and the high field strength minimizes electron attachment. It is apparent that there is order-of-magnitude agreement between the observations with photographic plates and with ionization chambers although the accuracy of the check leaves much to be desired.

For bursts of the smallest sizes, the frequency distribution curve rises again rather rapidly as is evident from Fig. 7. It seems unlikely, from the arguments presented in previous sections, that these bursts are also the result of nuclear processes. Some new phenomenon must be predominant in this range of sizes. Only a few electrons crossing the chamber are necessary to produce these bursts and it is suggested that these represent locally produced showers of electrons. In support of this contention, we can only point out that the number of single rays through the chambers amounts to 250 cm^{-2} $hr.^{-1}$. Since many of these are electrons and since there is an approximately equal number of photons, small showers occurring with the observed frequency are not unreasonable. It should be noticed that the number of these small showers appears to be proportional to the chamber area.

In conclusion, the authors wish to express their thanks to Mr. C. J. Abrams of the Climax Molybdenum Company and to Mr. E. F. Fahy and Professor Marcel Schein of the University of Chicago for their generous hospitality that made the observations at high altitude possible.

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Narrow Air Showers of Cosmic Rays*

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Observations are reported of the decoherence curves for air showers of cosmic rays at sea level, 1640 meters and 3510 meters elevation. The variation of the decoherence curves with counter area and the absorption of narrow showers were also measured at the highest elevation. It was found that narrow showers (that is, showers observed at small counter separations) differ from ordinary extensive air showers with respect to variation with elevation, variation with area of the counters, and penetrating power. Consideration of the effects of pressure on the spreading function show that the altitude variation cannot be taken as a certain indication that narrow showers differ in nature from extensive ones, nor can the variation with counter area. The relative penetration, however, of narrow showers in lead, iron, and aluminum cannot be explained except by differences in the nature of the particles involved.

INTRODUCTION

SHOWERS of cosmic rays which originate in the atmosphere have been the subject of many investigations since their discovery by Auger.[1] Auger and many others have measured the coincident counting rate of two or more counters as a function of the separation between them up to large distances. The lateral spread of such showers as measured by this "decoherence curve" agrees well with theoretical predictions[2,3] of the cascade theory of electron-photon showers for separations from 1 to 1000 meters. Between about 1 and 10 meters separation there is a well defined plateau in the decoherence curve, and Molière's calculations, as well as those of others, predict only a negligibly small increase of counting rate as the separation is decreased to zero. Observations, however, deviate from the theoretical predictions, and a sharp rise in counting rate is found which attains many times the expected value at separations of 20 or 30 cm. The showers that are responsible for this discrepancy at small separations we call "narrow air showers." Although this discrepancy was noted long ago, no detailed investigations were made concerning it until the experiments of Alichanian *et al.*[4] Many questions concerning narrow showers were still unanswered precisely and hence the present experiments were undertaken. The observations to be described consist of measurements of the decoherence curve for different elevations and different counting areas, some absorption measurements, and subsidiary experiments to test the importance of local showers.

THE DECOHERENCE CURVE

Most of the experiments were performed with the arrangement of counters indicated in Fig. 1. Four trays of four counters each were arranged as two telescopes of wide angle in order to minimize the accidental coincidences. The counters had copper cathodes 20.5 cm by 4.3 cm in glass envelopes and were filled with argon

* Part of a dissertation to be presented by the first author for the degree of Doctor of Philosophy at Yale University.
** Assisted by the Joint Program of the ONR and the AEC.
[1] Auger, Maze, Ehrenfest, and Freon, J. de phys. et rad. **10**, 39 (1939).
[2] G. Molière, *Lectures on Cosmic Radiation* (Dover Publications, New York, 1946), edited by W. Heisenberg.
[3] G. Cocconi, Phys. Rev. **72**, 350 (1947).
[4] A. Alichanian *et al.*, J. Phys. U.S.S.R. **10**, 296, 518 (1946); **11**, 16 (1947).

mixed either with alcohol or with ethyl ether vapor. The resultant efficiencies were more than 99 percent. Each tray thus had an effective area of 320 cm^2. Triple coincidences (T) between counters 123 and quadrupole coincidences (Q) were simultaneously recorded as a function of the distance d for three elevations: New Haven, Connecticut, 10 meters above sea level, Denver, Colorado, elevation 1640 m, and Climax, Colorado, elevation 3510 m. The counters were placed either in a truck or in small houses close to thin wooden roofs of thickness about 1.3 g/cm^2. The effect of such a covering is small, as discussed in more detail below.

