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objections (1) that it is based on the Dirac theory for the nucleons, including, in particular, the virtual role of the anti-nucleons and (2) that it employs the usual perturbation methods of field theory. If these objections are tentatively ignored, the calculation suggests that the symmetric coupling scheme is tenable only if the π -meson is pseudoscalar and β -decay coupling is at least partially pseudovector. On the other hand, it is possible that a correct calculation of the divergent integrals will not agree with experiment even in the pseudoscalar case. In that event it would seem natural to reject the symmetric coupling scheme in favor of direct π - μ -coupling.⁷

We wish to thank Dr. R. F. Christy for helpful discussion.

⁷ According to Lopes, Phys. Rev. 74, 1722 (1948) the hypothesis of *direct* π - μ -coupling demands a pseudoscalar meson with pseudovector coupling if one wishes to retain the Yukawa scheme for β -decay. Yukawa (Rev. Mod. Phys. 21, 474 (1949)) has pointed out that this peculiarity of the pseudoscalar case can also be inferred from the earlier work of Sakata. However, according to more detailed calculations the direct coupling scheme does not give satisfactory lifetimes for β -decay, even if the π -meson is pseudoscalar.

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Precise Measurement of the Gyromagnetic Ratio of He³

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The gyromagnetic ratio of He^3 was compared to that of H_2 gas by the magnetic resonance method with the result

 $|\gamma(\text{He}^3)|/|\gamma(\text{H}_2)| = 0.7617866 \pm 0.0000012.$

To obtain the nuclear gyromagnetic ratios, a diamagnetic correction must be made which raises this by approximately 38 parts in 10⁶ to

$$|\gamma_{\rm He^3}| / |\gamma_p| = 0.761815.$$

Intensity measurements were consistent with a spin of $\frac{1}{2}$. In nuclear magnetons, the magnetic moment $\mu_{\text{He}^3} = (-)2.12815$ based on the value $\mu_p = 2.79353$ obtained by Taub and Kusch. The negative sign has not yet been verified.

PRELIMINARY measurement of the gyromagnetic ratio of He³ has already been reported in this journal.¹ In the present paper, a new series of measurements also based on the magnetic resonance method^{2,3} is reported using the same sample of He³, but in which the capabilities of the method were more fully exploited.

MAGNETIC RESONANCE METHOD

The technique used was a refinement of the one which was used in a measurement of $H^{3,4}$ being quite similar to that developed by Bloembergen, Purcell, and Pound.⁵ The sample formed the core of a small coil which was one element of a radiofrequency bridge. The coil and sample were placed in the magnetic field between the poles of an electromagnet, with the axis of the coil at right angles to that of the poles of the magnet. Under these circumstances, a nucleus with spin I may orient itself in 2I+1 ways with respect to the direction of the magnetic field, each having a different energy value. Transitions between neighboring states will be induced when the frequency of the signal applied to the coil satisfies the resonance condition,

$$2\pi\nu = |\gamma| H. \tag{1}$$

Here, $\gamma = \mu/\hbar I$ is the ratio of the magnetic moment to the angular momentum of the nucleus, ν is the frequency in cycles per second, and H is the magnetic field intensity in gauss at the nucleus.

The magnetic field is modulated about the resonance value by a small amount at 30 cycles per second. Under suitable conditions,* the transitions which occur each time the resonance value of the field is traversed produce a change in the impedance of the coil and an unbalance of the bridge. Thus, the magnetic resonance effect produces a 30-cycle modulation in the radiofrequency signal which feeds the bridge, and this may be detected by means of an ordinary radio receiver. The extreme sharpness of the resonance which may be obtained makes possible the great accuracy of the method.

It is seen that a measurement of γ requires a measurement of both the frequency ν and the magnetic field H. Since ν can be measured with much greater accuracy than H, it is preferable and also convenient to compare the value of ν with that of some standard nucleus in

¹ H. L. Anderson and A. Novick, Phys. Rev. **73**, 919 (1948). ² Purcell, Torrey, and Pound, Phys. Rev. **69**, 37 (1946).

^a Bloch, Hansen, and Packard, Phys. Rev. **69**, 127 (1946); **70**, 474 (1946); F. Bloch, Phys. Rev. **70**, 460 (1946).
^a H. L. Anderson and A. Novick, Phys. Rev. **71**, 372 (1947).
^b Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).

