dividual terms on the right, this term is of relative magnitude:

$$\frac{1}{h^2}\frac{dp_d{}^2}{dx}AR\Big/\frac{2m}{h^2}\{W(x)+V(x)-E\}AQ$$

$$\sim\frac{m^*}{m}\left(\frac{a(dV/dx)}{W+V-E}\right), \quad (4.78)$$

since

$$R/Q\sim a, \quad (4.79)$$

near a band edge, where the effective mass is defined. The first of the remaining terms is the largest of these, and can be shown to be of the same order of magnitude as the term just considered. It thus appears that Eq. (4.67) gives a solution for which the error terms in the wave equation are small, in about the ratio of the change in perturbation energy across a single cell to the true value of the local kinetic energy. For a slowly varying perturbation the solution is locally very accurate indeed; the long-range effect of accumulated local errors cannot be discussed here.

Perhaps the most interesting qualitative result of this analysis is the confidence it gives in the use of the effective-mass wave equation for the calculation of stationary-state energies, when $V(x)$ varies slowly. To each quadratically integrable solution ϕ of this equation there corresponds a quadratically integrable ψ *with the same energy*, providing a good approximate solution of the perturbed-periodic equation. The error in the energy E as an approximation to a stationary-state energy corresponds to the small error in this ψ, and not to the roughness in the arguments usually employed in arriving at the effective-mass equation.

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The Radiation Spectra of Barium^{140} and Lanthanum^{140}

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The radiations of Ba^{140} and La^{140} have been measured with a small 180° spectrometer. For the case of Ba^{140} three gamma-rays are present, all of which are converted, with energies 0.16, 0.31, and 0.54 Mev. The beta-ray spectrum shows the presence of two groups with maximum energies of 0.48 (40 percent) and 1.022 (60 percent) Mev.

The radiations of La^{140} are complex. Six gamma-rays of energies 0.093?, 0.335, 0.49, 0.82, 1.60, and 2.5 Mev are present, the first three being internally converted. The beta-ray spectrum is resolved into three groups with end points at 1.32 (70 percent), 1.67 (20 percent), and 2.26 (10 percent) Mev. Decay schemes for these isotopes are suggested.

I. INTRODUCTION

THE radiations of 12.8-day Ba^{140} and its daughter, 40-hour La^{140}, have been re-investigated with the aim of formulating the mode of decay of these elements. These isotopes have been studied by numerous workers[1–7] and their results tend toward consistency insofar as energy values are concerned. However, previous investigations have been fragmentary and no integrated survey has been reported.

The present work was carried out with a small 180° spectrometer and very thin window G-M tube detection. Equilibrium samples of Ba^{140} and La^{140} obtained from Oak Ridge were used. Beta-ray sources were of the order of 0.1 mg/cm^2 in thickness. Photoelectron spectra were obtained with a lead radiator 30 mg/cm^2 thick. The procedure consisted of measuring the beta-ray and photoelectron spectra initially with equilibrium sources, and finally with chemically isolated sources of Ba^{140} and La^{140}.

II. EQUILIBRIUM MEASUREMENTS

The composite radiations were first measured by using sources in which the constituents were in equilibrium. Since the gamma-radiation of La^{140} is complex and the half-life relatively short, equilibrium photoelectron measurements furnish more reliable values of the energies. Subsequent runs with separated sources were, of course, necessary to identify the gamma-rays with the correct isotope.

Figure 1 shows the equilibrium photoelectron spectrum. The K and L photoelectron peaks associated with five gamma-rays are superimposed on a pronounced Compton-electron distribution. The photoelectron lines may be resolved into gamma-rays of energies 0.335,

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0.49, 0.54, 0.82, and 1.60 Mev. The dominant Compton-electron curve is primarily caused by the 1.60-Mev gamma-ray. The Compton-electron spectrum persists beyond 1.60 Mev and finally comes to zero in the region of 2.5 Mev. No marked evidence of a photoelectron line associated with this energy could be found. The use of a thorium radiator and much stronger sources did not add appreciably to the information obtained with the lead radiator. Evidently, a gamma-ray of about 2.5 Mev is present which is so low in intensity that the corresponding photoelectron peak cannot be reliably located in the presence of the background due to the numerous strong gamma-rays.

