Absolute Voltage Determination of Two Nuclear Resonances below 0.4 Mev

A. H. MORRISH
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
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An absolute voltage determination was made of the $F^{19}(p\alpha', \gamma)$ and the $B^{11}(p,\alpha)$ and $B^{11}(p,\gamma)$ resonances. A calibrated electrostatic analyzer was used to measure the proton beam energy from a Crockroft-Walton linear accelerator. The values obtained were 0.3404 ± 0.0004 Mev and 0.1628 ± 0.0002 Mev, respectively. An approximate estimate of the half-width appears to give 2.0 ± 1.0 kv for the fluorine and 4.5 ± 1.5 kv for the boron resonance.

INTRODUCTION

A NUMBER of absolute voltage measurements have been made of the $Li(p,\gamma)$ resonance,[1,2] and recently[3] absolute determinations of three resonances were made in a precise experiment. Shoupp, Jennings, and Jones[4] have made an absolute determination of the $Li(p,n)$ threshold using a radiofrequency ion speed gauge. Up to now, no highly precise absolute determination has been made below 0.4 Mev. In this work the $F(p\alpha',\gamma)$ and the $B(p,\alpha)$ and the $B(p, \gamma)$ resonances have been measured with the aid of a 90° electrostatic analyzer, calibrated with an electron gun. The advantage in using the analyzer derives from the fact that the voltage is scaled down by a factor, in this case about 15, and the resulting voltage accurately determined with the aid of a stack of wire-wound resistors.

The intense fluorine resonance has the advantage of a much narrower half-width as compared to the Li resonance at approximately 440 kv, and should find considerable use as a calibration point in the future. The gamma-resonance in boron is relatively weak, and perhaps not too well suited as a calibration point. However the alpha-resonance, at the same voltage, is readily detected with an ion chamber.

EXPERIMENTAL TECHNIQUE

A linear accelerator of the Cockcroft-Walton type ("kevatron"), with up to 0.4 million volts range, was used to accelerate hydrogen ions from a low voltage arc.[5] The ionic beam was magnetically analyzed, and either H^+ or H_2^+ ions were selected to impinge on the target. The targets were perforated with a slot along the horizontal axis (except for the pressed amorphous boron ones). Most of the beam hit the target, initiating the desired nuclear reaction. The gamma-rays were detected with a Geiger-Müller counter, the active volume of which was approximately 13 cm from the target. Part of the beam, however, passed through the slot, and entered an electrostatic analyzer. These ions were then collected in a Faraday cage when suitable potentials were applied to the cylindrical plates of the analyzer. In this way the beam energy could be determined to a high degree of precision. The target slots were beveled sharply in order to prevent possible errors in the measurement of the beam due to scattering. A schematic diagram of the experimental arrangement employed for the gamma-resonances is shown in Fig. 1. In the case of the boron alpha-resonance, an ion chamber replaced the G-M tube, and the target was placed at approximately 45° to the proton beam and to the ion chamber. In the case of the pressed boron target, the beam energy was measured by sliding the target up out of the ion beam by means of a shaft working through a Wilson seal. The G-M counter was 5 cm in diameter and 10 cm long, enclosed in a lead shield. The counter was filled with argon at 10 cm pressure, and ethyl acetate was used as the quenching agent. The ion chamber was a wide angle type, 1.43 cm in diameter, 1.71 cm deep, with its window 5.40 cm from the target center, and filled with argon at atmospheric pressure. The window used was a film of mica, 2.6 mg/cm², and a film of aluminum, 3.44 mg/cm² in thickness; the combination stopped all alphas of range less than 3.9 cm of air.

The pulses from the G-M tube and the ion chamber were recorded by a conventional scaling circuit. The ions collected in the Faraday cage were recorded by observing the deflection of a sensitive galvanometer. Since the ions so collected had an energy distribution (see Fig. 2), it was found inconvenient to read the beam energy this way at every point during the study of a resonance. A resistor of approximately 6.6×10^9 ohms was used to drain a small current from the kevatron, which was read on a good milliammeter[6] with a 0.03-ma range scale. When the milliammeter reading for the resonance peak had been obtained from a gamma- or alpha-profile, the kevatron voltage was immediately reset to reproduce this current drain, and the protons bent around the analyzer to establish their energy on the absolute scale. This, of course, also gave an absolute value for the resistance of the 10^9 ohm stack at the time and conditions of the experiment; but this

[1] Hafstad, Heydenburg, and Tuve, Phys. Rev. **50**, 504 (1936); Parkinson, Herb, Bernet, and McKibben, Phys. Rev. **53**, 642 (1938); A. O. Hanson and D. L. Benedict, Phys. Rev. **65**, 33 (1944).
[2] R. Tangen, Kgl. Norske. Vid. Sels. Skrifter, NRI (1946).
[3] Herb, Snowdon, and Sala, Phys. Rev. **75**, 246 (1949).
[4] Shoupp, Jennings, and Jones, O.N.R. report GR-151.
[5] S. K. Allison, Rev. Sci. Inst. **19**, 291 (1948).