^{*} A detailed exposition of the theory of the method and the conditions for observing the magnetic resonance effect may be found in the pioneer papers, references 2 and 5.

the same magnetic field. If, as is usual, the comparison is made with the light hydrogen nucleus, such an experiment measures

$$\frac{\gamma}{\gamma(\mathrm{H}^{1})} = \frac{\nu}{\nu(\mathrm{H}^{1})}.$$
 (2)

The absolute value of γ may be obtained by using the value of $\gamma(H^1) = (2.6752 \pm 0.0002) \times 10^4 \text{ gauss}^{-1} \text{ sec.}^{-1}$ obtained by Thomas, Driscoll, and Hipple.⁶ To obtain a result in units of the Bohr magneton, the value $\mu(H^1) = (15.2106 \times 10^{-4} \pm 0.005 \text{ percent})$ Bohr magnetons for the magnetic moment of the proton obtained by Taub and Kusch⁷ should be used.

SIGNAL INTENSITY

The signal from the magnetic resonance is small at ordinary temperatures because only a small fraction of the nuclei in the sample contributes to it. This fraction is the excess of nuclei which, in thermal equilibrium, exists in the lower of the two energy states between which the transitions occur. It has the value (1/2I+1) $\times h\nu/kT = 1.5 \times 10^{-6}$ for spin $I = \frac{1}{2}$ at 20 megacycles and room temperature, the conditions of the present experiments.

The occurrence of these transitions tends to equalize the population of the two energy states, and unless there is a mechanism which restores the thermal equilibrium rapidly enough, the magnitude of the signal will decay during the course of the observations. The nuclear magnetic relaxation effects, the processes which bring about thermal equilibrium in nuclear systems, have been studied in considerable detail by Bloembergen, Purcell, and Pound.⁵ Some of their results are used in the discussion which follows.

In He³ gas, the atoms are in an S state, and the only important relaxation mechanism is the interaction of the nuclear magnetic moments during collisions in the gas. The order of magnitude of the relaxation time may be estimated from the formula

$$T_1 \approx \lambda v \{ d^2 / \gamma \mu \}^2, \tag{3}$$

where γ is the gyromagnetic ratio of He³, μ the magnetic moment of the atom with which it collides, d is the distance of closest approach, v is the relative velocity of the colliding atoms, and λ the collision mean free path. For pure He³ at 10 atmospheres, this formula gives $T_1 \approx 10^6$ seconds. If, however, following a suggestion of Purcell, Pound, and Bloembergen,8 ten atmospheres of oxygen gas are introduced, the relaxation time is reduced to $\frac{1}{2}$ second. The amplitude of the signal which may be observed is increased more than a thousand-fold thereby.

The other factors which determine the detectability of the magnetic resonance effect are evident from the following formula for the signal-to-noise ratio which has been given by Bloembergen, Purcell, and Pound,⁵

$$\frac{A_s}{A_n} = \frac{V^{\frac{1}{2}}Q^{\frac{1}{2}}\alpha\zeta\hbar^2 N\gamma\nu^{\frac{1}{2}}T_2^{\frac{1}{2}}I(I+1)}{24kT(kTBF)^{\frac{1}{2}}T_1^{\frac{1}{2}}},$$
(4)

where V is the volume of the coil in cm^3 ; Q is the ratio of the reactance to the series resistance of the coil; α is a coefficient appropriate to the shape of the signal, the amount of the modulation swing, the shape of the modulating wave, and the characteristics of the indicating meter; ζ is the fraction of the coil volume occupied by the sample; h is Planck's constant; N is the number of He³ atoms per cm³; ν is the resonance frequency in cycles per second; T_2 is the inverse line width expressed in seconds; I is the nuclear spin; k is Boltzmann's constant; B is the equivalent bandwidth of the detecting system in cycles per second; F is the noise figure of the detecting system including the bridge; and T_1 is the relaxation time in seconds. The factor 24 appears in the denominator instead of the factor 48 of the original formula because phase unbalance rather than amplitude unbalance of the bridge was used in the present experiments.