The coincidence circuits were simple Rossi circuits with 6SJ7 tubes, and thyratrons were used to drive mechanical recorders. The resolving time was of the order of 8μ sec. and corrections to the counting rates were made for accidental coincidences.

The six curves obtained are shown on a log–log scale in Fig. 2. The long plateau and the increase in counting rate for small separations are evident in each case.

The portion of the curves for separations greater than about 1 meter results entirely from extensive cascade showers. Hence, if the separation is measured in radiation units in air at the pressure where the observations were taken, the three curves should be identical except for an intensity factor dependent on elevation. It is possible to compare the curves on this basis if the ordinates and abscissas are reduced by the factors given in Table I. The decoherence curves reduced in this fashion are shown in Fig. 3. It is apparent that for large separations all the observations can be considered to lie on a common curve. The variation of the occurrence of extensive showers with elevation as given in the last column of Table I is in good agreement with that determined by Kraybill.[5]

For small values of d the situation is quite different. The decoherence curves do not reduce to a common shape and hence disagree with the predictions of the cascade theory. We can separate the narrow showers from the extensive ones by extrapolating the observed plateaus of Fig. 2 to the smallest separations and subtracting. The resultant curves are shown in Fig. 4 on a linear scale. It has been assumed that the decoherence curve for extensive showers has zero slope for distances less than about 1.5 meters and the counting rate at this distance has been subtracted from those at smaller distances. The standard deviations of the points have been correspondingly increased. No reduction to compensate for a change in pressure has been made.

It is evident that the variation with elevation is much less for narrow showers than for extensive ones. The increase from sea level to 3510 m is only by a factor of 4 for $d=20$ cms instead of by a factor of 12 as for extensive showers. This slow variation with elevation is characteristic of only the hard component of cosmic radiation and suggests that a genetic relation exists. As will become evident, corrections for the pressure variation may modify this conclusion.

[5] H. L. Kraybill, Phys. Rev. **73**, 632 (1948) and private communication.

TABLE I. Factors by which d and the counting rate should be divided to reduce the decoherence curves to sea level.

Location	Elevation	Pressure	Factor for d	Factor for counting rate
New Haven	10 m	1030 g/cm^2	1	1
Denver	1640	865	0.84	4
Climax	3510	675	0.65	12

Some observations were also made at Climax of the decoherence curves for different sensitive areas in each counter tray. These data are presented in Fig. 5. It is evident that the narrow showers become relatively less prominent, compared with extensive showers, as the area is decreased.

PROBABILITY CONSIDERATIONS

To interpret experiments on showers, it is always essential to calculate the probability that a count be recorded when the desired phenomenon occurs. Let us consider a simple model that should apply to our observations. Assume first that the showers under consideration come from the vertical direction and have an axis such that the N rays in the shower are distributed at random in azimuth about this axis. The probability distribution with respect to distance r from the axis we shall denote by $\rho(r)$ and assume that $\rho(r)$ is normalized according to

$$\int_0^\infty 2\pi r\rho(r)dr=1. \quad (1)$$

Let the number of showers with N rays be $f(N)$ per unit area per second. Suppose two counters of areas S_1 and S_2 are arranged as shown in Fig. 6. It can be shown[6] that the probability P that a shower of N rays with an axis through the point $(r, 0)$ will give at least one ray through each counter is

$$P=1-(1-p_1)^N-(1-p_2)^N+(1-p_1-p_2)^N, \quad (2)$$

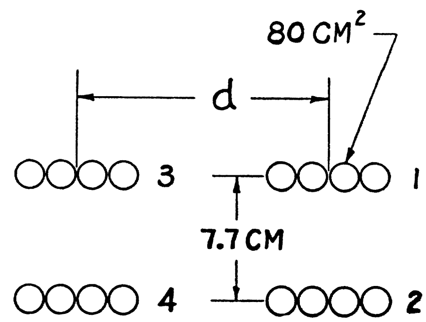

FIG. 1. Arrangement of 4 trays of counters to form 2 telescopes.