[6] Weston Model 622.

value was not used in computing the resonance voltage. Each time a resonance curve was taken the determination of the absolute energy of the hydrogen ion beam at the critical point in the resonance was repeated. The value of the resistance, also found at this time, was used in plotting the off-resonance points to establish the shape of the profile. The consistency of various runs shows that the 10^9 ohm resistor did not vary appreciably in the short time interval required to run a resonance. The resistor did, however, show measureable day to day variations.

THE ELECTROSTATIC ANALYZER

Since the details of the construction and calibration of the electrostatic analyzer are published elsewhere,[7] only a brief summary is included here. The instrument is of the cylindrical type, with an average radius of 15 cm, spacing 0.5 cm, and deflecting angle of 90°. The deflecting potential may be as high as 50 kilovolts, and therefore protons up to 0.75 Mev can be focused. Movable entrance and exit slits are incorporated. A set of calibration constants has been determined, so that given the coordinate of the centers of these slits, and the potentials applied to the deflecting plates, the energy of the focused ion beam can be calculated to within 0.25 percent.

The instrument was calibrated using an electron gun as a source. Electrons in the range 6 to 10 kilovolts were used, and precautions were taken to avoid errors due to stray magnetic fields. The deflector voltages for the calibration were obtained from stacks of B batteries, and measured with a potentiometer and two volt boxes.

The accelerating voltage for the electron gun was supplied by a 540-cycle, high voltage half-wave rectifier. The source of the 540-cycle current was a 5 hp synchronous motor—a.c. generator. The d.c. field of the generator was supplied by a storage battery. A 50-megohm stack of wirewound (Taylor) resistors, of 0.1 percent precision, was used for the accelerating voltage measurement. This stack was calibrated against another 50-megohm stack of Taylor resistors in order to detect any appreciable temperature effect. The two were found to be the same to within one part in a thousand. The resulting current drain was measured with a milliammeter. This meter was calibrated frequently with a potentiometer arrangement, and the