The following values, appropriate to the present experiment, were either measured or estimated in a manner which will be made clear in the sections to follow. V = 1.2 cm³, Q = 240, $\alpha = 0.4$, $\zeta = 0.46$, N = 2.7 $\times 10^{20}$ cm⁻³, $\gamma = 2.0 \times 10^{4}$ gauss⁻¹ sec.⁻¹, $\nu = 20 \times 10^{6}$ sec.⁻¹, $T_2 \simeq 1.6 \times 10^{-3}$ sec., $I = \frac{1}{2}$, $T = 300^{\circ}$ K, $B = \frac{1}{2}$ sec.⁻¹, $F \cong 4$, $T_1 \cong 1$ sec. These values give $A_s/A_n = 7$ in good



FIG. 1. Experimental arrangement for observing magnetic resonance of He³ and H₂O simultaneously. The mercury U tube is used to compress a mixture of He³, O₂, and H₂ gases into a small bulb in the center of the magnetic field. The small water comparison sample is close by but electrically shielded from the He³ sample.

⁶ Thomas, Driscoll, and Hipple, Phys. Rev. **75**, 902 (1949). ⁷ H. Taub and P. Kusch, Phys. Rev. **75**, 1481 (1949). ⁸ Purcell, Pound, and Bloembergen, Phys. Rev. **70**, 986 (1946).



FIG. 2. Twin "T" Bridge: This was used for observing the He³ resonance at 20.22 mc, and for the H₂ resonance at 26.54 mc. The bridge was first adjusted to null by varying C and C'. By shifting C slightly off balance, the dispersion signal was obtained. By shifting C' slightly off balance, the absorption signal was obtained.

accord with the signal-to-noise ratio which was obtainable with careful adjustment of the equipment.

A spin of $\frac{3}{2}$ would predict a signal-to-noise ratio 5 times greater. Thus, the observed intensity is a strong indication that the spin of He³ is $\frac{1}{2}$.

HELIUM SAMPLE

The He³ sample was obtained from the radioactive decay of tritium. The tritium was made by neutron bombardment of Li⁶ in the Hanford pile according to a method described elsewhere by Anderson, Novick, and Weil. Since He⁴ is also produced in this reaction, it was first necessary to separate the tritium from it. This was done by means of a palladium valve, through which the tritium was passed and collected in a Pyrex bulb. At a subsequent time, the tritium (and light hydrogen impurity) was removed from the bulb by passing it through another palladium valve, leaving the He³ gas behind. Gaseous impurities which may have accumulated in the course of these manipulations were removed by means of a charcoal trap. In this way, about 10 cm³ at N.T.P. of isotopically pure He³ gas was prepared.

In order to obtain a detectable signal from this sample, it was mixed with an equal amount of oxygen gas and then sealed off in a Pyrex glass system in which it was possible to compress the mixture into a small bulb whose volume was nearly 1 cm³. A sketch of this system is shown in Fig. 1. The sample was isolated from a tank of compressed nitrogen by a mercury U tube. The compression of the sample could be adjusted by means of a pressure-reducing valve on the nitrogen tank.

This sample of He³ was prepared by Dr. Aaron Novick of the Chemistry Division of the Argonne National Laboratory, who carried through the extraction of tritium from irradiated Li⁶. The detection of the magnetic resonance signal from this sample was reported shortly thereafter.¹

The initial experiments were made with a mixture of equal volumes of oxygen and He³ at a total pressure of 20 atmospheres. Later, hydrogen gas was added to this mixture through the mercury U tube by means of a fine stainless steel capillary which was flexible enough to be bent around the bend in the U tube. The volumes of the gases He³, O₂, and H₂ were then in the proportions 1:1:0.37, respectively, and the experiments were carried out at a pressure of 24 atmospheres.



FIG. 3. Block diagram of radiofrequency system for simultaneous observation of magnetic resonance in He³ and H₂O. The means for precise measurement of frequency are also shown.

EXPERIMENTAL METHOD

The He³ sample was located within a small r-f coil and placed between the poles of an electromagnet. The magnetic field was modulated by a small amount by means of a 30-cycle current in a pair of modulation coils wound around the pole tips. At a steady value of the field sufficiently near the correct value, the magnetic resonance effect could be expected to produce a 30-cycle modulation of the radiofrequency signal. The problem was to detect this signal and distinguish it from the noise background.

