

## Letters to the Editor

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### Theory of the Formation of Powder Patterns on Ferromagnetic Crystals

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**M**AGNETIC powder patterns or "Bitter patterns" are sometimes formed on the surfaces of ferromagnetic crystals when a liquid suspension of fine ferromagnetic particles is placed on the crystal. Powder patterns are the most powerful tool available for the study of the nature of ferromagnetic domains,<sup>1</sup> yet there has not been an adequate discussion of the mechanism by which the powder particles collect at domain boundaries to form the lines which are observed under the microscope. It is frequently said that the colloid particles collect in the regions in which the maximum inhomogeneity in the local magnetic field occurs. This is an inadequate statement, and in particular fails to account for the enhancement of some lines when a uniform magnetic field is applied normal to the surface under examination, and the disappearance of other lines under the same conditions.

The particles in the suspension are small enough so that they remain indefinitely in suspension. The particle distribution is accordingly governed by the Boltzmann distribution law. The magnetic energy of a particle of effective susceptibility  $\chi = (\mu - 1)/4\pi$  in a field  $H$  is  $-\frac{1}{2}\chi H^2 V$ , where  $V$  is the volume of the particle. In thermal equilibrium the density of particles is given by

$$p(H) = p(0)e^{\chi H^2 V/2kT}, \quad (1)$$

where  $p(0)$  is the density at a point for which  $H=0$ . It is seen that there will be a marked tendency for the particles to concentrate in regions in which  $H^2$  is large.

On the surfaces of crystals there are strong local fields of diverse origin. The fields of greatest interest are caused by long narrow strips of poles formed by the intersection of Bloch walls with the crystal surface. Suppose that the poles produce a magnetic field  $\Delta H$  in their vicinity; we then apply from outside a uniform field  $H$ . The particle density over the line of poles will be greater than that in other places by the factor

$$\exp[[2H \cdot \Delta H + (\Delta H)^2]\chi V/2kT].$$

When  $H$  is in the same direction as  $\Delta H$ , the line is enhanced with respect to the density in the absence of the applied field; if  $H$  is opposite to  $\Delta H$ , the line is weakened and may be made to disappear. The operation of this effect is beautifully illustrated by Fig. 17 in the paper by Williams, Bozorth, and Shockley.<sup>1</sup>

In the absence of an external field it is necessary that  $(\Delta H)^2 \chi V \gg 2kT$  for a well-defined line to collect. For magnetite spheres the effective susceptibility is essentially determined by the demagnetizing factor and we have  $\chi \cong \frac{3}{4}\pi$ . We require, for particles of volume  $10^{-12}$  cc,

$$|\Delta H| \gg [2kT/\chi V]^{\frac{1}{2}} \cong [4 \times 10^{-13}/10^{-12}]^{\frac{1}{2}} \cong 0.6 \text{ oersted},$$

which may be expected to be exceeded near a Bloch wall.

If the particles in the suspension are small enough to behave as single domains,<sup>2</sup> that is, as small permanent magnets, then the

distribution law will be

$$p(H) = p(0)[\sinh(\mu H/kT)]/(\mu H/kT), \quad (2)$$

where  $\mu$  is the magnetic moment of a particle. This relation is obtained by integrating the Boltzmann factor  $\exp[\mu H \cos\theta/kT]$  over all solid angles. The effect of an applied field is exhibited in some cases by this density function, although in a less striking manner. In the colloidal suspensions employed in practice the particle size is relatively large, so that distribution law (1) is applicable. An account of other aspects of the theory will be found in reference 3. In the derivation of both distribution functions we have neglected field inhomogeneities over the volume of a single particle, the effects of mutual interactions among the particles, and the reorientation of spins in a wall under the action of the external magnetic field. These approximations are not likely to change the general nature of the results.

<sup>1</sup>Williams, Bozorth, and Shockley, Phys. Rev. **75**, 155 (1949); H. J. Williams and W. Shockley, Phys. Rev. **75**, 178 (1949).

<sup>2</sup>C. Kittel, Phys. Rev. **70**, 965 (1946).

<sup>3</sup>C. Kittel, Rev. Mod. Phys. **21**, 541 (1949).