The combined beta-ray spectrum exhibits a number of conversion lines which can be identified as the *K* and *L* components of gamma-rays of energies 0.16, 0.335, 0.49, and 0.54 Mev. The shape of the continuous spectrum is complex and was resolved by a Fermi analysis into five groups of beta-rays with end points at 0.47, 0.98, 1.32, 1.68, and 2.19 Mev. While the accuracy of these values is subject to question because of the number of successive subtractions, the analysis nevertheless serves as a check on the values obtained with separated sources.

III. BARIUM^{140}

Lanthanum-free barium was prepared by adding NH_4OH to a solution of the chlorides of Ba and La. Repeated precipitation of the $La(OH)_3$ in this way assured a clean separation. The Ba was then precipitated as the carbonate by adding NH_4OH and finally converted to the chloride by adding HCl. Since very little Ba carrier was added, the specific activity of the source material was high. Measurements were taken as soon after the separation as possible to avoid the effect of the growth of the La.

Figure 2 shows the continuous spectrum of Ba^{140}. Three conversion lines appear which are associated with gamma-rays of energies 0.160, 0.306, and 0.540 Mev. Subsequent measurements taken during the growth of the La indicated that these gamma-rays should be attributed to Ba^{140}, since the intensities of the conversion lines did not increase. In addition, as growth proceeded, conversion lines corresponding to the 0.335- and 0.49-Mev gamma-rays found in the equilibrium run became evident. As equilibrium was approached, the conversion line due to the 0.306-Mev gamma-ray of the Ba was masked by the stronger line of the 0.335-Mev gamma-ray from the La.

A Fermi analysis of the continuous spectrum is shown in Fig. 3. Two groups of beta-rays are found with end points at 0.48 and 1.022 Mev. These values are in good agreement with two of those found in the equilibrium spectrum.

IV. LANTHANUM^{140}

Barium-free lanthanum, prepared by the chemical procedure outlined above, was used to complete the study of these radiations. All of the gamma-rays found in the equilibrium photoelectron spectrum, with the exception of the 0.54-Mev gamma-ray, are due to La^{140}. Figure 4 shows the beta-ray spectrum. All parts

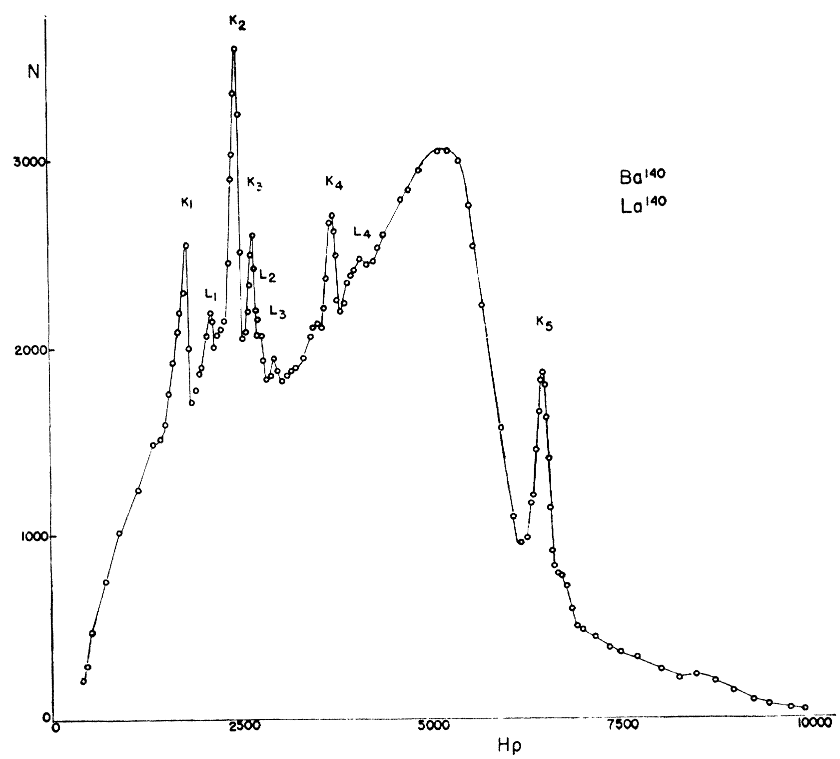

FIG. 1. Photoelectrons ejected from a Pb radiator 30 mg/cm² thick by the gamma-rays of Ba^{140} and La^{140} in equilibrium. The energy values are given in Table I.

of the spectrum decayed with the characteristic 40-hour half-life. No evidence of residual Ba^{140} or other impurity could be detected. In addition to conversion lines corresponding to gamma-rays at 0.335 and 0.49 Mev, observed previously in the growth from Ba^{140}, there appear three lines in the low energy region. The lowest line is most probably the Auger line associated with the conversion of the gamma-rays. The remaining two lines are weak and their identification uncertain. As K and L components they would correspond to a gamma-ray of energy 0.093 Mev. A gamma-ray of this energy could not be observed in the photoelectron spectrum.