The coil was made a part of a bridge circuit between an r-f source and detector. By balancing the bridge, the carrier signal level was reduced enough so that it would not overload the detector. A narrow band 30cycle lock-in amplifier was used which responded to the 30-cycle modulation produced by the magnetic resonance effect and discriminated strongly against noise and other undesired signals. This is the method of Bloembergen, Purcell, and Pound,⁵ and differs only in the type of bridge used. The weakness of the signal made it necessary to adjust the experimental parameters with considerable care. In what follows the experimental conditions will be elucidated in sufficient detail to permit a reasonable evaluation of all the factors which affect the signal to noise ratio (Eq. (4)).

The coil was made of 12 turns of No. 18 enameled copper wire, and fitted snugly over the outside of the small Pyrex bulb in which the sample was contained. The coil was 14 mm long and had an inside diameter of 9 mm. The sample bulb extended beyond the coil by about 2 mm at each end. Since the wall thickness of the bulb was about 1 mm, the filling factor ζ was 0.46. The volume of the coil V was 1.17 cm³.

In order to provide electrical shielding, this assembly was mounted inside a cavity in a copper block. The sample was centered in this cavity by means of small Bakelite wedges which served to reduce microphonic effects which would result from the vibration of the coil.

TWIN "T" BRIDGE

The coil was a part of a radiofrequency twin "T" bridge,⁹ connection to which was made with copper coaxial line, 1 foot long, $\frac{3}{8}$ -in. O.D., and having a capacity of 16 mmf. The circuit for the twin "T" bridge is shown in Fig. 2. This bridge has the advantage that the reactive and the resistance balance are controlled separately and independently by the variable condensers *C* and *C'*, respectively. The balance conditions are

reactive balance:
$$\frac{1}{\omega^2 L} = C + C_1 + C_2 \left(1 + \frac{C_1}{C_1'} \right), \quad (5)$$

resistive balance:
$$\frac{1}{R} = \omega^2 C_1 C_2 \left(1 + \frac{C'}{C_1'} \right) R'. \quad (6)$$

⁹ W. N. Tuttle, Proc. I.R.E. 28, 23 (1940).

Here, R is the equivalent shunt resistance of the coil, ω is the angular frequency. The Q of the coil is the ratio $R/\omega L$ and may be obtained from the circuit constants at balance. The resistor R_L was inserted to terminate properly the coaxial line from the oscillator and thereby facilitate input voltage measurements. The condensers C and C' were of the conventional variable type, but were provided with special trimmers. Each trimmer was made of two small disks arranged with a fine slowmotion screw which could advance one toward the other. These made it easy to obtain a balance of better than 1 part in 10⁴ for both the resistive and the reactive components. Since the balance of the bridge is frequency dependent, care was taken to avoid frequency modulation effects in the r-f generator. Best results were obtained with a crystal-controlled oscillator, and this was used in most of the work with the He³ sample.

The conditions of operation of the bridge were determined by balancing it at 20.22 megacycles and then measuring the capacity of C and C'. The values C=65mmf and C'=39 mmf were obtained. The first value includes 18 mmf of distributed capacity in the connections to the coil. Equations (5) and (6) lead to $L=0.89\mu h$ and Q=240. The last value is fairly approximate because of an uncertainty in the values of C_1 and C_2 . The r-f voltage across the coil may be obtained from a knowledge of the input voltage applied to the bridge by means of the following formula, obtained from an analysis of the bridge circuit at balance.

$$\frac{|e_e|}{|e_s|} = \left\{ \left(\frac{C_2}{C_1'}\right)^2 + \left[\omega C_2 \left(1 + \frac{C'}{C_1'}\right)R'\right]^2 \right\}^{-\frac{1}{2}}, \quad (7)$$

which, in the present case, yields $|e_c|/|e_s| = 1.74$.

An analysis of the twin "T" bridge showed that the signal-to-noise ratio at the coil is reduced by a factor of 1.3 at the output terminals. The signal from the bridge was detected with a National HRO-5A radio



FIG. 4. Photograph of the experimental arrangement: The magnet has a 5-in. diameter pole and a 1-in. gap, and is shown with the He^3 sample in place.

receiver. The noise figure of this receiver is reported to be about 12. This noise figure was improved by preceding the receiver with a single tuned preamplifier, using a 6AK5 tube so that a reasonable value for the noise figure of the detecting system including the bridge was 4.