[6] C. G. Montgomery and D. D. Montgomery, J. Frank. Inst. **221**, 59 (1936).

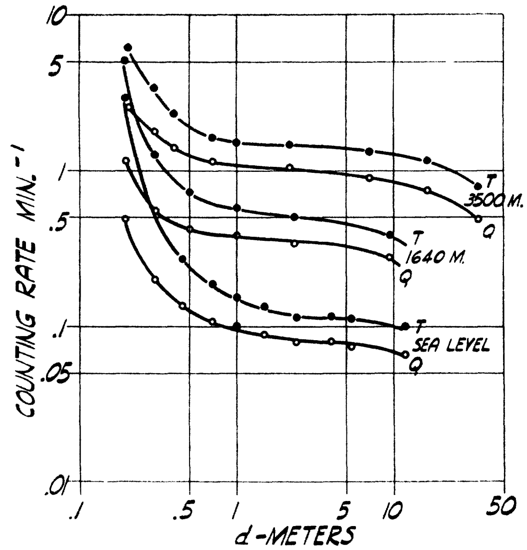

FIG. 2. Decoherence curves for triple and quadruple coincidences at three elevations.

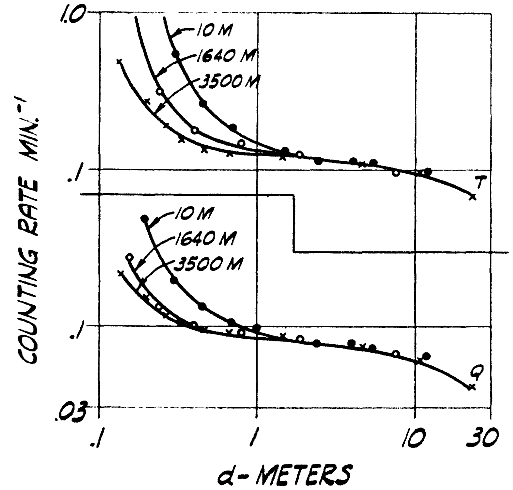

FIG. 3. The data of Fig. 2 reduced according to Table I.

where the *a priori* probabilities p_1 and p_2 are given by

$$p_1 = S_1\rho(r_1), \quad p_2 = S_2\rho(r_2). \tag{3}$$

The coincident rate C of the two counters is then

$$C = \int_{\text{plane}} rdrd\theta \sum_{N=2}^{\infty} Pf(N). \tag{4}$$

The lower limit of the sum could as well have been taken as 1 or 0 in Eq. (4) since P is zero for these values of N. If the probabilities p_1 and p_2 are not too large, P can be approximated by

$$P = (1 - e^{-p_1 N})(1 - e^{-p_2 N}), \tag{5}$$

and the sum in (4) replaced by an integral. Hence

$$C = \int_{\text{plane}} rdrd\theta \int_{N=2}^{\infty} (1 - e^{-p_1 N})(1 - e^{-p_2 N}) f(N) dN. \tag{6}$$

It is customary to represent the integral distribution of shower sizes by a power law. Thus let

$$F(N) = \int_N^{\infty} f(N) dN = A/N^{\gamma} \tag{7}$$

and

$$f(N) = \gamma A / N^{\gamma+1}. \tag{8}$$

If this function is inserted in Eq. (6) and the substitution $x = p_1 N = \rho(r_1) S_1 N$ is made, Eq. (6) becomes

$$C = \int_{\text{plane}} rdrd\theta \int_{x=(r_1)S_1}^{\infty} (1 - e^{-x})(1 - e^{-(P_2/P_1)x}) \times \frac{\gamma A}{x^{\gamma+1}} [\rho(r_1) S_1]^{\gamma} dx. \tag{9}$$