A Fermi plot of the beta-ray spectrum (Fig. 5) yields three groups of beta-rays with end points at 1.32, 1.67, and 2.26 Mev. These values are in good agreement with three of the five groups found in the equilibrium spectrum.

V. RESULTS

These experiments make it clear that the disintegration of Ba^{140} involves the emission of two groups of beta-rays with maximum energies of 0.48 and 1.022 Mev, and three gamma-rays having energies 0.160, 0.306, and 0.540 Mev. The higher energy beta-group comprises 60 percent of the transitions. The 0.54-Mev

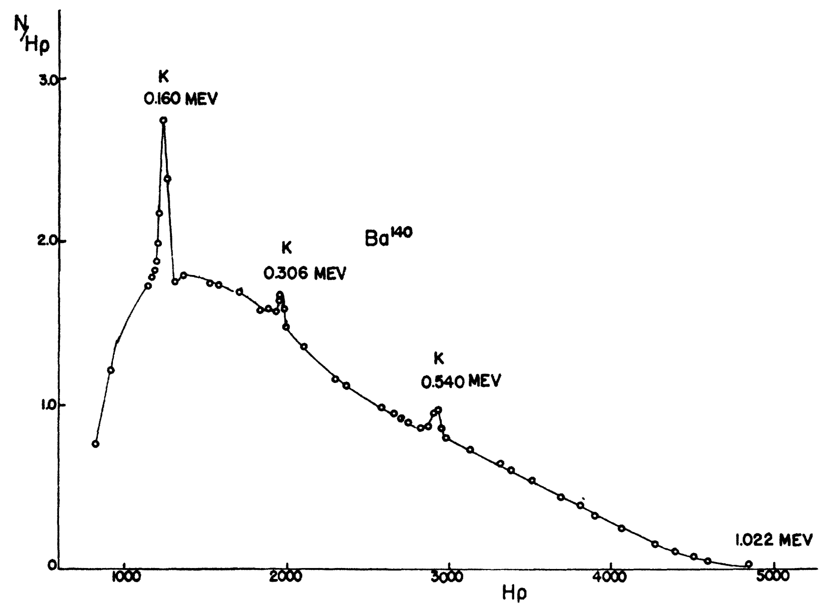

FIG. 2. Beta-ray spectrum of Ba^{140}. The energies given are those of the gamma-rays, not the conversion lines.

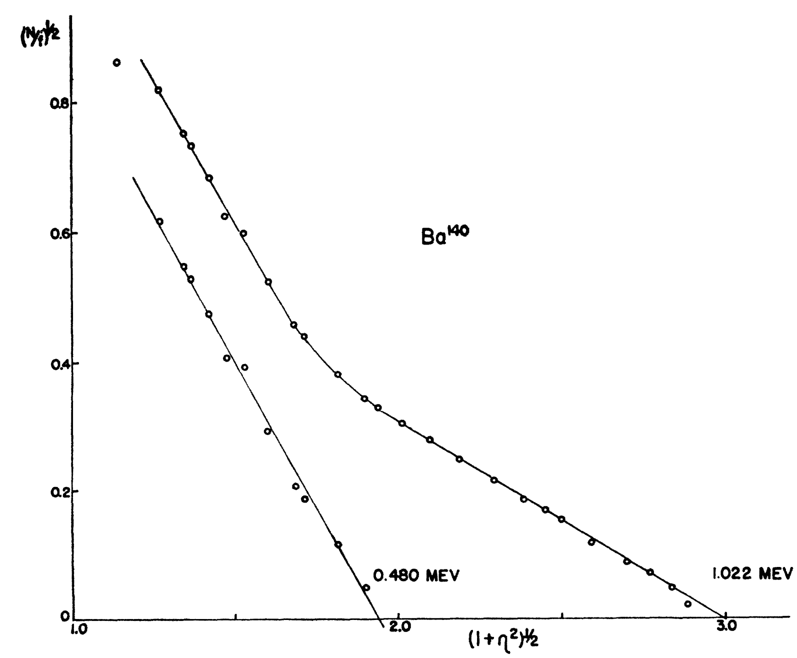

FIG. 3. Conventional Fermi plot of the beta-ray spectrum of Ba^{140}.