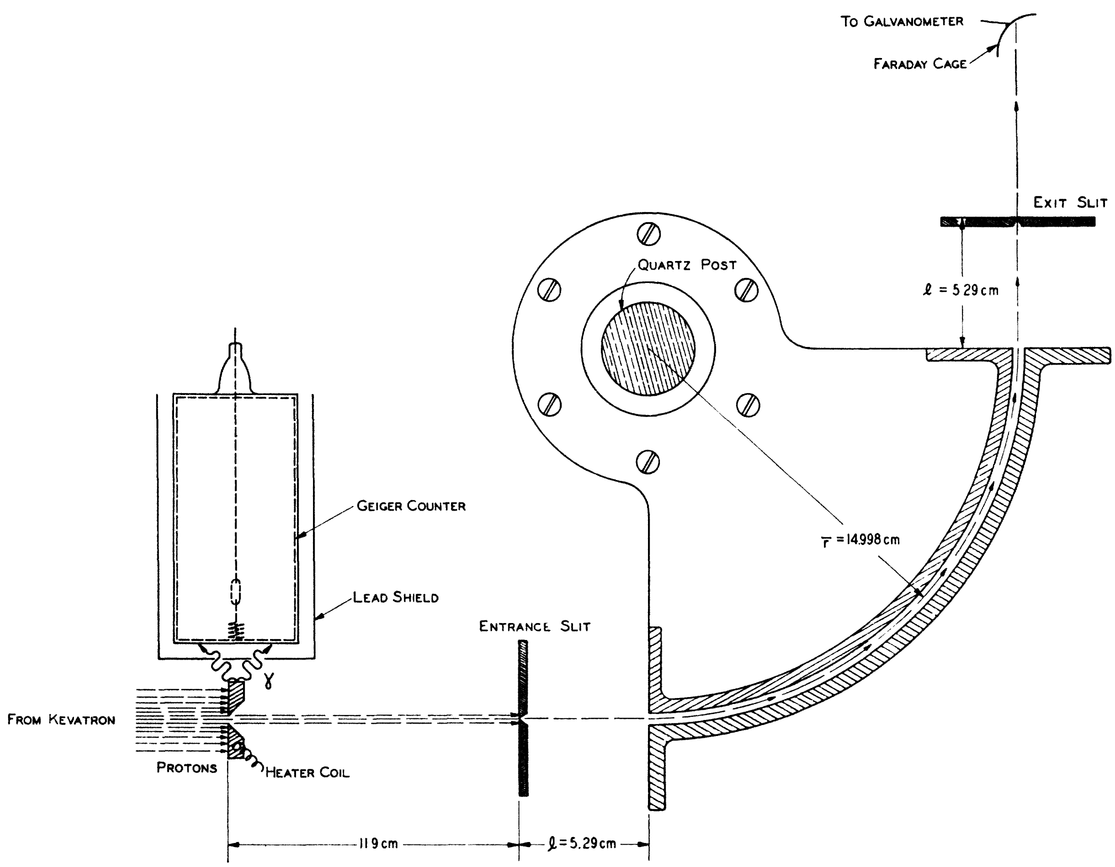

FIG. 1. Schematic diagram of the experimental arrangement for measuring gamma-ray resonances. For the boron alpha-resonance, the G-M counter was replaced by an ion chamber, and the target was at 45° to the proton beam and the chamber.

[7] Allison, Frankel, Hall, Montague, Morrish, and Warshaw, Rev. Sci. Inst., 20, 735 (1949).

standard cell used in this calibration was checked against another standard cell. The drain was about 200 microamperes, and the 540-cycle ripple was suppressed to less than 0.1 percent with a 0.05-microfarad filtering condenser. An electron profile obtained with the instrument, with a certain slit width, could then be used as a measure of the resolving power of the instrument, and later proton profiles from the kevatron were corrected for this effect using the electron curves.

The analyzer constant k, defined as $W_0/\{-(V_2-V_1)\}$, where W_0 is the kinetic energy of the ion, and V_1 and V_2 the deflecting potentials, is given in Table I, with the slits in the positions shown in Fig. 1. Inner charge means that the outer deflecting plate is at ground potential, and all the voltage is applied to the inner cylinder. Outer charge refers to the opposite case, and symmetric charge means that equal voltages, of opposite sign, were applied to the deflecting plates. The constant has been corrected for electron relativistic effects. (At 10 kv this is about 1 percent).

The geometry of the entrance and exist apertures of the deflecting channel is such that the stray field may be calculated analytically. This has been done for the case in which the deflecting plates are symmetrically charged. The predictions have been compared with experiments. Agreement with the calculations is demonstrated within the accuracy of the experiments, which is about 0.3 percent.

VOLTAGE REGULATION AND RIPPLE

The analyzer deflector voltages for the proton beam were obtained from the 540-cycle high voltage half-wave rectifier discussed in the section on the electrostatic analyzer as the electron accelerating voltage.

A Sola constant voltage transformer[8] was installed in the primary of the kevatron circuit. It is estimated that the kevatron voltage was maintained to about one part in 500 or 600 as read by the milliammeter in series with the 6.6×10^9 ohm resistor.

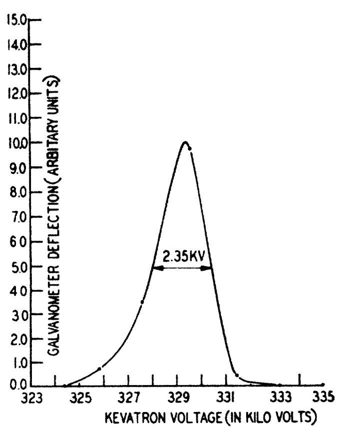

FIG. 2. H_2^+ energy profile as seen through the electrostatic analyzer with slits 1 mm wide. The indicated half-width has to be corrected for the resolving power of the analyzer itself.

[8] Model 30M818, Sola Electric Company, Chicago, Illinois.

TABLE I. Values of the analyzer constant k.

	Arrangement	k
A	Symmetric charge	15.31 ± 0.02
B	Inner charge	14.81 ± 0.02
C	Outer charge	15.81 ± 0.02