### Nuclear Moments of Mg<sup>25</sup>

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**T**HE Mg I lines 5184, 5173, and 5167A ( $3s3p\ ^3P_{2,1,0} - 3s4s\ ^3S_1$ ) and the Mg II resonance line 2796A ( $3s\ ^2S_{1/2} - 3p\ ^2P_{3/2}$ ) have been excited in an atomic beam light source and their hyperfine structures resolved with a Fabry-Perot etalon. The structures show that for Mg<sup>25</sup>  $I=5/2$  and the nuclear magnetic moment is negative and equal to  $-0.96 \pm 0.07$  nuclear magnetons.

The line 5167A ( $3s3p\ ^3P_0 - 3s4s\ ^3S_1$ ) was studied with 21-, 54-, and 73-mm etalons. The interferometer plates were coated with evaporated silver<sup>1</sup> to give a reflectivity of 80 percent and a transmission of 17.5 percent in the green. The 21-mm etalon patterns showed no structure. The 54-mm patterns showed two weak components which are assigned to Mg<sup>25</sup>, a strong component due to Mg<sup>24</sup>, and indications of a component due to Mg<sup>26</sup> very close to Mg<sup>24</sup>. The separations of the Mg<sup>25</sup> components relative to the Mg<sup>24</sup> components are  $+0.0303 \pm 0.0004$  and  $-0.0341 \pm 0.0006$  cm<sup>-1</sup> where the positive sign indicates a higher wave number.

For all three lines the Mg<sup>24</sup> and Mg<sup>26</sup> components were resolved with a 73-mm etalon in which the silver films had a 90 percent reflectivity. The separations of the Mg<sup>26</sup> component relative to the Mg<sup>24</sup> component are  $-0.0138 \pm 0.0003$ ,  $-0.0122 \pm 0.0015$ , and  $-0.0138 \pm 0.0004$  cm<sup>-1</sup> for the lines 5167, 5173, and 5184A, respectively. As predicted by the specific mass effect, the isotope shifts in all three lines are equal within the assigned errors.

The resolution of the Mg<sup>24</sup> and Mg<sup>26</sup> components in 5167A permits an accurate determination of the center of gravity of the three components due to Mg<sup>25</sup>, on the assumption that it is midway between the Mg<sup>24</sup> and Mg<sup>26</sup> components. Thus the interval rule can be used to determine the  $I$  value. The separations of the two Mg<sup>25</sup> components, measured in the 54-mm pattern, from their center of gravity are  $-0.0272$  and  $+0.0372$  cm<sup>-1</sup>. The ratio of these separations is  $1.37 \pm 0.07$ . This ratio should be 1.40 for  $I=5/2$  and 1.29 for  $I=7/2$ . Thus the spin of Mg<sup>25</sup> is  $5/2$  and the inverted splitting of  $^3S_1$  shows that the magnetic moment is negative.

The over-all separation of the Mg<sup>25</sup> components,  $0.0644 \pm 0.0010$  cm<sup>-1</sup>, is equal to  $6A(^3S_1)$ , where  $A(^3S_1)$  is the h.f.s. interval factor. Thus  $A(^3S_1) = 0.0107$  cm<sup>-1</sup>, and gives  $\mu(\text{Mg}^{25}) = -0.97 \pm 0.05$  n.m. by the method used for P IV by Crawford and Levinson.<sup>2</sup>

The line 2796A ( $3s\ ^2S_{1/2} - 3p\ ^2P_{3/2}$ ) was studied with 21-, 30-, and 50-mm etalons using aluminum coated plates with reflectivities of 80 percent. The 21-mm pattern showed a strong com-

ponent due to  $Mg^{24}$  and a weak component  $+0.095\text{ cm}^{-1}$  from the  $Mg^{24}$  component. The 30-mm pattern showed the weak component resolved into two, the stronger of which had the higher frequency. The separations of these from the  $Mg^{24}$  component are  $+0.1021 \pm 0.0004$  and  $+0.0786 \pm 0.0007\text{ cm}^{-1}$ . The stronger is assigned to  $Mg^{26}$  and the weaker is one of the two components expected from  $Mg^{26}$ . The 50-mm pattern indicated a faint component on the high frequency side of the strong component which in this pattern arises from an overlap of the  $Mg^{24}$  and  $Mg^{26}$  components in adjacent orders; but its position could not be accurately measured.