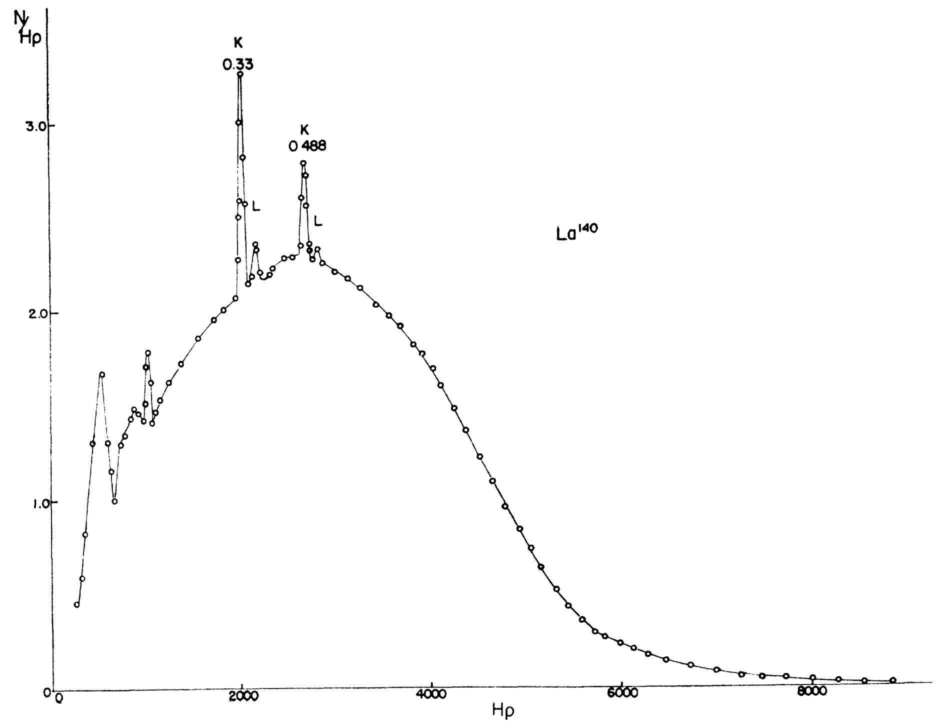

FIG. 4. Beta-ray spectrum of La^{140}. The energies given are those of the gamma-rays, not the conversion lines.