A proton profile taken with the electrostatic analyzer is a measure of the inhomogeneity of the beam energy. The inhomogeneity is caused entirely by the voltage ripple in the kevatron, the probe of the arc contributing a negligible (6 volts) amount. A rough calculation of the ripple using the profile of Fig. 2 and others (not illustrated), together with plausible assumptions, may be made in the following way. The electron profiles, discussed in the analyzer section, give a value of 30 volts full width at half maximum at 9430 volts beam energy, using slits 1 mm wide. Taking this as the limit of resolution of the instrument, a value is calculated for the instrumental half-width at 341 kilovolts, the voltage at which the H_2^+ profile was taken, assuming that dE/E is constant. Assuming the profiles are approximately Gaussian, and using the equation $W_{\text{proton}}=(W^2_{\text{obs}}-W^2_{\text{electron}})^{\frac{1}{2}}$, where W_{proton} is the true proton half-width, W_{obs} is the observed proton half-width, and W_{electron} is the electron or instrumental half-width, we obtain the value 2.1 kv for the true half-width of the H_2^+ profile. If the ripple were sinusoidal, and the analyzer presented a narrow energy "window"[9] to the beam, a profile of two peaks should be observed, since the voltage is at the extreme of the sine curve for the longest time in a cycle. A very wide "window," ($\Delta E>2A$), would give a flat topped profile. The limiting case of $\Delta E=2A$ gives one sharp peak, and corresponds most closely with the observed profiles, as illustrated in Fig. 2. A more triangular shaped ripple, which is what one has with a Cockcroft-Walton, gives a curve in fairly good agreement with the observed ones. For this curve the half-width is clearly $2A$, and therefore the 2.1 kilovolts obtained as above may be taken as the value of the total voltage fluctuation due to ripple, to an accuracy of probably 10 to 20 percent.

An analysis of the circuit, making simplifying assumptions concerning the actions of the electronic valves, gives the ripple as 270 volts per 100 microamperes drain. The value 2.1 kilovolts corresponds to a drain of about 800 microamperes, a not too unreasonable figure since the beam and resistor drain are about 400 microamperes, leaving the other 400 to be accounted for by leakage down the supports, the glass accelerator tube, etc.

The proton profiles obtained on different days were all sharply peaked, and of roughly the same half-width,

[9] That is, if the analyzer transmitted particles of energy distribution $\Delta E\ll A$, where A is the amplitude of the ripple.

TABLE II. Values of the resonance obtained from a number of targets.

Target	E (kv)	ΔE (kv)	E_R (kv)	Γ (kv)
PbF_2	340.4	thick	340.4	3.0
PbF_2	340.4	thick	340.4	3.5
PbF_2	340.6	0.57	340.3	4.0
PbF_2	340.8	1.26	340.2	4.2
PbF_2	340.7	0.45	340.5	3.7
PbF_2	340.5	0.52	340.3	3.7
CaF_2	340.4	thick	340.4	3.1
CaF_2	340.6	thick	340.6	3.3
CaF_2	340.5	0.70	340.2	2.4
CaF_2	340.7	0.52	340.5	3.5
CaF_2	340.7	1.40	340.0	3.9
		Average E_R 340.4 kv		

so it is believed the value 2.1 kilovolts may be taken as the ripple in all the experimental work reported in this paper.

TARGET TECHNIQUE

For the fluorine gamma-resonance, targets of CaF_2 and PbF_2 were used. These are non-hygroscopic fluorides which evaporate, in vacuum, at temperatures of the order of 1000°C, apparently without decomposition. Both thick and thin targets of these salts were prepared by vacuum evaporation.

The thin boron targets were prepared by evaporating either amorphous boron or B_2O_3 in vacuum, on steel slotted targets. The B_2O_3 targets had a short life under bombardment, however, and the boron targets were found to be the most satisfactory. The thick targets were made by pressing amorphous boron into a steel cap.

Considerable time was spent in the installation of a new pumping system on the kevatron, to eliminate oil deposits on the targets. A large liquid air trap incorporating two right-angle bends, was placed between the pumps and the acceleration tube. An auxiliary pumping system on the electrostatic analyzer had a trap, cooled with solid CO_2, in an analogous position. Vanes between the traps and the kevatron or deflector were only opened during a run, when the traps were cold. The high consistency of the results leads to the conclusion that the oil problem had been solved, since it is hardly likely that a constant thickness of oil film would be deposited on all targets.

THIN TARGET CORRECTIONS

The point of inflexion of the forward slope of a thick target yield curve gives the value of the resonance. However, for a thin target the peak of the resonance curve is shifted higher with increased target thickness. This thickness may be obtained with the aid of the equation:

$$A(\Delta E)=\int YdE=\Delta E Y_{\max}(\infty), \tag{1}$$

where A is the area under the excitation curve for a target whose thickness corresponds to a loss of energy ΔE, and $Y_{\max}(\infty)$ is the maximum yield of a corresponding thick target curve. It has been shown[10] that this result is independent of the homogeneity of the beam, and the exact shape of the yield curve.