Now if the counting rate resulting from showers containing only a few rays is small, the lower limit of the second integral can be replaced by a small constant. In this case it is evident that, if the counter areas are altered by the same factor, so that p_2/p_1 remains constant, the counting rate C is proportional to S^{γ}. The observations shown in Fig. 5 can be used to test this relation and to determine the parameter γ. In Fig. 7 the data have been replotted on a double logarithmic scale. It can be seen that the variation with area can indeed be approximated by a power law, but with an exponent γ which is different for the narrow and the extensive showers. For the latter γ is 1.4 in agreement with other observers,[5] but for the narrow showers the observed slope corresponds to a value of γ between 3.0 and 3.6 with a most probable value of 3.2. Again in

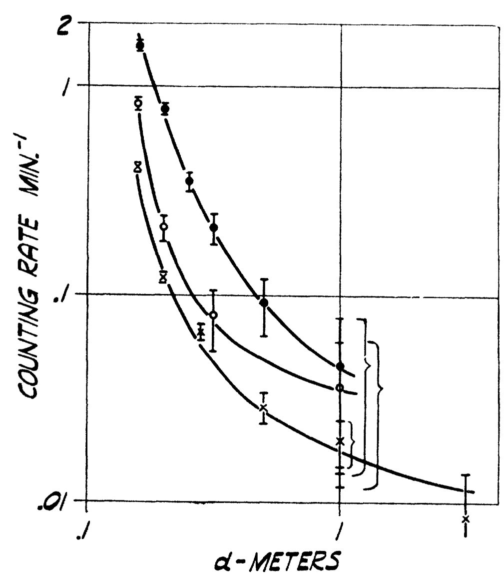

FIG. 4. Decoherence curves with extensive showers subtracted.

this respect the behavior of narrow showers appears to be characteristically different from that of extensive ones. Caution must be observed here, however. For such a large value of γ it may not be permissible to neglect the effect of the lower limit of the second integral in Eq. (9). In fact if the lower limit were zero, the integral diverges at this limit for $\gamma > 2$. In a rigorous calculation Eqs. (2) and (4) should be employed.

It is perhaps suggestive that γ so determined is close to 3. Such an exponent in the power-law distribution of the frequency of showers of N rays would be expected for cascade showers produced by knock-on and disintegration electrons of mesotrons.

Equation (9) can also be employed to find the shape of the decoherence curve. For extensive showers the function $\rho(r)$ has been calculated by Molière[2] and can be expressed as shown by Bethe[7] by the approximate analytic form

$$\rho(r)=\frac{0.45}{r_0 r}\left(1+4\frac{r}{r_0}\right)\exp\left[-4\left(\frac{r}{r_0}\right)^{\frac{1}{3}}\right],\quad \frac{r}{r_0}<0.5, \tag{10}$$

where the unit length r_0 is given by

$$r_0=\frac{E_s X_0}{E_c}, \tag{11}$$

E_c and X_0 being the familiar cascade parameters and E_s the characteristic scattering energy[8] which is equal to 21 Mev.

The occurrence of the singularity in ρ at the origin prompts one to suspect that a peak also occurs in the decoherence curve at the origin, contrary to Molière's calculations.[9] It should be noticed that ρ occurs raised

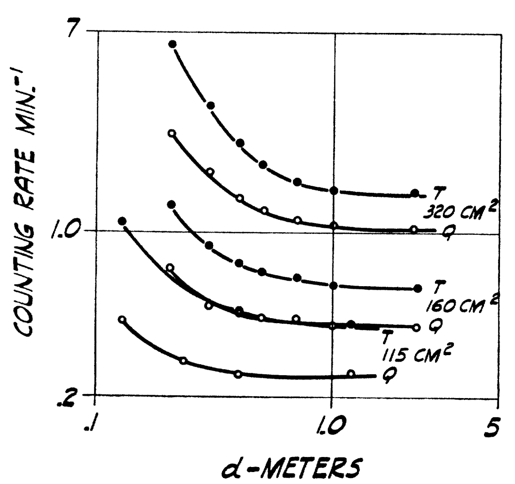

FIG. 5. Decoherence curves for different sensitive areas.

