respect to Q,

$$\mathrm{K}(p)=\frac{1}{32\pi^2}\int da\int db\exp\left[i(a+b)\kappa_0^2+i\frac{ab}{a+b}p_\lambda^2\right]\times\frac{\epsilon(b)\epsilon(a+b)}{(a+b)^2}\left[-\frac{ab}{(a+b)^2}p_\lambda^2-\kappa_0^2+\frac{2i}{(a+b)}\right]. \tag{4.21}$$

Symmetrizing with respect to a and b,

$$\epsilon(b)=\tfrac{1}{2}(\epsilon(a)+\epsilon(b)). \tag{4.22}$$

Making the substitution,

$$a=\tfrac{1}{2}z(1+y),\quad b=\tfrac{1}{2}z(1-y). \tag{4.23}$$

Then

$$\tfrac{1}{2}(\epsilon(a)+\epsilon(b))=\begin{cases}\epsilon(z) & \text{for } |y|<1,\\ 0 & \text{for } |y|>1,\end{cases} \tag{4.24}$$

and

$$dadb=\tfrac{1}{2}z\epsilon(z)dzdy. \tag{4.25}$$

Thus

$$\mathrm{K}(p)=\frac{1}{64}\left(\frac{1}{2\pi}\right)^2\int_{-1}^{+1}dy\int_{-\infty}^{\infty}dz\frac{\epsilon(z)}{z}\times\exp\left[iz\left(\kappa_0^2+\frac{1-y^2}{4}p_\lambda^2\right)\right]\times\left\{\frac{8i}{z}-(1-y^2)p_\lambda^2-4\kappa_0^2\right\}. \tag{4.26}$$

Let

$$z=(\omega/\kappa_0^2), \tag{4.27}$$

and

$$\mu=(\kappa_0/\kappa). \tag{4.28}$$

Then substituting for p_λ^2 from (4.9),

$$\mathrm{K}(\kappa)=\frac{\kappa^2}{64}\left(\frac{1}{2\pi}\right)^2\int_{-1}^{+1}dy\left[\frac{8i\mu^2}{\omega}-4\mu^2+1-y^2\right]\times\int_{-\infty}^{\infty}d\omega\frac{1}{|\omega|}\exp\left[i\omega\left(1-\frac{1-y^2}{4\mu^2}\right)\right] \tag{4.29}$$

$$=\frac{\kappa^2}{64}\left(\frac{1}{2\pi}\right)^2\int_{-1}^{+1}dy\left(A+\frac{iB}{\omega}\right)\times\int_{-\infty}^{\infty}d\omega\frac{1}{|\omega|}\exp[iC\omega], \tag{4.30}$$

where

$$A=1-y^2-4\mu^2,\quad B=8\mu^2,\quad C=1-(1-y^2)/4\mu^2. \tag{4.31}$$

The imaginary terms in the integrand are odd and can be regarded as giving no contribution to the integral. Thus

$$\int_{-\infty}^{\infty}\left(A+\frac{iB}{\omega}\right)\frac{1}{|\omega|}\exp[iC\omega]d\omega=2\int_0^{\infty}(A-BC)\frac{\cos C\omega}{\omega}d\omega-2BC, \tag{4.32}$$

and

$$\mathrm{K}(\kappa)=\frac{\kappa^2}{32}\left(\frac{1}{2\pi}\right)^2\int_{-1}^{+1}\left\{(A-BC)\times\left(\log\frac{1}{\gamma\omega_0}-\log C\right)-BC\right\}dy \tag{4.33}$$

$$=\frac{\kappa^2}{32\pi^2}\left[(1-6\mu^2)\log\frac{1}{\gamma\omega_0}+\text{finite terms}\right]. \tag{4.34}$$

Therefore,

$$\eta(x)=\varphi(x)\left(\frac{\kappa^2f^2}{4\pi\hbar c}\right)\left(\frac{-1}{8\pi}\right)\times\left[(1-6\mu^2)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \tag{4.35}$$

And by (4.2),

$$\mathcal{H}_{13}^s(x)=2\kappa\kappa_1\{\varphi^2(x)\}_1, \tag{4.36}$$

where

$$\kappa_1=\kappa\left(\frac{f^2}{4\pi\hbar c}\right)\left(\frac{-1}{8\pi}\right)\left[(1-6\mu^2)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \tag{4.37}$$

The scalar meson g self-energy is given by

$$\mathcal{H}_{23}^s(x)=(-i/8\hbar c^3\kappa^2)\int_{-\infty}^{\infty}\langle[j_\mu(x),j_\nu(x')]\rangle_0\times\{\varphi_\mu(x),\varphi_\nu(x')\}_1\epsilon(\sigma,\sigma')d\omega' \tag{4.38}$$

$$=\{\varphi_\mu(x),\eta_\mu^s(x)\}_1, \tag{4.39}$$

where by (2.36),

$$\eta_\mu^s(x)=(-i/8\hbar c^3\kappa^2)\int_{-\infty}^{\infty}\langle[j_\mu(x),j_\nu(x')]\rangle_0\times\varphi_\nu(x')\epsilon(\sigma,\sigma')d\omega' \tag{4.40}$$

$$\begin{aligned}&=(g^2/\hbar c\kappa^2)\int_{-\infty}^{\infty}[\bar\Delta_{0\mu}(x-x')\Delta_{0\nu}^{(1)}(x-x')\\&\quad+\bar\Delta_{0\nu}(x-x')\Delta_{0\mu}^{(1)}(x-x')\\&\quad-\delta_{\mu\nu}\{\bar\Delta_{0\lambda}(x-x')\Delta_{0\lambda}^{(1)}(x-x')\\&\quad+\kappa_0^2\bar\Delta_0(x-x')\Delta_0^{(1)}(x-x')\}]\varphi_\nu(x')d\omega'\end{aligned}$$

$$=(g^2/\hbar c\kappa^2)\int_{-\infty}^{\infty}\mathrm{K}_{\mu\nu}^s(x-x')\varphi_\nu(x')d\omega'. \tag{4.41}$$

Evaluating for a particular component of $\varphi(x)$,

$$\begin{aligned}\eta_\mu^s(x)&=(g^2/\hbar c\kappa^2)(1/2\pi)^4\int d\omega_\xi'\int d^4k\mathrm{K}_{\mu\nu}^s(k)\varphi(p)ip_\nu\\&\quad\times\exp[ip_\lambda x_\lambda]\exp[i\xi_\sigma\cdot k_\sigma-p_\sigma]\\&=(g^2/\hbar c\kappa^2)\varphi(p)ip_\nu\exp[ip_\lambda x_\lambda]\mathrm{K}_{\mu\nu}^s(p)\\&=(g^2/\hbar c\kappa^2)\varphi_\nu(x)\mathrm{K}_{\mu\nu}^s(p).\end{aligned} \tag{4.42}$$