For a target of thickness Δx, corresponding to loss of energy ΔE, we may say

$$\begin{aligned} Y&=k\int_0^{\Delta x}\sigma(E)dx \\ &=k\int_E^{E-\Delta E}\sigma(E)(\partial x/\partial E)dE, \end{aligned} \tag{2}$$

where $\sigma(E)$ is the cross section for disintegration, and k a constant. The assumption is made the $\partial x/\partial E$ is constant over the resonance, valid if the resonance is sharp. The expression (2) neglects, however, the inhomogeneity of the beam energy, and taking this into account gives

$$Y=k'\int_{E-e}^{E+e}\int_E^{E-\Delta E}\sigma(E)f(E-E')dEdE',$$

where $f(E-E')$ is a function determined by the ripple of the kevatron, and represents that fraction of ions between E' and $E'+dE'$ in energy, and e is the amplitude of the ripple. The function $f(E-E')$ was not known.

The Breit-Wigner resonance equation[11]

$$\sigma=\frac{K}{(E-E_R)^2+(\Gamma/2)^2}$$

is used, neglecting the wave-length and penetration factor. Substitution for σ, and neglecting constants and the inhomogeneity of the beam, gives

$$Y\sim\int_E^{E-\Delta E}\frac{dE}{(E-E_R)^2+(\Gamma/2)^2},$$

Γ being the half-width. This gives

$$Y=\tan^{-1}\left(\frac{E-E_R}{\Gamma/2}\right)-\tan^{-1}\left(\frac{E-\Delta E-E_R}{\Gamma/2}\right). \tag{3}$$

For a thick target we get, putting $\Delta E=E$,

$$Y=\tan^{-1}\left(\frac{E-E_R}{\Gamma/2}+\frac{\pi}{2}\right). \tag{4}$$

Now the maximum of (3) lies at

$$E_{\max}=E_R+(\Delta E/2), \tag{5}$$

and at this value $Y_{\max}(\Delta E)=2\tan^{-1}(E/\Gamma)$. For a

[10] Bernet, Herb, and Parkinson, Phys. Rev. **54**, 398 (1938).
[11] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

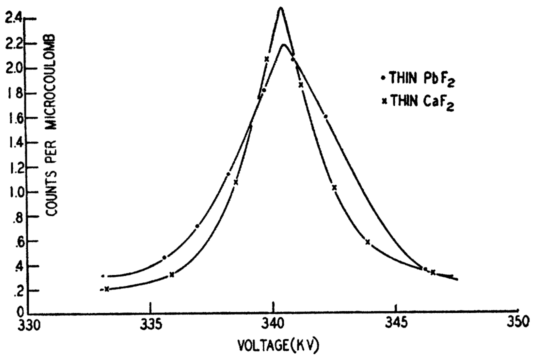

FIG. 3. The fluorine resonance for two thin targets. The resonance value had to be corrected for finite target thickness.

thick target we get $Y_{\max}(\infty)=\pi$. Therefore:

$$\frac{Y_{\max}(\Delta E)}{Y_{\max}(\infty)}=\frac{2\tan^{-1}(E/\Gamma)}{\pi}. \tag{6}$$

Also, if the half-width of the experimental curve is Γ', then

$$\Gamma=(\Gamma'^2-\Delta E^2)^{\frac{1}{2}}. \tag{7}$$

Graphs of Eqs. (6) and (7) may be found in the literature.[12]

In this work, the target corrections applied were obtained from (1) and (5). Values of ΔE obtained from (6) and (7) would have lowered E_R by approximately 3 or 4 tenths of a kilovolt from the values obtained with (1) for the fluorine resonance. This is undoubtedly due to the measured half-width's dependence on the voltage ripple. The results so obtained were in good agreement with the thick target resonance values, as will be seen later.

However, the value of Γ, the width at half maximum, is given as obtained from (6) and (7), and gives an indication of the resonance width, at least in the case of the boron curves, since the half-width was much larger than the ripple. The half-width obtained for fluorine, however, was just slightly wider than the

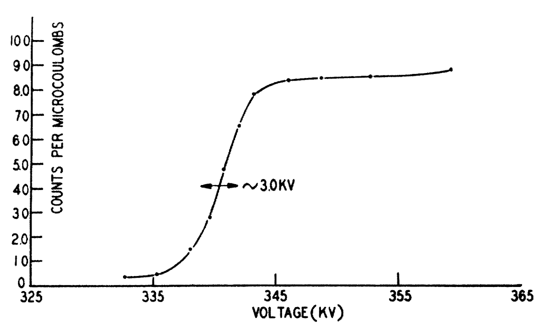

FIG. 4. A thick fluorine yield curve. The line with arrow is a measure of the half-width of the resonance, and should be corrected for the kevatron 2.1-kv ripple.