[7] H. A. Bethe, Phys. Rev. **72**, 172 (1947).
[8] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).
[9] Similar conclusions have been drawn for ionization chambers by J. M. Blatt, Phys. Rev. **75**, 1584 (1949). The authors are indebted to Dr. Blatt for allowing them to see before publication a manuscript dealing with this question.

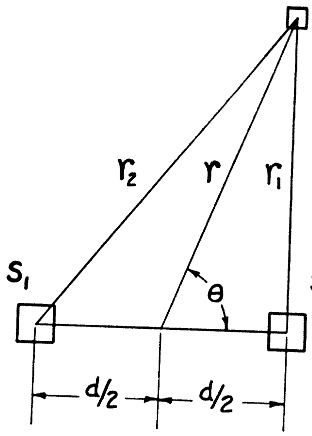

FIG. 6. Definition of symbols.

to the γ-power in the integrand of Eq. (9). Consequently, the exact manner in which the probability P approaches zero and the lower limit of the integral are important in the determination of the value of the counting rate C.

The interpretation of the variation of the counting rate with elevation is complicated by the change in atmospheric pressure. From Eq. (11) it is evident that the unit length r_0 is proportional to the cascade parameter X_0 which is in turn inversely proportional to the pressure. Comparison at two elevations should not, therefore, be made for the same values of the counter separation d nor for the same counter area A at the two elevations. The counting rate must depend on pressure, by Eqs. (6) and (8), as

$$C=C_0(p^2S, pd)A(p). \tag{12}$$

If C is proportional to S^γ, then

$$C=p^{2\gamma}S^\gamma A(p)g(pd), \tag{13}$$

where g is the function giving the decoherence curve. To find the ratio of the intensities A at two elevations with pressures p_1 and p_2, such values of d should be chosen that pd and therefore $g(pd)$ remains constant. If the area A is also taken constant, then

$$\frac{A(p_1)}{A(p_2)}=\frac{C(p_1)}{C(p_2)}\left(\frac{p_2}{p_1}\right)^{2\gamma}. \tag{14}$$

For extensive showers, taking for p_1 and p_2 the pressures at Climax and sea level, respectively, we have $\gamma=1.4$ and

$$A(p_1)/A(p_2)=12(1030/675)^{2\times1.4}=39.$$

For the narrow showers, from the data of Fig. 4, the counting rate at $d=50$ cm at Climax can be compared with that for $d=33$ cm at sea level. The exponent $\gamma=3.2$ but may be as small as 3.0 or as large as 3.6. The intensity ratio is thus most probably

$$\frac{A(p_1)}{A(p_2)}=1.9\left(\frac{1030}{675}\right)^{2\times3.2}=29,$$

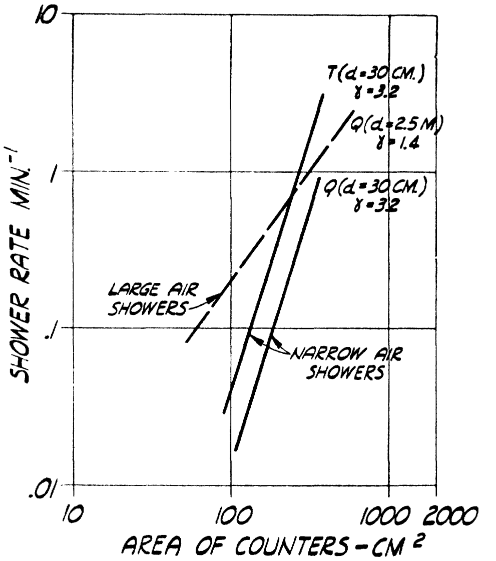

FIG. 7. Dependence of shower rate on counter area.