In addition to (4.9), there is the relation

$$p_\mu p_\nu\varphi(p)ip_\nu\exp[ip_\lambda x_\lambda]=-\kappa^2\varphi(p)ip_\mu\exp[ip_\lambda x_\lambda]=-\kappa^2\varphi_\mu(x). \tag{4.43}$$

Thus for terms in the integrand of $\mathrm{K}_{\mu\nu}^s(p)$,

$$p_\mu p_\nu=-\kappa^2\delta_{\mu\nu}. \tag{4.44}$$

$K_{\mu\nu}{}^s(p)$ can be integrated by the same method as $K(p)$. Introducing the variables a, b, and Q and integrating over Q,

$$K_{\mu\nu}{}^s(p)=\frac{1}{32\pi^2}\int da\int db \times\exp\left[i(a+b)\kappa_0{}^2+i\frac{ab}{a+b}p_\lambda{}^2\right]\frac{\epsilon(b)\epsilon(a+b)}{(a+b)^2} \times\left[\delta_{\mu\nu}\left\{\frac{-i}{(a+b)}+\frac{ab}{(a+b)}p_\lambda{}^2-\kappa^2\right\} -p_\nu p_\mu\frac{2ab}{(a+b)^2}\right]. \quad (4.45)$$

Introducing variables y and ω and eliminating p_λ by (4.9) and (4.44)

$$K_{\mu\nu}{}^s(x)=\frac{\kappa^2}{64}\left(\frac{1}{2\pi}\right)^2\delta_{\mu\nu}\int_{-1}^{+1}dy\left[\frac{-4i\mu^2}{\omega}-4\mu^2+1-y^2\right] \times\int_{-\infty}^{\infty}d\omega\frac{1}{|\omega|}\exp\left[i\omega\left(1-\frac{1-y^2}{4\mu^2}\right)\right], \quad (4.46)$$

which can be evaluated like (4.30) with

$$A=1-y^2-4\mu^2,\quad B=-4\mu^2,\quad C=1-(1-y^2)/4\mu^2. \quad (4.47)$$

In this case $A-BC=0$, giving the finite result

$$K_{\mu\nu}{}^s(\kappa)=\frac{\kappa^2}{32\pi^2}\delta_{\mu\nu}[2\mu^2-\tfrac{1}{3}]. \quad (4.48)$$

Thus

$$\eta_\mu{}^s(x)=\varphi_\mu(x)\left(\frac{g^2}{4\pi\hbar c}\right)\left(\frac{1}{8\pi}\right)[2\mu^2-\tfrac{1}{3}], \quad (4.49)$$

and

$$\mathcal{H}_{23}{}^s(x)=-2(\kappa_2/\kappa)\{\varphi_\mu{}^2(x)\}_1, \quad (4.50)$$

where

$$\kappa_2=\kappa\left(\frac{g^2}{4\pi\hbar c}\right)\left(\frac{1}{8\pi}\right)[\tfrac{1}{3}-2\mu^2]. \quad (4.51)$$

Now

$$\varphi(x)=\sum_k \varphi^{(+)}(k)\exp[i(k_rx_r-k_0x_0)] +\varphi^{(-)}(k)\exp[i(k_rx_r+k_0x_0)], \quad (4.52)$$

where

$$k_0=(k_r{}^2+\kappa^2)^{\frac{1}{2}}. \quad (4.53)$$

Consider $\{\varphi^2(x)\}_1$ as an expression in terms of $\varphi(k)$. Since it is a term in an energy density, only terms with zero exponential in the space variables need be included. Also $\{\varphi^2(x)\}_1$ operates on one meson which must be annihilated and subsequently recreated by the two operators $\varphi(x)$. Thus the exponential in the time factor is also zero and

$$\{\varphi^2(x)\}_1=\sum_k \varphi^{(-)}(k)\varphi^{(+)}(k). \quad (4.54)$$

Now

$$(\partial\varphi(x)/\partial x_r)=\sum ik_r\varphi^{(+)}(k)\exp[i(k_rx_r-k_0x_0)] +ik_r\varphi^{(-)}(k)\exp[i(k_rx_r+k_0x_0)], \quad (4.55)$$

and

$$(\partial\varphi(x)/\partial x_4)=\sum -k_0\varphi^{(+)}(k)\exp[i(k_rx_r-k_0x_0)] +k_0\varphi^{(-)}(k)\exp[i(k_rx_r+k_0x_0)]. \quad (4.56)$$

Thus

$$\{\varphi_\mu{}^2(x)\}_1=\sum (k_r{}^2-k_0{}^2)\varphi^{(-)}(k)\varphi^{(+)}(k) =-\kappa^2\{\varphi^2(x)\}_1. \quad (4.57)$$

So that the g self-energy of the scalar meson can also be written in the form

$$\mathcal{H}_{23}{}^s=2\kappa_2\kappa\{\varphi^2(x)\}_1. \quad (4.58)$$

The Schrödinger Eq. (3.4) can now be written

$$i\hbar c\delta\Psi/\delta\sigma=\{\mathcal{H}_e{}^s(\varphi)+\mathcal{H}_m{}^s(\varphi)\}\Psi, \quad (4.59)$$

where

$$\mathcal{H}_m{}^s(\varphi)=2(\kappa_1+\kappa_2)\kappa\varphi^2(x), \quad (4.60)$$

and

$$\mathcal{H}_e{}^s(\varphi)=\mathcal{H}_1{}^s(\varphi)+\mathcal{H}_2{}^s(\varphi) +\mathcal{H}_3{}^s(\varphi)+\mathcal{H}_4{}^s(\varphi)-\mathcal{H}_m{}^s(\varphi). \quad (4.61)$$

The field variable φ of which the various expressions $\mathcal{H}(\varphi)$ are functions, satisfies the equation of motion of the free meson field determined by the Hamiltonian (1.30). To obtain an equation from which the self-energies $\mathcal{H}_m(\varphi)$ have been eliminated make the transformation

$$\Psi(\sigma)=U_0(\sigma)\bar{\Psi}(\sigma), \quad (4.62)$$

where

$$i\hbar c\delta U_0/\delta\sigma=\mathcal{H}_m{}^s(\varphi)U_0. \quad (4.63)$$

Then

$$i\hbar c\delta\bar{\Psi}/\delta\sigma=\mathcal{H}_e{}^s(\bar{\varphi})\bar{\Psi}, \quad (4.64)$$

where

$$\bar{\varphi}=U_0{}^{-1}\varphi U_0. \quad (4.65)$$

Define $R_\varphi(\sigma)$ by the equation

$$i\hbar c\delta R_\varphi/\delta\sigma=\mathcal{H}_{\text{free}}{}^s(\varphi_s)R_\varphi, \quad (4.66)$$