A panoramic adapter connected to the output of the first detector allowed visual observations of the carrier signal and proved to be exceedingly useful in balancing the bridge and in the rapid adjustment of the circuits. The audio output of the receiver had to be modified slightly to improve its response at 30 cycles. It was fed into a narrow band 30-cycle amplifier followed by a lock-in amplifier copied from one described by Dicke.¹⁰ This system had an equivalent band width of about $\frac{1}{2}$ cycle, and its response was recorded by means of an Esterline-Angus recording milliammeter. A block diagram of the general arrangement is shown in Fig. 3.

COMPARISON SAMPLE

All of this equipment was duplicated to permit simultaneous observations of the proton resonance in water. Various water samples were used, but in the last and best measurements, 0.06 cm^3 of water was used with a manganous ion concentration of 6.7×10^{17} ions/cm³. According to the data of Bloembergen, Purcell, and Pound,⁵ Fe⁺⁺⁺, which should have the same paramagnetic behavior as Mn⁺⁺, in this concentration produces a relaxation time of about 0.1 second. The center of this sample was 1.4 cm from the center of the He³ sample and electrically shielded from it. The r-f coil which was wound on the outside of the glass tube had 8 turns of No. 18 enameled copper with an inside diameter of 4 mm. This coil was operated at 26.54 megacycles.

The oscillator for the water sample was not crystal controlled as was that for the He³, but could be adjusted in frequency so that both resonances could be recorded simultaneously. The water signal was strong enough to be seen visually on a cathode-ray oscilloscope connected to the audio output of the radio receiver. The horizontal sweep of this oscilloscope was driven by the same 30-cycle oscillator which was used to modulate the magnet. Visual observation of the water signal was used to find the resonance value of the magnetic field quickly. A preamplifier was not needed for the water sample.

ELECTROMAGNET

The electromagnet used in this work is shown in the photograph of Fig. 4. It has 5-in. diameter poles with a 1-in. plane parallel gap. A high voltage winding was used to allow the use of a rectified and filtered d.c. supply capable of delivering 1 ampere at 2500 volts. This winding was cooled by circulating oil. In the experiments described here, it was operated at 8640 gauss with 0.695 ampere and at 6240 gauss with 0.430 ampere.

The magnet power supply and regulator is shown schematically in Fig. 5. The rectified output was well filtered to reduce to an unnoticeable extent the ripple in the current from the rectifier. The magnet current was regulated by means of a d.c. vacuum tube amplifier, the reference voltage for which was obtained from a manganin-wound 185-ohm resistor in series with the magnet. This resistor was cooled by the same oil which



FIG. 5. Electronically stabilized magnet current supply. At 2500 volts, the total magnet current was 1 ampere, and the magnetic field in the gap was 10,000 gauss.

¹⁰ R. H. Dicke, Rev. Sci. Inst. 17, 268 (1946).



FIG. 6. Typical records of simultaneous observation of the He³ and H₂O magnetic resonance signals with reactive unbalance of the bridge. The 30-cycle modulation amplitude was $H_m = 0.064$ gauss. The resonance was traversed four times in the above portion of the record, twice from above and twice from below. The jogs which appear in the pattern are due to an unsteadiness in the magnetic field of about 1 part in 10⁵. Other irregularities were due to noise. The H₂O receiver was more noisy than usual when this particular record was taken.



FIG. 7. Typical records of simultaneous observations of the H_2 and H_2O magnetic resonance signals with reactive unbalance of the bridge.

circulated through the magnet windings. The magnet current was adjusted by selecting the proper voltage from a battery box which bucked the voltage developed across this resistor, and then raising the input voltage of the high voltage power supply to the point where the regulator operated in the sensitive part of its range. A motor-driven tubular potentiometer in this battery circuit was arranged so that the bucking voltage could be automatically varied slowly and linearly through the resonance value of the field. In some cases, the rate was as small as 0.005 volt/cm corresponding to a magnetic field change of 0.25 gauss/min. in the vicinity of 6240 gauss. When the resonance value of the field was traversed in this way, the Esterline-Angus instruments would record the amplitude of the 30-cycle component of the magnetic resonance signal. The fluctuation in the regulated magnetic field which occurred during the traversal of a resonance was usually less than 1 part in 10⁵. These fluctuations are responsible for some of the irregularities in the resonance patterns which are evident in the sample records shown in Figs. 6 and 7.