The ratio of the intensities of the  $Mg^{26}$  component and the neighboring  $Mg^{25}$  component, corrected for their mutual influence and the background due to  $Mg^{24}$ , was measured from the 30-mm pattern and is  $1.7 \pm 0.2$ . The assigned error is large because the background intensity due to  $Mg^{24}$  is comparable to the intensity of both the  $Mg^{25}$  and  $Mg^{26}$  components. Using the known relative abundances<sup>3</sup> of the isotopes and the intensity formula, values of this ratio can be predicted for both positive and negative moments and various  $I$  values. For a negative moment and  $I=1/2, 3/2, 5/2,$  and  $7/2$  the predicted values are 1.49, 1.78, 1.91, and 1.99, respectively. Positive values of the moment give ratios greater than 2.55. Thus the measured value confirms a negative magnetic moment. It is not sufficiently accurate to determine  $I$ , but is compatible with  $I=5/2$ .

If the center of gravity of the  $Mg^{26}$  components is assumed to be midway between the  $Mg^{24}$  and  $Mg^{26}$  components the interval factor of  ${}^2S_{1/2}$  of Mg II can be evaluated and used to calculate the magnetic moment. The position of the  $Mg^{25}$  component, corrected for the influence of the  $Mg^{26}$  component, relative to the center of gravity is  $0.0274 \pm 0.0015\text{ cm}^{-1}$ . For  $I=5/2$  this separation gives  $a_{3s}=0.0231\text{ cm}^{-1}$  when allowance is made for the small structure of  ${}^2P_{3/2}$ .  $\mu(Mg^{25}) = -0.93 \pm 0.05\text{ n.m.}$  is obtained from  $a_{3s}$  by the Goudsmit formula.<sup>4</sup> However, the uncertainty in  $\mu$  would be increased to 10 percent if the assumed position of the center of gravity is in error by  $0.001\text{ cm}^{-1}$ .

The values of the magnetic moment determined from the h.f.s. splittings of  ${}^3S_1$  of Mg I and  ${}^2S_{1/2}$  of Mg II agree within the assigned errors. The weighted mean is  $-0.96 \pm 0.07$  nuclear magnetons. The Fermi-Segrè<sup>5</sup> correction has been applied and the effect of the finite size of the nucleus<sup>6,7</sup> is negligible.

The spin and the negative sign of the magnetic moment agree with the predictions of the shell structure in nuclei.<sup>8,9</sup> The magnitude of the magnetic moment falls near the average curve for similar nuclei in a Schmidt diagram,<sup>8</sup> but as for most of these nuclei it does not agree with the value predicted by an individual particle model.

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<sup>1</sup> S. Tolansky, *Multiple Beam Interferometry of Surfaces and Films* (Oxford University Press, New York, 1948).

<sup>2</sup> M. F. Crawford and J. Levinson, *Can. J. Research* **A27**, 156 (1949).

<sup>3</sup> J. R. White and A. E. Cameron, *Phys. Rev.* **74**, 991 (1948).

<sup>4</sup> S. Goudsmit, *Phys. Rev.* **43**, 636 (1933).

<sup>5</sup> E. Fermi and E. Segrè, *Zeits. f. Physik* **82**, 729 (1933).

<sup>6</sup> J. E. Rosenthal and G. Breit, *Phys. Rev.* **41**, 459 (1932).

<sup>7</sup> M. F. Crawford and A. L. Schawlow, *Phys. Rev.* **76**, 1310 (1949).

<sup>8</sup> E. Feenberg and K. C. Hammack, *Phys. Rev.* **75**, 1877 (1949).

<sup>9</sup> L. W. Nordheim, *Phys. Rev.* **75**, 1894 (1949).

half-width,  $0.1\text{ cm}^{-1}$ , of the Y III resonance line 2817A ( $5s\ {}^2S_{1/2} - 5p\ {}^2P_{3/2}$ ) Wittke<sup>8</sup> estimated an upper limit of  $0.05\text{ cm}^{-1}$  for the h.f.s. splitting of  $5s\ {}^2S_{1/2}$ , and from this calculated  $\mu \leq 0.1\text{ n.m.}$ , on the supposition that  $I=3/2$ .

In the present investigation, the Y III resonance line 2817A ( $5s\ {}^2S_{1/2} - 5p\ {}^2P_{3/2}$ ) has been resolved into two components. The observed intensity ratio is 3:1, and the fainter component is of lower frequency. Thus  $I=3/2$ , and the magnetic moment is negative. The magnetic moment calculated from the observed separation of the two components is  $\mu = -0.14\text{ n.m.}$

The Y III spectrum was excited in the electrodeless discharge described by Crawford and Levinson.<sup>5</sup>  $YCl_3$  was introduced into the quartz discharge tube. The tube was then evacuated, and argon admitted. The pressures used were in the range of 0.1 to 1.0 mm Hg at room temperature ( $26^\circ\text{C}$ ). Gaseous impurities in the discharge tube were removed by calcium shavings placed in a side branch. Graded seals connected the discharge tube to the pumping system, and liquid air traps placed between the stopcocks and the discharge tube prevented contamination by vapors from the stopcock grease. The discharge tube, with the exception of the windows, was immersed in liquid air. Even with liquid air cooling the Y III lines were easily excited.