$\mathcal{H}_{\text{free}}{}^s(\varphi_s)$ being given by (1.30). The variables φ can be regarded as the Heisenberg variables of the free field and R_φ is the transformation which takes them into the Schrödinger variables of the free field by the relation

$$\varphi=R_\varphi{}^{-1}\varphi_sR_\varphi. \quad (4.67)$$

Thus

$$\bar{\varphi}=V^{-1}\varphi_sV, \quad (4.68)$$

where

$$V=R_\varphi U_0. \quad (4.69)$$

By (4.63), (4.66), and (4.69),

$$i\hbar c\delta V/\delta\sigma=\mathcal{H}_{\text{free}}{}^s(\varphi_s)V+R_\varphi\mathcal{H}_m{}^s(\varphi)U_0 =\{\mathcal{H}_{\text{free}}{}^s(\varphi_s)+\mathcal{H}_m{}^s(\varphi_s)\}V. \quad (4.70)$$

Thus by (4.68), $\bar{\varphi}$ satisfies the equation of motion determined by the Hamiltonian $\mathcal{H}_{\mathrm{free}}^{s}+\mathcal{H}_{m}^{s}$. But by (4.60) and (1.30) this is equal to $\mathcal{H}_{\mathrm{free}}^{s}$ with κ replaced by $\kappa+\kappa_1+\kappa_2$. Thus the field variable of Eq. (4.64) from which the meson self-energy terms have been eliminated satisfies the equation of motion of the free meson field with the renormalized mass $\kappa+\kappa_1+\kappa_2$ (κ_1, κ_2 small). This is only a formal renormalization since κ_1 is given infinite by the theory. The renormalized mass is put equal to the observed mass since it is the "bare" particle plus the vacuum effects which is actually observed.

The vector meson f self-energy arises from

$$\mathcal{H}_{13}{}^{v}(x)=(-i/8\hbar c^3)\int\langle[j_\mu(x),\,j_\nu(x')]\rangle_0 \times\{\varphi_\mu(x),\,\varphi_\nu(x')\}_1\epsilon(\sigma,\sigma')d\omega' \quad (4.71)$$

$$=\{\varphi_\mu(x),\,\eta_\mu{}^v(x)\}_1, \quad (4.72)$$

where

$$\eta_\mu{}^v(x)=(-i/8\hbar c^3)\int\langle[j_\mu(x),\,j_\nu(x')]\rangle_0 \times\varphi_\nu(x')\epsilon(x-x')d\omega'$$

$$=(f^2/\hbar c)\int \mathrm{K}_{\mu\nu}{}^v(x-x')\varphi_\nu(x')d\omega'. \quad (4.73)$$

$\mathrm{K}_{\mu\nu}{}^v(x)$ is the same expression as occurred in (4.41) in the scalar g self-energy. As for (4.8),

$$\eta_\mu{}^v(x)=(f^2/\hbar c)\mathrm{K}_{\mu\nu}{}^v(p)\varphi_\nu(x). \quad (4.74)$$

From the supplementary condition

$$(\partial\varphi_\nu/\partial x_\nu)\Psi=0,\quad ip_\nu\varphi_\nu(p)\exp[ip_\lambda x_\lambda]\Psi=0. \quad (4.75)$$

Thus, in this case, for the terms in the integrand of $\mathrm{K}_{\mu\nu}{}^v(p)$

$$p_\mu p_\nu=0. \quad (4.76)$$

This relation replaces (4.44) in the scalar g case. Equation (4.45) is still valid. Introducing the variables y and ω and eliminating p_λ by (4.9) and (4.76),

$$\mathrm{K}_{\mu\nu}{}^v(\kappa)=\frac{\kappa^2}{64}\left(\frac{1}{2\pi}\right)^2\delta_{\mu\nu}\int_{-1}^{+1}dy\left[\frac{-4i\mu^2}{\omega}-4\mu^2-1+y^2\right] \times\int d\omega\frac{1}{|\omega|}\exp\left[i\omega\left(1-\frac{1-y^2}{4\mu^2}\right)\right]. \quad (4.77)$$

This integral is of the standard form (4.30) with

$$A=-1+y^2-4\mu^2,\quad B=-4\mu^2,\quad C=1-(1-y^2)/4\mu^2. \quad (4.78)$$

Thus

$$\mathrm{K}_{\mu\nu}{}^v(\kappa)=\frac{\kappa^2}{32}\left(\frac{1}{2\pi}\right)^2\delta_{\mu\nu}\left[-\frac{8}{3}\log\frac{1}{\gamma\omega_0}+2\int_{-1}^{+1}(1-y^2) \times\log\left(1-\frac{1-y^2}{4\mu^2}\right)dy+8\mu^2-\frac{4}{3}\right] \quad (4.79)$$

$$=-\frac{\kappa^2}{16\pi^2}\delta_{\mu\nu}\left[\frac{1}{3}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right] \quad (4.80)$$

and

$$\eta_\mu{}^v(x)=\varphi_\mu(x)\left(\frac{\kappa^2f^2}{4\pi\hbar c}\right)\left(\frac{-1}{4\pi}\right)\left[\frac{1}{3}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (4.81)$$

Hence

$$\mathcal{H}_{13}{}^v(x)=2\kappa\kappa_3\{\varphi_\mu{}^2(x)\}_1, \quad (4.82)$$

and

$$\kappa_3=\kappa\left(\frac{f^2}{4\pi\hbar c}\right)\left(\frac{-1}{4\pi}\right)\left[\frac{1}{3}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (4.83)$$

In the particular case when $\kappa=0$ the only non-zero term in $\mathcal{H}_{13}{}^v(x)$ comes from the term $8\mu^2$ in the square bracket of (4.79). Then

$$\kappa_{\mu\nu}{}^v(\kappa)=\frac{\kappa_0{}^2}{4}\left(\frac{1}{2\pi}\right)^2\delta_{\mu\nu},$$

which leads to

$$\mathcal{H}_{13}{}^v(x)=\kappa_0{}^2\left(\frac{f^2}{4\pi\hbar c}\right)\frac{1}{2\pi}\{\varphi_\mu{}^2(x)\}_1. \quad (4.84)$$

This is the finite value for the photon self-energy obtained by Wentzel.[6]

The strongest singularities of the Δ-functions are

$$\bar{\Delta}(x)=(-1/4\pi)\delta(x_\mu{}^2)+\cdots, \quad (4.85)$$

$$\Delta^{(1)}(x)=(-1/2\pi x_\mu{}^2)+\cdots. \quad (4.86)$$

Thus the integral of a product $\bar{\Delta}(x)\Delta^{(1)}(x)$ through the origin is not regular and will give different values for different methods of integration. For unobservable effects such as meson self-energies, which are to be transformed away, only the form of the expression is really significant. However, for observable effects some precise definition of the integrals is necessary. The vector meson f self-energy is an observable effect when $\kappa=0$, since it then reduces to the photon self-energy which must be zero. Pauli[7] has proposed a procedure for regularizing the integrals which will be given here in some detail.