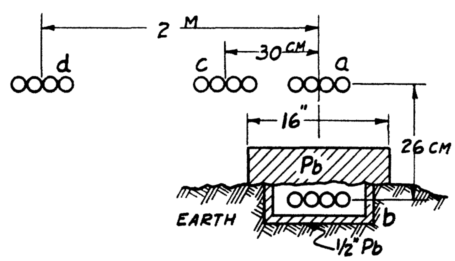

FIG. 8. Arrangement of counters for absorption experiment.

but may be as small as 24 or as large as 40 depending on the value of γ chosen. Thus when the variation of the spreading function ρ with pressure is taken into account, a change in the shape of the decoherence curve with elevation can be understood and the large apparent difference in altitude variation of narrow and extensive showers becomes much reduced and perhaps eliminated.

It is evident that more detailed calculations of the counting rate C are needed. If, however, the narrow showers differ in character from the extensive ones, then such calculations require a knowledge of the spreading function for narrow showers, as well as of the distribution $f(N)$. Even if ρ for narrow showers is different from that given by Eq. (10), it should depend upon pressure in the same manner if elastic scattering is the cause of the angular divergence of the showers.

THE ABSORPTION OF NARROW SHOWERS

Some experiments have also been performed to test the penetrating power of the narrow showers. Four counter trays each of area 320 cm² were arranged as shown in Fig. 8. The lower tray was placed below the level of the ground and shielded below and on the side by $\frac{1}{2}$ inch of lead in addition. The whole arrangement was placed inside a box with a thin top of 0.5 g/cm² located immediately over trays a, c, d. Triple coincidences were recorded for counters a, b, c and for counters a, b, d. The difference in the counting rates as a function of the thickness of the lead between trays a and b is taken as a measure of the absorption of the narrow showers; the counting rate a, b, d is the result of the extensive showers alone. The results of this experiment are shown in the semilogarithmic plot of Fig. 9. The absorption of the extensive showers is in agreement with that measured by other observers.[10] The penetrating power of the narrow showers is greater than that of the extensive ones. The absorption corresponds to a mean free path of 9 cm of lead or about 100 g/cm²; for extensive showers the mean free path is 67 g/cm².

We have also measured the absorption of 41 g/cm² of aluminum and that of 73 g/cm² of aluminum. These points have been plotted in Fig. 9, first on the assumption that the absorption is mass proportional and second with the thickness measured in radiation units. For extensive showers the iron and aluminum points agree approximately with the absorption in lead when the thicknesses of the absorbers are expressed in radiation units. For the narrow showers the converse is true and the iron and aluminum points fall approximately on the lead curve only if the thicknesses are expressed in g/cm².

There are thus two important differences between the absorption of narrow and extensive showers. (1) The narrow showers are more penetrating than extensive ones and (2) the absorption is mass proportional for narrow showers but for extensive showers equal absorption occurs for equal thicknesses in radiation units.

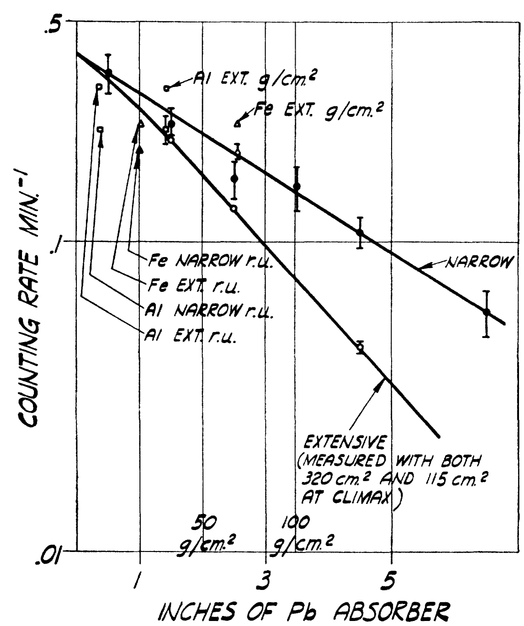

FIG. 9. Absorption of narrow and extensive showers.