[12] Fowler, Lauritsen, and Lauritsen, Rev. Mod. Phys. **20**, 236 (1948).

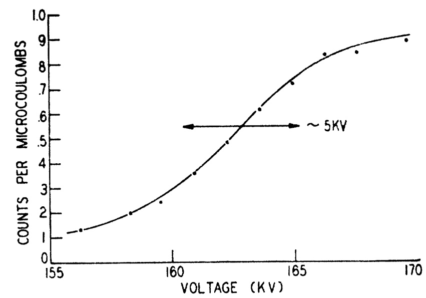

FIG. 5. Thick boron gamma-curve. The half-width, uncorrected for ripple is indicated.

width of the beam energy inhomogeneity, and the values of Γ obtained are not nearly as reliable. Half-widths were also obtained by differentiating the forward slope of the thick yield curves, and were somewhat smaller than the half-widths obtained from the thin targets. The differentiated curves should be corrected for beam energy inhomogeneity, but this was not possible in this work. However, it is believed that an estimate of the half-width to within 1 to 2 kilovolts may be made with the data.

THE FLUORINE RESONANCE

The fluorine gamma-rays were observed first by McMillan.[13] It has been shown[14] that a short range group of alpha-particles occur at the same resonance as the gamma-yield curve, with a range corresponding to the energy available. The reaction, then, is

$$F^{19}+H^1\rightarrow Ne^{20*}\rightarrow O^{16*}+He^4+1.8\text{ Mev}$$
$$O^{16*}\rightarrow O^{16}+h\nu.$$

Values of the resonance obtained by various workers is found in Table IV and discussed in a later section.

The H^+ beam was used in this work, and no difficulty was experienced in obtaining 15 to 25 microamperes of beam on the target. Typical experimental curves obtained are shown in Figs. 3 and 4. Table II lists the values of the resonance obtained from a number of targets. The average is seen to be 340.4 kv. Each determination is believed to be good to 0.25 percent, and the internal consistency good to 0.1 percent.

TABLE III. Complete summary of the data.

Target	Product observed	E (kv)	ΔE (kv)	E_R (kv)	Γ (kv)
Boron	Gamma	162.8	thick	162.8	5
Boron	Gamma	162.8	thick	162.8	5
Boron	Alpha	162.8	thick	162.8	5
Boron	Alpha	163.1	0.60	162.8	5.5
B_2O_3	Alpha	166.1	6.5	162.9	
			Average E_R 162.8 kv		

[13] E. McMillan, Phys. Rev. **46**, 325 (1934).

[14] McLean, Becker, Fowler, and Lauritsen, Phys. Rev. **55**, 796 (1939); W. E. Burcham and C. L. Smith, Nature, **143**, 795 (1939); Dee, Curran, and Strothers, Nature, **143**, 759 (1939); W. E. Burcham and S. Devons, Proc. Roy. Soc. London **(A) 173**, 555 (1939).

TABLE IV. Values of the resonances given by other authors.

Authors	Fluorine Resonance (kv)	Fluorine Half-width (kv)	Boron Resonance (kv)	Boron Half-width (kv)
Hafstad and Tuve[a]	320			
Hafstad, Heydenburg, and Tube[b]	328	<4		
Bernet, Herb, and Parkinson[c]	334	8		
Williams, Wells, Tate, and Hill[d]			180(α)	
Bothe and Gentner[e]			180($\alpha+\gamma$)	
Haxby, Allen, and Williams[f]			160(α)	
Waldmann, Waddel, Callihan, and Schneider			$165\pm4(\alpha+\gamma)$	<6
Bowersox[g]			$172\pm5(\alpha)$	
McLean, Young, Whitson Plain, and Ellet[h]			$160\pm5(\alpha)$	
v. Ubisch[i]			165(α)	
Jacobs and McLean[j]			$158\pm3(\alpha)$	
Marvin[k]			$162\pm1(\alpha)$	<11
Tangen[l]	339 ± 2	1.5 ± 0.5	$162\pm1(\gamma)$	5.3 ± 1
Fowler, Lauritsen, and Lauritsen[m]	338	4 ± 1		
Fowler and C. C. Lauritsen[n]	340 ± 2	3.2		

[a] L. R. Hafstad and M. A. Tuve, Phys. Rev. **48**, 306 (1935).
[b] See reference 1.
[c] See reference 10.
[d] See reference 15.
[e] See reference 16.
[f] Haxby, Allen, and Williams, Phys. Rev. **55**, 140 (1939).
[g] R. B. Bowersox, Phys. Rev. **55**, 323 (1939).
[h] See reference 18.
[i] See reference 19.
[j] Jacobs and McLean, Proc. Iowa Acad. Sci. **48**, 304 (1941).
[k] J. F. Marvin, Phys. Rev. **68**, 228 (1945).
[l] See reference 2.
[m] See reference 12.
[n] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **76**, 314 (1949).