It can be shown that both $\bar{\Delta}(x)$ and $\Delta^{(1)}(x)$ are functions of $\kappa^2x_\mu{}^2$ so that a regular function $\mathbf{K}$ can be obtained from $\mathrm{K}(\kappa^2x_\mu{}^2)$ defined by

$$\mathbf{K}=\int_0^\infty\rho(\alpha)\mathrm{K}(\alpha x_\mu{}^2)d\alpha, \quad (4.87)$$

where

$$\int_0^\infty \rho(\alpha)d\alpha=0. \tag{4.88}$$

and

$$\int_0^\infty \alpha\rho(\alpha)d\alpha=0. \tag{4.89}$$

Also

$$\rho(\alpha)=\delta(\alpha-\kappa^2)+\rho_1(\alpha), \tag{4.90}$$

and $\rho_1(\alpha)=0$ for finite α and $\rho_1(\alpha)$ is such that in the limit

$$\int_0^\infty (\rho_1(\alpha)/\alpha)d\alpha=0. \tag{4.91}$$

If $\rho(\alpha)$ is expressed in terms of discreet masses

$$\rho_1(\alpha)=\sum C_i\delta(\alpha-\kappa_i^2). \tag{4.92}$$

Then

$$1+\sum C_i=0, \tag{4.93}$$

and

$$\kappa^2+\sum C_i\kappa_i^2=0. \tag{4.94}$$

The regularization of the matrix element for any effect can thus be regarded as subtracting the matrix element for the same effect due to an interaction with similar particles with masses corresponding to κ_i and weight factors C_i and in the final limit letting the masses of the subsidiary particles tend to infinity. The procedure is thus a generalization of the ideas of Podolsky[10] and Feynman.[11] In this case, however, the subsidiary fields are introduced purely as a mathematical device with no suggestion of physical reality.

To regularize a term $\exp[ix\kappa_0^2]$ in $\kappa_{\mu\nu}$ it must be replaced by

$$R(x)=\int_0^\infty \rho(\alpha)\exp[i\alpha x]d\alpha, \tag{4.95}$$

where by (4.88) and (4.89),

$$R(0)=0, \tag{4.96}$$

and

$$R'(0)=0. \tag{4.97}$$

Also

$$R'(x)=\int_0^\infty i\alpha\rho(\alpha)\exp[i\alpha x]d\alpha. \tag{4.98}$$

Thus a term $\kappa_0^2\exp[ix\kappa_0^2]$ occurring in $\mathbf{K}_{\mu\nu}$ is regularized by replacing it by $-iR'(x)$. Applying this procedure to (4.45) gives

$$\mathbf{K}_{\mu\nu}(p)=\frac{1}{32\pi^2}\int da\int db\frac{\epsilon(a)\epsilon(a+b)}{(a+b)^2}\exp\left[ip_\lambda^2\frac{ab}{a+b}\right]$$
$$\times\left\{\delta_{\mu\nu}\left[\left(\frac{-i}{a+b}+\frac{ab}{(a+b)^2}p_\lambda^2\right)R(a+b)\right.\right.$$
$$\left.\left.+iR'(a+b)\right]-p_\nu p_\mu\frac{2ab}{(a+b)^2}R(a+b)\right\}. \tag{4.99}$$

By symmetrizing with respect to a and b and introducing the variables y and z

$$\mathbf{K}_{\mu\nu}(p)=\frac{1}{32}\cdot\frac{1}{2\pi^2}\int_{-1}^{+1}dy\int_{-\infty}^{\infty}dz\epsilon(z)$$
$$\times\left\{\delta_{\mu\nu}\frac{d}{dz}\left[i\frac{R(z)}{z}\exp\left[\frac{i}{4}(1-y^2)p_\lambda^2z\right]\right]\right.$$
$$+\tfrac{1}{2}(p_\lambda^2\delta_{\mu\nu}-p_\mu p_\nu)(1-y^2)\frac{R(z)}{z}$$
$$\left.\times\exp\left[\frac{i}{4}(1-y^2)p_\lambda^2z\right]\right\}$$
$$=\frac{1}{32}\cdot\frac{1}{2\pi^2}\int_{-1}^{+1}dy\int_{-\infty}^{\infty}dz\left\{\delta_{\mu\nu}\left(\frac{-d\epsilon(z)}{dz}\right)\right.$$
$$\times\left[i\frac{R(z)}{z}\exp\left[\frac{i}{4}(1-y^2)p_\lambda^2z\right]\right]$$
$$+\tfrac{1}{2}\epsilon(z)(p_\lambda^2\delta_{\mu\nu}-p_\mu p_\nu)(1-y^2)\frac{R(z)}{z}$$
$$\times\exp\left[\frac{i}{4}(1-y^2)p_\lambda^2z\right]. \tag{4.100}$$

By (2.49) the first term is

$$-2i(R(z)/z)_{z=0}=-2R'(0).$$

This is zero by (4.97). Thus

$$\mathbf{K}_{\mu\nu}(p)=\frac{1}{32}\cdot\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dz\frac{\epsilon(z)}{z}R(z)\int_{-1}^{+1}dy(1-y^2)$$
$$\times\exp\left[\frac{i}{4}(1-y^2)p_\lambda^2z\right](\delta_{\mu\nu}p_\lambda^2-p_\mu p_\nu)$$
$$=\frac{1}{32}\cdot\frac{1}{4\pi^2}(\delta_{\mu\nu}p_\lambda^2-p_\mu p_\nu)F(p_\lambda^2). \tag{4.101}$$

Thus

$$\eta_\mu^v(x)=\varphi_\nu(x)\frac{f^2}{\hbar c}\frac{1}{32}\cdot\left(\frac{1}{2\pi}\right)^2(\delta_{\mu\nu}p_\lambda^2-p_\mu p_\nu)F(p_\lambda^2) \tag{4.102}$$
$$=-\frac{f^2}{\hbar c}\frac{1}{32}\cdot\left(\frac{1}{2\pi}\right)^2\kappa^2F(-\kappa^2)\varphi_\mu(x). \tag{4.103}$$

This is zero in the photon case ($\kappa=0$) as required and is infinite in the meson case ($\kappa\neq0$).