A great deal of effort was devoted to the improvement of the uniformity of the magnetic field. Most of this was done by means of a lapping tool. The uniformity of the field was checked after each lapping operation by means of a small water sample having 7×10^{17} manganous ions per cm³. This probe was mounted on a micro manipulator and could be moved about between the poles and so explore the field. The spatial variation of the field was followed visually by observing the position of the magnetic resonance signal on the cathode-ray oscillograph screen. The best uniformity which was achieved was a variation of 1 part in 10⁵ over an area 1 inch in diameter. The mechanical stability of the magnet was not sufficient to maintain this degree of uniformity for very long. When the magnet was turned on again on the following day, the region which was uniform to this amount was noticeably smaller. Some improvement could be obtained by recycling the magnet, but the extended region of uniformity could only be regained by further lapping. Since even a fairly slight pressure on the magnet pole could alter the value of the field by 1 part in 10⁵, it was clear that it would be necessary to redesign the magnet to obtain a further improvement in the uniformity.

It was possible to obtain a measure of the inhomogeneity of the magnetic field over the He³ sample in the following way: A sample of water having the same size as the He³ sample was placed in the same position as was standard for the He³. Moreover, the radiofrequency coil was made the same as for He³. The water sample had added to it 7×10^{17} manganous ions per cm³ so that the relaxation time was about $\frac{1}{10}$ second. The proton resonance signal from this sample could be observed on the cathode-ray oscilloscope screen. These signals showed a number of oscillations. These occur when the inverse line width T_2 is not too small.^{6,11} The oscillations are due to beats between the radiofrequency signal from the oscillator and the magnetic resonance signal. The frequency of the latter varies as the magnetic field is shifted off resonance, and its amplitude decreases as $\exp(t/T_2)$. By analyzing the decay of the oscillations, it was found that the inverse line width was $T_2=1.6\times10^{-3}$ second. This is small compared to T_1 and, provided the r-f input to the bridge is small enough, it must be due almost entirely to an inhomogeneity in the magnetic field of 3.8 parts in 10⁶.

AD JUSTMENT

With proper adjustment of the amplifiers, the response of the Esterline-Angus instrument is proportional to the 30-cycle Fourier coefficient of the input signal. In the present experiments, the dispersion signal (obtained by reactive unbalance of the bridge) was used. This signal increases with increasing values of the rotating component of the r-f field H_1 . It is desirable to set $(\gamma H_1)^2 T_1 T_2 > 1$ for the largest signal amplitude. However, excessive values of H_1 broaden the signal in a manner unsuited to a precise location of the center. For any given value of H_1 , there is an optimum value for H_m , the amplitude of the 30-cycle modulation field. Again, a large response requires $(\gamma H_m)^2 T_2^2 > 1$. With excessive values of H_m , however, the signal as it appears on the Esterline-Angus record is broadened. There is also an optimum adjustment of the phase of the modulation current relative to the lock-in signal.

Because of the weakness of the He³ signal, it was necessary to accept some amount of broadening in the interest of signal amplitude. What seemed to be an optimum compromise was found by empirical adjustment of H_1 and H_m . In the final experiments, the values were found to be $H_1=0.005$ gauss, $H_m=0.064$ gauss. The value of H_1 was obtained from a measurement of the r-f voltage output to the bridge ($e_s=0.05$ volt r.m.s.) and by making use of (7) and the approximate formula,

$$H_{\mathbf{i}} \cong \left(\frac{2\pi e_c^2 \times 10^7}{\omega^2 LV}\right)^{\frac{1}{2}}.$$
(8)

The value of H_m was obtained by measuring with an audio frequency vacuum tube voltmeter the 30-cycle voltage induced in a pick-up coil of known number of turns and area inserted between the poles of the magnet at the sample position.

A reliable measurement of the relaxation time was not made in the present work. However, the response obtained with the above value of T_2 and several values of H_1 and H_m were consistent with $T_1 \cong 1$ second. These values lead to $a^2 = (\gamma H_m)^2 T_2 = 4$, $b^2 = (\gamma H_1)^2 T_1 T_2$

¹¹ B. A. Jacobsohn and R. K. Wangsness, Phys. Rev. 73, 942 (1948).