Since the half-width is probably narrower than any of the values shown in the table, due to the ripple of 2.1 kv, a value of about 2 kv for this resonance width would seem to be a good estimate.

THE BORON RESONANCE

A number of reactions are possible when boron is bombarded with protons. The reactions are usually taken to be [2,15–19]

$$B^{11}+H^1\rightarrow C^{12*}\rightarrow Be^{8*}+He^4\rightarrow 3He^4$$
$$B^{11}+H^1\rightarrow C^{12**}\rightarrow C^{12}+h\nu$$
$$B^{11}+H^1\rightarrow C^{12***}\rightarrow Be^8+He^4\rightarrow 3He^4.$$

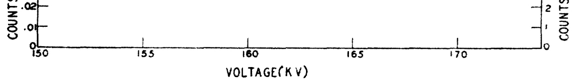

FIG. 6. Thick and thin boron alpha-yield curves.

[15] Williams, Wells, Tate, and Hill, Phys. Rev. **51**, 434 (1937).
[16] W. Bothe and W. Gentner, Zeits. f. Physik **104**, 685 (1937).
[17] Waldmann, Waddel, Callihan, and Schneider, Phys. Rev. **54**, 1017 (1938).
[18] McLean, Young, Whitson, Plain, and Ellet, Phys. Rev. **57**, 1083 (1940).
[19] H. v. Ubisch, K.N.V.S. Forh. **XV**, 71 (1942).

The alphas are found to consist of a homogeneous group of 4.5-cm range, and a short range continuous group. The long range alphas and the gammas were found[16,17] to have a common resonance. Also, some workers[18,19] have found the short range alphas to have a resonance at this same energy. They are usually taken to result from the first reaction above. Work on the gamma-spectrum[20] revealed lines at 16.6, 11.8 and 4.3 Mev. It is, however, possible that weak lines could be due to the reaction

$$B^{11}+H^1\rightarrow C^{12}\rightarrow Be^{8*}+He^4\rightarrow 3He^4+h\nu$$

at resonance. The 16.6 Mev line was measured at an energy above resonance, and, as pointed out by Tangen,[2] it is not certain that the line actually occurs in the resonance radiation. The origin of all the alpha-particles is not certain yet.

In this work a beam of H_2^+ ions were used, and about 20 microamperes were obtained on the target. Lead shielding $\frac{1}{8}$-inch thick was placed between the G-M counter and the target. A typical gamma-ray curve is shown in Fig. 5, and thick and thin alpha-curves are shown in Fig. 6. A complete summary of the data is given in Table III. It is seen that the average value of the resonance for both the gammas and the alphas is 162.8 kv. The accuracy is probably as good as in the fluorine case. The half-width would seem to be about 4.5 kv, within 1.5 kv.

DISCUSSION

Table IV lists values of the resonances obtained by other workers. Bernet, Herb, and Parkinson obtained their value using a rotating disc voltmeter. Tangen's result of 339 ± 2 kv for fluorine is based on a direct determination of the Li resonance at 440 kv, using a calibrated high resistance stack. He obtains, on weighing his results with other writers, the value 340 kv. Fowler and Lauritsen's value was obtained by using Herb, Snowdon, and Sala's recently determined value for the 873.5 kv fluorine resonance. The half-widths reported by various authors is higher than the estimated value obtained in this work, with the exception of Tangen's result, which is lower.

The previous work on boron is also summarized in the table. Waldman, Waddel, Callihan, and Schneider used a resistance calibrated against a stack of Taylor resistors in their work. The most recent value was obtained by Tangen in the same manner as he determined the fluorine resonance.

ACKNOWLEDGMENTS

It is a pleasure to thank S. K. Allison for extending the facilities of his laboratory to the author for this investigation, and for his interest, suggestions, and assistance. Also, I wish to thank J. H. Montague for

[20] Fowler, Gaerttner, and Lauritsen, Phys. Rev. **53**, 628 (1939).

discussions and assistance, and D. S. Bushnell, who aided in the collection of much of the data.

APPENDIX

Tangen[2] believes his accuracy of the Li(p,γ) resonance to be 0.5 percent, but believes his voltage scale has a much higher relative accuracy. The linearity of his voltmeter scale was tested with protons and diatomic ions, and he got excellent correspondence with two reactions below 450 kev. As noted previously, he obtained 339 kev for the F($p\ \alpha',\gamma$) and 162 for the B(p,γ) resonances.