Taking the regularized form of $\mathrm{K}_{\mu\nu}(p)$ by (4.9) and (4.44)

$$\eta_\mu^s(x)=0. \tag{4.104}$$

Thus the regularized scalar meson g self-energy is

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[11] R. P. Feynman, Phys. Rev. **74**, 1430 (1948).

zero. The scalar meson f self-energy and the vector meson g self-energy remain infinite with or without regularization, unless extra conditions are imposed.

The vector meson g self-energy is given by

$$\mathcal{H}_{23}{}^{v}(x)=(-i/8\hbar c\kappa^2)\int\langle[m_{\mu\nu}(x),\, m_{\pi\rho}(x')]\rangle_0 \times\{\varphi_{\nu\mu}(x),\, \varphi_{\rho\pi}(x')\}_1\epsilon(x-x')d\omega' \quad (4.105)$$

$$=\{\varphi_{\nu\mu}(x),\, \eta_{\mu\nu}{}^{v}(x)\}_1, \quad (4.106)$$

where

$$\eta_{\mu\nu}{}^{v}(x)=(-i/8\hbar c\kappa^2)\int\langle[m_{\mu\nu}(x),\, m_{\pi\rho}(x')]\rangle_0 \times\varphi_{\rho\pi}(x')\epsilon(x-x')d\omega'$$

$$=(g^2/\hbar c\kappa^2)\int \mathrm{K}_{\mu\nu\pi\rho}{}^{v}(x-x')\varphi_{\rho\pi}(x')d\omega'$$

$$=(g^2/\hbar c\kappa^2)\mathrm{K}_{\mu\nu\pi\rho}{}^{v}(p)\varphi_{\rho\pi}(x). \quad (4.107)$$

For factors in the integrand of $\kappa_{\mu\nu\pi\rho}{}^{v}(p)$, Eq. (4.9) is still valid and in place of (4.44),

$$p_\pi p_\mu=-\kappa^2\delta_{\pi\mu}. \quad (4.108)$$

Also from the supplementary condition, in the integrand of $\kappa_{\mu\nu\pi\rho}{}^{v}(p)$ effectively

$$p_\rho=0. \quad (4.109)$$

From (2.37),

$$\mathrm{K}_{\mu\nu\pi\rho}{}^{v}(x)=\delta_{\rho\nu}\mathrm{K}_{\pi\mu}{}'(x)-\delta_{\rho\mu}\mathrm{K}_{\pi\nu}{}'(x)+\delta_{\pi\mu}\mathrm{K}_{\rho\nu}{}'(x) -\delta_{\pi\nu}\mathrm{K}_{\rho\mu}{}'(x)+(\delta_{\rho\mu}\delta_{\pi\nu}-\delta_{\pi\mu}\delta_{\rho\nu})\mathrm{K}(x), \quad (4.110)$$

where K(x) is defined in (4.10) and, with the notation (2.34),

$$\mathrm{K}_{\mu\nu}{}'(x)=\Delta_{0\mu}{}^{(1)}(x)\bar{\Delta}_{0\nu}(x)+\Delta_{0\nu}{}^{(1)}(x)\bar{\Delta}_{0\mu}(x). \quad (4.111)$$

By the usual method

$$\mathrm{K}_{\mu\nu}{}'(p)=\frac{1}{64\pi^2}\int_{-1}^{+1}dy\int_{-\infty}^{\infty}dz\epsilon(z) \times\exp\left[iz\left(\kappa_0{}^2+\frac{1-y^2}{4}p_\lambda{}^2\right)\right] \times\left\{\frac{i\delta_{\mu\nu}}{z^2}-\frac{1-y^2}{2z}p_\mu p_\nu\right\}, \quad (4.112)$$

and from (4.26),

$$\mathrm{K}(p)=\frac{1}{64\pi^2}\int_{-1}^{+1}dy\int_{-\infty}^{\infty}dz\epsilon(z) \times\exp\left[iz\left(\kappa_0{}^2+\frac{1-y^2}{4}p_\lambda{}^2\right)\right] \times\left\{\frac{2i}{z^2}-\frac{\kappa_0{}^2}{z}-\frac{(1-y^2)}{4z}p_\lambda{}^2\right\}. \quad (4.113)$$

Introducing ω and substituting for p_λ from (4.9), (4.108), and (4.109)

$$\mathrm{K}_{\mu\nu\pi\rho}{}^{v}(\kappa)=-\frac{\kappa^2}{64}\left(\frac{1}{2\pi}\right)^2\int_{-1}^{+1}dy(4\mu^2+1-y^2) \times\int_{-\infty}^{\infty}d\omega\frac{1}{|\omega|}\exp\left[i\omega\left(1-\frac{1-y^2}{4\mu^2}\right)\right] \times(\delta_{\rho\mu}\delta_{\pi\nu}-\delta_{\pi\mu}\delta_{\rho\nu}). \quad (4.114)$$

This is of the standard form (4.30) with

$$A=4\mu^2+1-y^2,\quad B=0,\quad C=1-(1-y^2)/4\mu^2, \quad (4.115)$$

and yields

$$\mathrm{K}_{\mu\nu\pi\rho}{}^{v}(p)=\frac{-\kappa^2}{32\pi^2}\left[(\tfrac{1}{3}+2\mu^2)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right] \times(\delta_{\rho\mu}\delta_{\pi\nu}-\delta_{\pi\mu}\delta_{\rho\nu}). \quad (4.116)$$

Thus

$$\eta_{\mu\nu}{}^{v}(x)=-\left(\frac{g^2}{4\pi\hbar c}\right)\left(\frac{1}{8\pi}\right)\left[(\tfrac{1}{3}+2\mu^2)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right] \times(\varphi_{\mu\nu}(x)-\varphi_{\nu\mu}(x)), \quad (4.117)$$

and

$$\mathcal{H}_{23}{}^{v}(x)=-2(\kappa_4/\kappa)\{(\varphi_{\mu\nu}(x)-\varphi_{\nu\mu}(x))^2\}_1, \quad (4.118)$$

where

$$\kappa_4=\kappa\left(\frac{g^2}{4\pi\hbar c}\right)\left(\frac{1}{8\pi}\right)\left[(\tfrac{1}{3}+2\mu^2)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (4.119)$$