[10] For example, G. T. Reynolds and W. D. Hardin, Phys. Rev. 74, 1549 (1948).

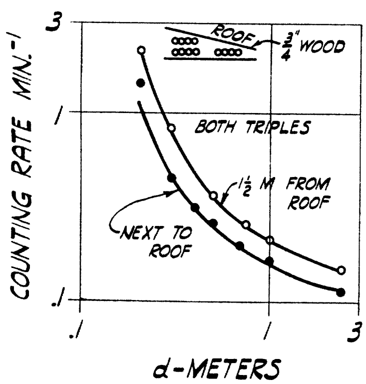

FIG. 10. Test of roof effect.

SUBSIDIARY EXPERIMENTS

Several additional observations were also made. In order to test the effect of the roof of the counter housing, an experiment was made in a house with a roof of $\frac{3}{4}$ inch of wood. Triple coincidences were recorded from three counter trays arranged as shown in the insert to Fig. 10. The decoherence curve was measured with the three trays placed as close to the roof as possible and then with the trays 1.5 meters below the roof. The counting rates when the trays were far from the roof were larger by a constant factor than those with the counters close. It seems likely that some production of ionizing rays occurs in the roof. With the roof 1.5 meters above the counters, the rays produced diverge and separate sufficiently to produce additional counts; when the roof is close the spreading is not large enough to cover both trays. We conclude that roof effects are best minimized by placing the counters as close as possible to the roof.

A rough check was made to find the effects of changing the number of counters in coincidence and hence of changing the minimum number of rays necessary to record a shower. In all cases so far described, only two rays were necessary. Four coincidence trays of 320 cm^2 area were used and in addition an anticoincidence tray of 640 cm^2 area was placed at a distance of 2.5 meters to eliminate the extensive showers. The four trays were arranged either as two telescopes (A), as one telescope and two single trays (B) or as four single trays (C). The average distance between any two trays was about 30 cm. Table II gives the counting rates observed. The observations were made at sea level.

The change in counting rate is thus a very sensitive function of the number of counters. This is another indication that the shower distribution function decreases rapidly with increasing number of rays. If a power-law expression is possible, the exponent γ must be large. It is not possible to determine γ quantitatively in this manner without a knowledge of the spreading function ρ because of the effect of the lower limit of the integral in Eq. (9). For extensive showers the lower limit can be taken as zero, and the values of γ obtained in this way agree, as Cocconi has shown, with the values found from the variation with area.

TABLE II. Narrow showers as a function of number of counters.

Arrangement (see text)	Minimum number of rays	Counting rate min.$^{-1}$	Standard deviation min.$^{-1}$
A	2	0.151	0.008
B	3	0.024	0.003
C	4	0.010	0.0014

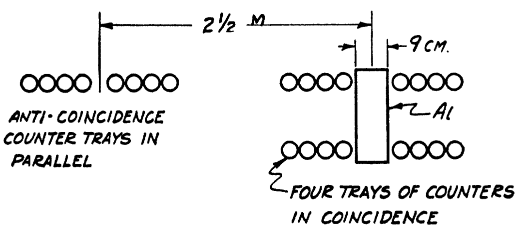

FIG. 11. Arrangement of counters to test effect of material between telescopes.

Since the variation with the number of counters is so rapid, it seems likely that the telescopic arrangement of trays used does have some directional selectivity and that the counters are not often discharged by rays coming at large angles with the vertical. This fact, in turn, agrees with the small ratio in the number of triple and quadruple counts as shown by the data of Fig. 2. From simple geometrical estimates, a ratio as small as that observed is possible only if the rays from narrow showers are concentrated about the vertical direction.

Finally an experiment was performed with the counters placed as shown in Fig. 11, similar to arrangement A of Table II. Nine cm of aluminum was interposed between the two telescopes as shown. The counting rate was found to be unchanged within the 7 percent statistical error of the measurements. We again conclude that the rays causing the counts are concentrated about the vertical direction.

ACKNOWLEDGMENTS

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