The author's value of boron was found at 162.8±0.2 kev and assuming a linear scale, we get 340.7 kev for the fluorine resonance when Tangen's value of 339 kev is used. The value actually obtained was 340.4±0.4 kev, adding weight to the belief that Tangen's resistor was truly ohmic in character. A linear extrapolation from the fluorine value, using Tangen's result of 440 kev for the lithium resonance, gives 441.8 kev. A linear extrapolation from the boron value gives 442.2 kev, and the average of the two is 442.0 kev. This is in good agreement with the value 441.4±0.5 kev obtained by Fowler and Lauritsen,[n] and the value 442.4 kev obtained by Hudspeth and Swann,[21] both obtained using the F($p\ \alpha',\gamma$) resonance at 873.5 kev as recently determined by Herb, Snowdon, and Sala[3] in an absolute measurement.

Recently, N. P. Heydenburg has informed me he has made absolute determinations using a new, carefully calibrated resistor in his high resistance voltmeter. He obtained for $B^{11}(p,\gamma)$ 161.7 ±0.2 kev; for F($p\ \alpha',\gamma$) 339.7±0.2 kev; and for $Li^7(p,\gamma)$ 440.8 ±0.5 kev. These values are 1.1 and 0.7 kev lower than the ones made in this laboratory, and 1.2 kev lower than the estimate for lithium. If an average of these values is taken, one obtains the following table, in which extra weight has been given to Heydenburg's $Li^7(p,\gamma)$ value:

	kev
$B^{11}(p,\gamma)$	162.2
$F(p\ \alpha',\gamma)$	340.0
$Li^7(p,\gamma)$	441.2
$F(p\ \alpha',\gamma)$	873.5

These values are quite close on a percentage basis with the results from each laboratory, and one has some confidence they are good to ±0.5 kev, a fairly satisfactory situation.

[21] E. L. Hudspeth and C. P. Swann, Phys. Rev. 75, 1272 (1949).

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The Application of the Tomonaga-Schwinger Theory to the Interaction of Nucleons with Neutral Scalar and Vector Mesons

P. T. MATTHEWS
Clare College, Cambridge University, Cambridge, England
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The Tomonaga-Schwinger theory is applied to the interaction of neutral scalar and vector mesons with nucleons. The Hamiltonians are derived in the interaction representation and virtual effects are transformed away. The meson and nucleon self-energies are calculated. It is shown that they are invariant and, like the electron self-energy, can be transformed away by formal renormalizations of the meson and nucleon masses. All effects are independent of the direction of the general space-like surface in spite of the occurrence in the Hamiltonian of terms explicitly dependent on this direction.

INTRODUCTION

THE generalized Schrödinger equation in the interaction representation was first given by Tomonaga[1] for a Hamiltonian which commuted with itself at all points on a general space-like surface. When this condition was not immediately satisfied, it was found necessary to add certain terms explicitly dependent on the normal to the space-like surface and then to verify that this new Hamiltonian led to an integrable equation and that the generalized equation reduced to the ordinary formalism for a special choice of the general surface. By this method the Hamiltonians have been obtained by Myamoto[2] for the cases to be considered here, namely, the interaction of nucleons with scalar or vector mesons. It has been shown by the present author[3] that the generalized Schödinger equation in the interaction representation can be derived by a development of the work of Weiss.[4] The Hamiltonians of this equation for the nucleon-meson interactions are here deduced by an application of this theory.

The equations so obtained are then transformed by the methods which Schwinger[5] used for the interaction of the electron with the electrodynamic field. Besides real effects, the transformed Hamiltonians contain terms which give rise to infinite self-energies of both types of particle. These are evaluated and it is shown that they can be transformed away, leaving an equation in terms of field variables which satisfy the free field equations of the separate meson and nucleon fields with renormalized masses. Thus the observed free particle is taken to be the "bare" particle plus the vacuum effects.

The main difference between the electromagnetic case dealt with by Schwinger and those dealt with here is that now interactions which contain derivatives of the

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[2] Y. Myamoto, Prog. Theor. Phys. 3, 124 (1948).
[3] P. T. Matthews, Phys. Rev. 75, 1270 (1949). See also T. S. Chang, Phys. Rev. 75, 967 (1949).
[4] P. Weiss, Proc. Roy. Soc. A 169, 102 (1938).
[5] J. Schwinger, Phys. Rev. 74, 1492 (1948); 75, 615 (1949).