$\nu(H_2)$ cycles/sec.	ν(He³) cycles/sec.	v(He³)/v(H2)	H(He ³) – H(H ₂) gauss
26.543.725	20.221.180	0.7618064	+0.173
3.755	1,180	0.7618056	+0.184
4,648	1,180	0.7617799	-0.046
4,677	1,180	0.7617791	-0.065
4,702	1,180	0.7617784	-0.058
2,670	1,180	0.7618367	+0.391
2,683	1,180	0.7618363	+0.402
2,700	1,180	0.7618358	+0.388
3,816	1,180	0.7618038	+0.151
3,850	1,180	0.7618028	+0.132
3,880	1,180	0.7618020	+0.132
3,909	1,180	0.7618011	+0.132
3,944	1,180	0.7618001	+0.115
3,963	1,180	0.7617996	+0.110
3,982	1,180	0.7617990	+0.108
4,720	1,180	0.7617779	-0.081
4,745	1,180	0.7617771	-0.113
4,760	1,180	0.7617767	-0.081
4,765	1,180	0.7617766	-0.079

TABLE I. The difference in the resonance value of the magnetic field for He^3 and H_2 for various frequencies.

=16, and

$$\alpha = \frac{1}{\pi} \int_{0}^{2\pi} \frac{ab \sin^2 p dp}{a^2 \sin^2 p + b^2 + 1} = 0.4,$$
(9)

the first harmonic Fourier coefficient of the Bloch dispersion formula. A somewhat larger signal was obtained with $H_m = 0.12$ gauss. This setting was used in some of the measurements. In this case, the value of α is 0.56.

The signal obtained from the hydrogen gas in the sample was considerably stronger than the He³ signal. This is due to the shorter relaxation time in H₂. A preamplifier was not used in detecting the H₂ signal. The H₂ signal disappeared completely when the pressure in the sample bulb was reduced to 1 atmosphere. This was also the case for the He³ signal. Thus, there was no doubt that these signals originated in these gases and not in any of the surrounding material.

FREQUENCY MEASUREMENT

Precise measurements of frequency were made using a General Radio secondary frequency standard. This unit contains a temperature-controlled 100-kilocycle crystal oscillator followed by a set of multivibrators which produce square wave outputs at 100 kc and 10 kc, and also at 1000 cycles and 100 cycles. The output of the 100-kc multivibrator contains useful harmonics of 100 kc beyond 36 megacycles. These harmonics were readily detected with a Hallicrafters SX42 communications receiver. By mixing the output of the 10-kc multivibrator with the 100-kc multivibrator, it was easy to interpolate at 10-kc intervals between the 100-kc markers. Finally, mixing one of the oscillators at the input to the receiver produced an audible beat of this signal with each of the two neighboring 10-kc markers. This beat signal was connected to the vertical plates of an oscilloscope. By adjusting the frequency of an audio oscillator connected to the horizontal plates of this oscilloscope, it was possible to obtain a circular pattern which symbolizes frequency equality of the two signals. The calibration of the audio oscillator was checked against the 100-cycle and 1000-cycle multivibrator outputs of the frequency standard; the frequency standard itself was checked against the 10megacycle transmissions of WWV. In this way, the magnetic resonance frequencies could be determined as closely as the audio oscillator dial could be read. No attempt was made to improve this reading to better than ± 10 cycles. The block diagram of this arrangement is shown in Fig. 3.

The drift in the frequency of the crystal oscillator was usually not detectable. The frequency of the variable frequency oscillator was measured every two minutes during a run. A drift of about 50 cycles in 10 minutes was typical and easily followed.