Now

$$\varphi_\mu(x)=\sum_k\varphi_\mu(k)\exp[ik_\mu x_\mu]. \quad (4.120)$$

As in the case of (4.57)

$$\{\varphi_{\mu\nu}{}^2(x)\}_1=\sum k_\nu{}^2\varphi_\mu{}^{(-)}(k)\varphi_\mu{}^{(+)}(k) =-\kappa^2\{\varphi_\mu{}^2(x)\}_1. \quad (4.121)$$

Similarly

$$\{\varphi_{\nu\mu}(x)\varphi_{\mu\nu}(x)\}_1=\sum k_\mu k_\nu\{\varphi_\nu{}^{(+)}(k)\varphi_\mu{}^{(-)}(k) +\varphi_\nu{}^{(-)}(k)\varphi_\mu{}^{(+)}(k)\}. \quad (4.122)$$

But by the supplementary condition

$$k_\nu\varphi_\nu{}^{(+)}(k)=0,\quad\text{and}\quad k_\nu\varphi_\nu{}^{(-)}(k)=0. \quad (4.123)$$

Thus

$$\{(\varphi_{\mu\nu}(x)-\varphi_{\nu\mu}(x))^2\}_1 =2\{\varphi_{\mu\nu}{}^2(x)-\varphi_{\mu\nu}(x)\varphi_{\nu\mu}(x)\}_1=-2\kappa^2\{\varphi_\mu{}^2(x)\}_1. \quad (4.124)$$

By a similar argument it can be shown that

$$\kappa^2\{\varphi_\mu{}^2(x)\}_1=\kappa^2\{A_\mu{}^2(x)\}_1+\kappa^2\{B^2(x)\}_1, \quad (4.125)$$

which is of the form of the mass dependent terms in $\mathcal{H}_{\text{free}}^v(x)$ by (1.58). Define

$$\mathcal{H}_0^v(x)=2(\kappa_3+\kappa_4)\kappa\varphi_\mu^2(x). \quad (4.126)$$

Then by replacing the label s by the label v in the derivation of (4.64), the wave equation can be transformed into

$$i\hbar c\delta\Psi/\delta\sigma=\mathcal{H}_e^v(\bar{\varphi}_\mu)\Psi, \quad (4.127)$$

which is free from the infinities due to meson self-energies and in which the meson field variables satisfy the equations of motion of the free field with renormalized mass, $\kappa+\kappa_3+\kappa_4$.

The cross terms which arise from $\mathcal{H}_3^s(x)$ and $\mathcal{H}_3^v(x)$ have not yet been considered. These give rise to the fg meson self-energy through $\mathcal{H}_{33}^s(x)$ and $\mathcal{H}_{33}^v(x)$. From (2.38) it follows immediately that $\mathcal{H}_{33}^s(x)$ is zero. It can be shown after a short calculation using the properties of the Δ-functions, (2.6), and (2.39) that $\mathcal{H}_{33}^v(x)$ is also zero.

V. THE NUCLEON SELF-ENERGIES

The nucleon f self-energy caused by interaction with scalar mesons is the one-nucleon part of $\mathcal{H}_{1,2}^s(x)$ which is given by (3.5), (3.10) and the discussion following these equations.

$$\mathcal{H}_{fse}^s(x)=(-i/8c^2)\times\int_{-\infty}^{\infty}[\omega(x),\omega(x')]\Delta^{(1)}(x-x')\epsilon(x-x')d\omega' -(1/4c^2)\int_{-\infty}^{\infty}\{\omega(x),\omega(x')\}_1\times\bar{\Delta}(x-x')d\omega'. \quad (5.1)$$

By (2.31 and (2.45)

$$\begin{aligned}\mathcal{H}_{fse}^s(x)=(f^2/8)\int_{-\infty}^{\infty}\{&[\bar{\psi}(x),S^{(1)}(x-x')\psi(x')]\bar{\Delta}(x-x')\\&+[\bar{\psi}(x),\bar{S}(x-x')\psi(x')]\Delta^{(1)}(x-x')\\&+[\bar{\psi}(x')S^{(1)}(x-x'),\psi(x)]\bar{\Delta}(x-x')\\&+[\bar{\psi}(x')\bar{S}(x'-x),\psi(x)]\Delta^{(1)}(x-x')\}d\omega' \quad (5.2)\\=\tfrac{1}{4}[\bar{\psi}(x),\zeta_1(x)]&+[\bar{\zeta}_1(x),\psi(x)], \quad (5.3)\end{aligned}$$

where

$$\zeta_1(x)=\tfrac{1}{2}f^2\int\{S^{(1)}(x-x')\bar{\Delta}(x-x')+\bar{S}(x-x')\Delta^{(1)}(x-x')\}\psi(x')d\omega' \quad (5.4)$$

$$=\tfrac{1}{2}f^2\int \mathrm{K}_1(x-x')\psi(x')d\omega'. \quad (5.5)$$

The last equation defines $\mathrm{K}_1(x-x')$. As for (4.8)

$$\zeta_1(x)=\tfrac{1}{2}f^2\psi(x)\kappa_1(p). \quad (5.6)$$

Since $\psi(x)$ satisfies the Dirac Eq. (1.18),

$$i\gamma_\mu p_\mu=-\kappa_0, \quad (5.7)$$

and

$$p_\lambda^2=-\kappa_0^2. \quad (5.8)$$

Expanding the S-functions, it follows from (5.5) with the notation (2.34) that

$$\mathrm{K}_1(\xi)=\gamma_\lambda[\bar{\Delta}(\xi)\Delta_{0\lambda}^{(1)}(\xi)+\Delta^{(1)}(\xi)\bar{\Delta}_{0\lambda}(\xi)] -\kappa^0[\bar{\Delta}(\xi)\Delta_0^{(1)}(\xi)+\Delta^{(1)}(\xi)\bar{\Delta}_0(\xi)]. \quad (5.9)$$

Substituting for $\bar{\Delta}$ and $\Delta^{(1)}$ from (4.11) and (4.12),

$$\mathrm{K}_1(p)=\left(\frac{1}{2\pi}\right)^3\int d_4q\left\{\left[\frac{\delta(q_\lambda^2+\kappa_0^2)}{(p_\lambda-q_\lambda)^2+\alpha^2}+\frac{\delta((p_\lambda-q_\lambda)^2+\kappa^2)}{q_\lambda^2+\kappa_0^2}\right](i\gamma_\lambda q_\lambda-\kappa_0)\right\}. \quad (5.10)$$

This can be evaluated by the transformations (4.15), (4.16) (4.18), and (4.23) used for the meson self-energy. Eliminating p_λ by (5.7) and (5.8),

$$\mathrm{K}_1(\kappa_0)=\frac{-\kappa_0}{64\pi^2}\int_{-1}^{+1}dy(3-y)\int_{-\infty}^{\infty}d\omega\frac{1}{|\omega|}\times\exp\left[i\omega\left\{\left(\frac{1+y}{2}\right)^2+\frac{1-y}{2\mu^2}\right\}\right]. \quad (5.11)$$