The structure of 2817A was resolved with a 3-cm Fabry-Perot etalon, with aluminum coated plates. The etalon was placed between the collimator and the dispersing prism of the spectrograph, which had an  $F/4$  off-axis paraboloidal mirror as the camera objective. The structure of the other resonance line of Y III could not be measured because an argon line was superimposed on it.

The effects of different excitation conditions and argon pressures on the half-intensity width and the intensity of the components were studied. It was possible to obtain half-widths of  $0.05\text{ cm}^{-1}$  with intensity adequate to give in a two-hour exposure a photographic density suitable for accurate intensity measurements. The plates were calibrated for intensity measurements by the step-slit continuous spectrum method. The intensity ratio of the two components was found to be independent of excitation conditions and argon pressures. Thus the structure observed must be h.f.s.

The separation of the components, when corrected for their small overlap as determined from the analysis of the intensity contours, is  $0.060 \pm 0.005\text{ cm}^{-1}$ . This separation is very nearly equal to the difference between the splittings of the  ${}^2S_{1/2}$  and  ${}^2P_{3/2}$  levels. A calculation shows that the splitting of the  ${}^2P_{3/2}$  level is 1/15 that of the  ${}^2S_{1/2}$  level. Therefore the  ${}^2S_{1/2}$  splitting is  $0.064 \pm 0.005\text{ cm}^{-1}$ . The magnetic moment calculated from this separation by the Goudsmit formula,<sup>6</sup> with a Fermi-Segrè correction,<sup>7</sup> is  $\mu = -0.14\text{ n.m.}$  The correction for the finite size of the nucleus<sup>8</sup> is negligible compared to the experimental error in  $\mu$ .

The spin and the sign of the magnetic moment of  ${}_{39}Y^{89}$  are consistent with the predictions of the single particle model<sup>9-11</sup> for a configuration in which 50 neutrons form closed shells and the last 5 protons are in the  $3p$  shell. Thus one concludes that at  $Z=39$  the  $2s$  and  $3p$  proton levels have not crossed. The magnitude of the magnetic moment of  ${}_{39}Y^{89}$  is in fair agreement with that predicted for the single particle model, and is additional evidence that the observed and predicted magnetic moments are in much better agreement for nuclei with completed shells plus or minus one proton, or plus or minus one neutron than for other nuclei.  $Pb^{207}$  and  $Bi^{209}$  are the two outstanding exceptions and they are adjacent to the radioactive elements.

<sup>1</sup> H. Schüller and T. Schmidt, *Naturwiss.* **22**, 838 (1934).

<sup>2</sup> D. T. Williams and L. P. Graneth, *Phys. Rev.* **54**, 338 (1938).

<sup>3</sup> H. Wittke, *Zeits. f. Physik* **116**, 547 (1940).

<sup>4</sup> P. C. Kruger and C. N. Challacombe, *Phys. Rev.* **47**, 509 (1935); **48**, 111 (1935).

<sup>5</sup> M. F. Crawford and J. Levinson, *Can. J. Research* **A27**, 156 (1949).

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<sup>7</sup> E. Fermi and E. Segrè, *Zeits. f. Physik* **83**, 729 (1933).

<sup>8</sup> M. F. Crawford and A. Schawlow, *Phys. Rev.* **76**, 1310 (1949).

<sup>9</sup> E. Feenberg and K. C. Hammack, *Phys. Rev.* **75**, 1877 (1949).

<sup>10</sup> L. W. Nordheim, *Phys. Rev.* **75**, 1894 (1949).

<sup>11</sup> Maria G. Mayer, *Phys. Rev.* **75**, 1969 (1949); Haxel, Jensen, and Suess, *Phys. Rev.* **75**, 1766 (1949).

## Nuclear Moments of ${}_{39}Y^{89}$

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LINEs in the first, second, and third spectra of yttrium have been examined for hyperfine structure,<sup>1-4</sup> but none has been observed. The only result obtained from these investigations was that the magnetic moment of  ${}_{39}Y^{89}$  is small. From the observed