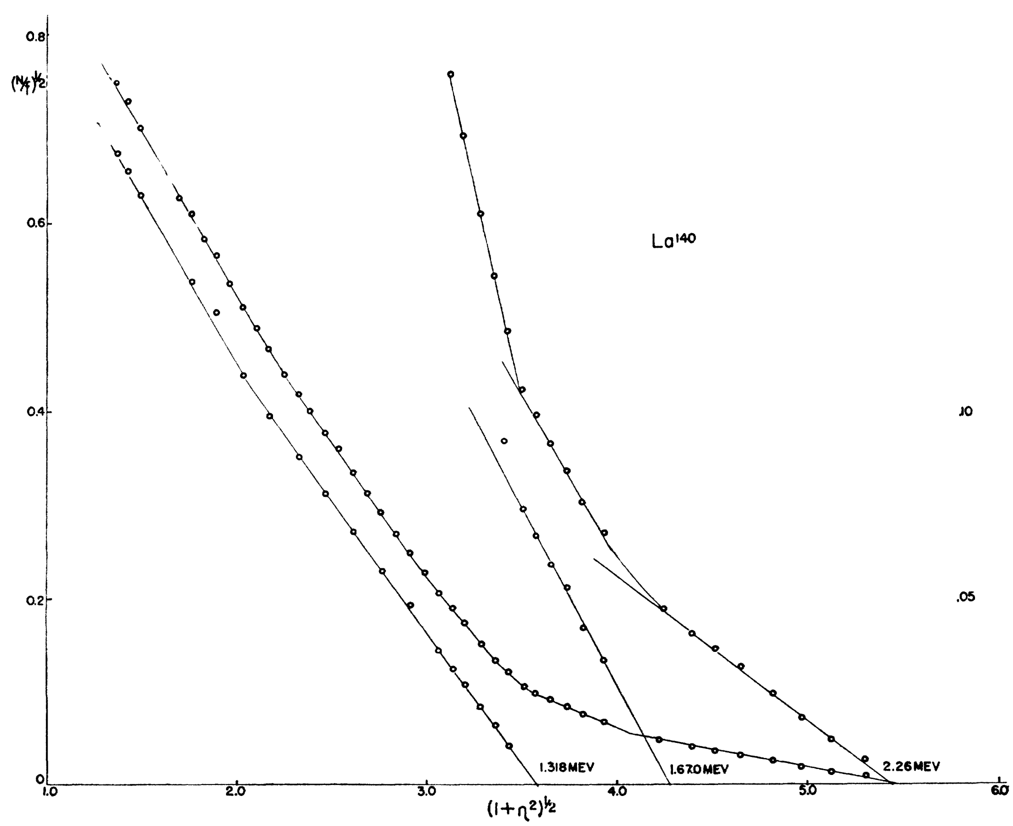

FIG. 5. Conventional Fermi plot of the beta-ray spectrum of La^{140}.

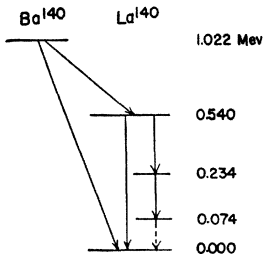

FIG. 6. Proposed decay sheme of Ba^{140}.

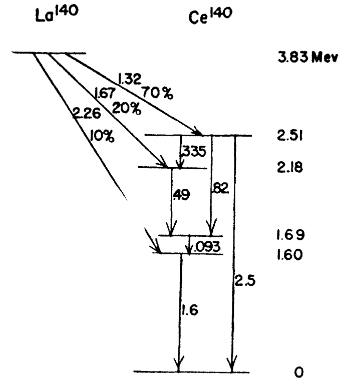

FIG. 7. Tentative decay scheme of La^{140}.

gamma-ray is more intense than either of the other two and is, therefore, probably not in cascade with them. Coincidence experiments done in this laboratory to verify this were inconclusive because of the masking effect of the La^{140}. A tentative decay scheme is given in Fig. 6. To complete the scheme, it would be necessary to prove the existence of a weak 70-kev gamma-ray. In this case, such a gamma-ray would be difficult to find because of its low energy.

The radiations of La^{140} are summarized in the decay scheme given in Fig. 7. Three beta-ray groups having end points at 1.32, 1.67, and 2.26 Mev and six gamma-rays of energies 0.093, 0.335, 0.49, 0.82, 1.60, and 2.5 Mev are present. The energies and intensities of the various components make the scheme remarkably consistent and no alternative scheme seems possible. The 1.60-Mev gamma-ray is the most intense, while the 2.5-Mev gamma-ray is probably the weakest. Perhaps the grestest uncertainty is the assignment of the gamma-ray of energy 0.093 Mev. If this is to follow three gamma-rays, as shown, it must be present in about 90 percent of the disintegrations and therefore must be strong. Since the assignment is based on two weak conversion lines, one is led to the questionable assumption that the conversion of the gamma-ray is very small. The 2.5-Mev gamma-ray was estimated from the visual end point of the Compton-electron distribution and is not as accurately determined as the rest. However, the value obtained is consistent with the decay scheme suggested in Fig. 7 and the value which Wattenberg[7] has obtained by quite different means $(D-\gamma-n)$.

TABLE I. Summary of data.

	Beta-energy (Mev)	Gamma-energy (Mev)*	Gamma-ray No.**
Ba^{140}	0.480 (40%)	0.160 (IC)	
	1.022 (60%)	0.306 (IC)	
		0.540 (IC)	3
La^{140}	1.32 (70%)	0.093? (IC)	
	1.67 (20%)	0.335 (IC)	1
	2.26 (10%)	0.490 (IC)	2
		0.820	4
		1.600	5
		2.50	

* (IC) signifies that internal conversion of the gamma-ray was observed.
** The number in this column refers to the legend of Fig. 1.

A summary of the data is given in Table I. Relative intensities of the gamma-rays are not given since estimates must necessarily be arbitrary for such complex disintegrations. The values given by Rall and Wilkinson[1] and Miller and Curtiss[6] are, however, thought to be a fair measure of the relative intensities. Conversion coefficients have not been calculated since this requires a knowledge of the branching ratios of the gamma-rays. The results given in Table I are thought to be accurate to one percent.

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