Now

$$(3-y)=\frac{1}{2}\frac{d}{dy}(7-y)(1+y). \quad (5.12)$$

Integrating (5.11) by parts,

$$\begin{aligned}\mathrm{K}_1(\kappa_0)&=-\frac{\kappa_0}{32\pi^2}\left[6\int_0^{\infty}\frac{\cos\omega}{\omega}d\omega+\int_{-1}^{+1}\frac{(7-y)(1+y)\{(1+y)\mu^2-1\}}{(1+y)^2\mu^2+1-y}dy\right] \quad (5.13)\\&=-\frac{\kappa_0}{32\pi^2}\left[6\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right] \quad (5.14)\end{aligned}$$

since the integrand in the second integral in (5.13) is finite. Thus

$$\begin{aligned}\zeta_1(x)&=-\hbar c\kappa_0\left(\frac{f^2}{4\pi\hbar c}\right)\left[\frac{3}{8\pi}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]\psi(x) \quad (5.15)\\&=-\hbar c\kappa_0^{(1)}\psi(x), \quad (5.16)\end{aligned}$$

where

$$\kappa_0^{(1)}=-\kappa_0\left(\frac{f^2}{4\pi\hbar c}\right)\left[\frac{3}{8\pi}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (5.17)$$

Thus by (5.3)

$$\mathcal{H}_{fse}{}^{s}(x)=\tfrac{1}{2}hc\kappa_0{}^{(1)}(\bar{\psi}(x)\psi(x)-\psi(x)\bar{\psi}(x)). \quad (5.18)$$

This term can be eliminated from the Schrödinger Eq. (3.4) by the same transformation as was used for the meson self-energies, (4.62) and (4.63), with $\mathcal{H}_m{}^s(x)$ replaced by $\mathcal{H}_{fse}{}^s(x)$. By the same argument as that which lead to (4.70) the new field variable ψ will satisfy the equation of motion determined by the Hamiltonian $\mathcal{H}_{\text{free}}(x)+\mathcal{H}_{fse}(x)$. But the mass terms in $\mathcal{H}_{\text{free}}(x)$ are

$$\tfrac{1}{2}hc\kappa_0(\bar{\psi}(x)\psi(x)-\psi(x)\bar{\psi}(x)),$$

as can be seen from the Lagrangian (1.16). Thus the new field variables will satisfy the equation of the free field with the renormalized mass $\kappa_0+\kappa_0{}^{(1)}$.

The nucleon g self-energy due to scalar mesons is the one particle part of $\mathcal{H}_{22}{}^s(x)+\mathcal{H}_4{}^s(x)$ which was derived in (3.13). The required commutators are given by (2.31) and (2.45). A calculation similar to that outlined above, complicated slightly by the additional γ_μ factors, gives

$$\mathcal{H}_{gse}{}^{s}(x)=\tfrac{1}{2}hc\kappa_0{}^{(2)}(\bar{\psi}(x)\psi(x)-\psi(x)\bar{\psi}(x)), \quad (5.19)$$

where $\kappa_0{}^{(2)}$ is finite and remains finite in the limit $\mu\rightarrow 0$ that is, when the meson mass becomes infinite. The scalar fg nucleon self-energy can be shown to be zero by a similar calculation.

The vector f nucleon self-energy is the one-nucleon part of $\mathcal{H}_{12}{}^v(x)+\mathcal{H}_{41}{}^v(x)$ which was derived in (3.21). This splits into two parts, one containing pure Δ-functions and the other containing double differentials of Δ-functions. The latter is identical with the scalar g nucleon self-energy which is finite. Thus the infinity is contained in the first part. Substituting for the commutators from (2.31) and (2.45) and evaluating as above,

$$\mathcal{H}_{fse}{}^{v}(x)=\tfrac{1}{2}hc\kappa_0{}^{(3)}(\bar{\psi}(\kappa)\psi(x)-\psi(x)\bar{\psi}(x)), \quad (5.20)$$

where

$$\kappa_0{}^{(3)}=\kappa_0\left(\frac{f^2}{4\pi hc}\right)\left[\frac{3}{4\pi}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (5.21)$$

It is important that $\kappa_0{}^{(1)}$ and $\kappa_0{}^{(3)}$ differ in their infinite term only by a factor $-\frac{1}{2}$ so that $2\kappa_0{}^{(1)}+\kappa_0{}^{(3)}$ is finite. This is the basis of the Pais f-field theory[8] in which a finite electron self-energy is obtained by adding a scalar meson field to the photon field with the coupling constant,

$$f^2=2e^2. \quad (5.22)$$

Also, since the infinite part of $\kappa_0{}^{(3)}$ is independent of κ and the finite part remains finite when $\kappa\rightarrow\infty$, a finite electron self-energy is obtained by subtracting a vector meson field with coupling constant

$$f=e. \quad (5.23)$$

This is the basis of the theories of Podolsky[10] and Feynman.[11] Difficulties arise over the interpretation of the meson field if it is regarded as having any physical reality since the theory necessarily involves either negative energies or negative "probabilities."

The vector g and fg nucleon self-energies, given by the one-nucleon parts of $\mathcal{H}_{22}{}^v(x)+\mathcal{H}_{42}{}^v(x)$, (3.24), and $\mathcal{H}_{32}{}^v(x)$, respectively, are

$$\mathcal{H}_{gse}{}^{v}(x)=\tfrac{1}{2}hc\kappa_0{}^{(4)}(\bar{\psi}(x)\psi(x)-\psi(x)\bar{\psi}(x)), \quad (5.24)$$

and

$$\mathcal{H}_{fgse}{}^{v}(x)=\tfrac{1}{2}hc\kappa_0{}^{(5)}(\bar{\psi}(x)\psi(x)-\psi(x)\bar{\psi}(x)), \quad (5.25)$$

where

$$\kappa_0{}^{(4)}=\kappa_0\left(\frac{g^2}{4\pi hc}\right)\frac{\mu^2}{4\pi}\left[\left(8+\frac{3}{\mu^2}\right)\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right], \quad (5.26)$$

and

$$\kappa_0{}^{(5)}=-\kappa_0\left(\frac{fg}{4\pi hc}\right)\left[\frac{3}{2\pi\mu}\log\frac{1}{\gamma\omega_0}+\text{f. t.}\right]. \quad (5.27)$$

All the nucleon self-energies are of the same form as (5.18) and can be eliminated from the Schrödinger Eqs. (3.4) or (3.14) by transformations of the type described in the discussion following Eq. (5.18). The new nucleon field variable, $\psi(x)$, will satisfy the equation of motion of the free field with renormalized mass, in the scalar case,

$$\bar{\kappa}_0=\kappa_0+\kappa_0{}^{(1)}+\kappa_0{}^{(2)}, \quad (5.28)$$

and in the vector case

$$\bar{\kappa}_0'=\kappa_0+\kappa_0{}^{(3)}+\kappa_0{}^{(4)}+\kappa_0{}^{(5)}. \quad (5.29)$$

These are equated to the experimental mass.

The transformations for eliminating the meson self-energies and the nucleon self-energies are independent. If $\mathcal{H}_n{}^s(x)$ is defined by

$$\mathcal{H}_n{}^s(x)=\mathcal{H}_{fse}{}^s(x)+\mathcal{H}_{gse}{}^s(x), \quad (5.30)$$

and both transformations are performed, the Schrödinger equation for the transformed state vector, $\Psi(\sigma)$, is

$$ihc\delta\Psi/\delta\sigma(x)=\mathcal{H}^s(x)\Psi(\sigma), \quad (5.31)$$

where

$$\mathcal{H}^s(x)=\mathcal{H}_e{}^s(x)-\mathcal{H}_n{}^s(x). \quad (5.32)$$

$\mathcal{H}_e{}^s(x)$ is defined by (4.61). The matrix elements of $\mathcal{H}^s(x)$ are free from infinites. $\mathcal{H}^s$ is a function of the transformed variable $\varphi(x)$ and $\psi(x)$, each of which satisfies its respective free-field equation with renormalized mass. A similar equation can be obtained in the vector case. Since all the integrals for the mass renormalizations are independent of the choice of n_μ, this is an invariant result. The transformations do not alter the form of the supplementary condition.

VI. THE YUKAWA POTENTIALS

One real effect which of course has not been eliminated from (5.31) is the Yukawa potentials between nucleons. These have been known for many years but it is of interest that the derivation of the expression for the potential in configuration space from the interaction operator in field theory, which has previously been rather awkward, is very neat when the theory is developed in this Schwinger form.

The f scalar nucleon interaction is the two-nucleon part of $\mathcal{H}_{12}{}^s(x)$ (5.1). Since, by (2.31), $[\omega(x), \omega(x')]$ is a one-nucleon operator, only the second term need be considered.

$$\mathcal{H}_{f\mathrm{int}}{}^s(x) = -(1/4c^2)\int_{-\infty}^{\infty} \{\omega(x), \omega(x')\}_2\bar{\Delta}(x-x')d\omega'. \quad (6.1)$$

Now $\omega(x)$ can be expressed as

$$\omega(x) = fc\bar{\psi}(x)\psi(x) + \text{c. number}. \quad (6.2)$$

Thus

$$\mathcal{H}_{f\mathrm{int}}{}^s(x) = -(f^2/4)\int \{\bar{\psi}(x)\psi(x), \bar{\psi}(x')\psi(x')\}_2 \times\bar{\Delta}(x-x')d\omega' \quad (6.3)$$

$$= (f^2/2)\int \bar{\psi}(x)\bar{\psi}(x')\psi(x)\psi(x') \times\bar{\Delta}(x-x')d\omega'. \quad (6.4)$$

In the second equation a one-nucleon term has been dropped. Substituting for $\bar{\Delta}(x-x')$ from (2.56),

$$\mathcal{H}_{f\mathrm{int}}{}^s(x) = \frac{f^2}{2}\left(\frac{1}{2\pi}\right)^4 \int d^4x' \int d^4k \times\bar{\psi}(x)\bar{\psi}(x')\psi(x)\psi(x')\frac{\exp[ik_\mu\cdot x_\mu - x_\mu']}{k_\mu{}^2+\kappa_0{}^2}. \quad (6.5)$$

If recoil is neglected, $\bar{\psi}(x')\psi(x')$ is independent of x_0' and the integration over x_0 and k_0 can be performed

$$\mathcal{H}_{f\mathrm{int}}{}^s(x) = \frac{f^2}{2}\left(\frac{1}{2\pi}\right)^3 \int d^3x' \int d^3k \times\bar{\psi}(x)\bar{\psi}(x')\psi(x)\psi(x')\frac{\exp[i\mathbf{k}\cdot\mathbf{x}-\mathbf{x}']}{|\mathbf{k}|^2+\kappa_0{}^2}. \quad (6.6)$$

But[12]

$$\left(\frac{1}{2\pi}\right)^3 \int \frac{\exp[i\mathbf{k}\cdot\mathbf{x}-\mathbf{x}']}{|\mathbf{k}|^2+\kappa_0{}^2}d^3k = \frac{1}{4\pi}\frac{\exp[-\kappa_0|\mathbf{x}-\mathbf{x}'|]}{|\mathbf{x}-\mathbf{x}'|} = J(|\mathbf{x}-\mathbf{x}'|), \quad (6.7)$$

and

$$\bar{\psi}(x)\bar{\psi}(x')\psi(x)\psi(x') = \psi^*(x)\psi^*(x')\gamma_4\gamma_4'\psi(x)\psi(x'), \quad (6.8)$$

where γ_4 operates on $\psi(x)$ and γ_4' on $\psi(x')$. Thus

$$\mathcal{H}_{f\mathrm{int}}{}^s(x) = (f^2/2)\int \psi^*(x)\psi^*(x')\gamma_4\gamma_4'\psi(x)\psi(x') \times J(|\mathbf{x}-\mathbf{x}'|)d^3x'. \quad (6.9)$$

This field theory operator expresses the interaction between nucleons in terms of annihilation and creation operators. The Yukawa potential in configuration space is that from which $\mathcal{H}_{f\mathrm{int}}{}^s(x)$ can be derived by the process of second quantization. If $V(\mathbf{x}-\mathbf{x}')$ is the Yukawa potential then

$$\int_{-\infty}^{\infty} \mathcal{H}_{f\mathrm{int}}(x)d^3x = \frac{1}{2}\int \psi^*(x)\psi^*(x')V(\mathbf{x}-\mathbf{x}')\psi(x)\psi(x')d^3xd^3x'. \quad (6.10)$$

Thus by (6.9)

$$V_f'(\mathbf{x}-\mathbf{x}') = f^2\gamma_4\gamma_4'J(|\mathbf{x}-\mathbf{x}'|), \quad (6.11)$$

which agrees with result given by Kemmer.[13] The g scalar and the f and g vector potentials can be derived similarly. Exchange effects have been neglected.

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