PROCEDURE

Measurements were made by observing the magnetic resonance signals of He³ and of H₂O on the Esterline-Angus recorders, at the same time. The frequencies of the He³ and the H₂O oscillators were fixed so that the two resonance signals would appear at almost the same value of the magnetic field. The magnet current was then slowly and automatically varied through the two resonances by means of the motor-driven potentiometer of Fig. 5. The two resonances were traversed first with the magnet current increasing and then with the magnet current decreasing. This was repeated a number of times. The measured quantity is the displacement between the centers of symmetry of the two signals as recorded on the Esterline-Angus instruments. A typical pair of records is shown in Fig. 6. This displacement was measured alternately with field increasing and with field decreasing, always taking the average of these. By proceeding in this way, certain systematic errors were eliminated which might arise from time lags in



FIG. 8. Ratio of frequency of He³ and H₂ oscillators plotted against difference in the resonance value of the magnetic field. The straight line is the least squares solution. The intercept at $H(He^3) - H(H_2) = 0$ is the ratio of the gyromagnetic ratios of He³ and H₂.

the apparatus or from non-linear inhomogeneities in the magnet field at the two samples.

The H_2O oscillator was then connected to both bridges so that the signal from the hydrogen gas which was mixed with the He³ sample could be compared with the signal from the water sample. This is essentially a measurement of the difference in the value of the magnetic field at the positions of the two samples. The significant quantity is the difference between the displacements He³-H₂O and H₂-H₂O. This difference is a true measure of the difference in the resonance values of the magnetic field for He^3 and H_2 for any given pair of frequencies. Since both the He³ and the H₂ signals were recorded with the same apparatus, possible instrumental errors were largely eliminated. The frequency of the H₂ oscillator was adjusted so as to give both positive and negative values of the He^3-H_2 difference. Thereby, the frequency ratio for which the resonance value of the magnetic field for He³ and H₂ was the same could be established by interpolation. Observations made in this way on March 8 and 9, 1949, are recorded in Table I. The measurements were taken with the magnetic field near 6234 gauss. Each difference in the resonance value of the magnetic field for He³ and H₂ was obtained by taking the average of the pair of displacements found for $He^3 - H_2O$ with increasing and with decreasing magnet currents, and subtracting the average displacement found for H₂ $-H_2O$ which was -0.079 gauss. The value of $\nu(\text{He}^3)/$ $\nu(H_2)$ at $H(He^3) - H(H_2) = 0$, was found by a least squares solution to be

$$\frac{\nu_0(\text{He}^3)}{\nu_0(\text{H}_2)} = 0.76178649 \pm 0.00000045.$$

The precision measure indicated above is merely a measure of the self-consistency of the observations. The experimental error is certainly greater than this.

A search was made for obvious systematic errors in the experiment. One possibility was an effect of the slight asymmetry in the signal as recorded on the Esterline-Angus instrument. This was due in part to friction in the recording pen and in part to residual charge effects in the condensers of the lock-in amplifier. The presence of the asymmetry made it uncertain how to judge correctly the center of the magnetic resonance pattern. In the above experiment, the signal due to H_2 was about twice as large as that from He³, thereby opening the possibility of a small systematic error in the judgment of the centers of the two patterns.

Accordingly, a new experiment was carried out and care was taken to adjust the receiver gain when observing H₂ to give a signal amplitude more nearly equal to that of He³. Several other parameters were changed as well. The 30-cycle modulation amplitude was reduced from 0.123 gauss to 0.064 gauss, and the rate at which the magnetic field was varied was reduced from 0.600 gauss/minute to 0.252 gauss/minute. Typical

TABLE II. The difference in the resonance value of the magnetic field of He^3 and H_2 for various frequencies.

$\nu(H_2)$ cycles/sec.	$\nu({\rm He^3})$ cycles/sec.	$ u(H_2)/\nu(He^3) $	$H(He^3) - H(H_2)$ gauss
26,544,266	20,221,180	0.7617909	+0.037
240	í í180	916	+0.051
225	180	921	+0.053
215	180	924	+0.038
205	180	927	+0.039
900	180	727	-0.111
900	180	727	-0.119
900	180	727	-0.127
900	180	727	-0.115
900	180	727	-0.107

records for He^3 and H_2O and for H_2 and H_2O are given in Figs. 6 and 7, respectively. The observations are collected in Table II. In this case, the result was

$$\frac{\nu_0(\text{He}^3)}{\nu_0(\text{H}_2)} = 0.76178650 \pm 0.00000025,$$

in excellent agreement with the previous result. This argues against the presence of instrumental systematic errors due to obvious causes.

In Fig. 8, all the above measurements are plotted, together with the least squares straight line solution. The result is

$$\frac{\gamma(\text{He}^3)}{\gamma(\text{H}_2)} = 0.7617866 \pm 0.0000012.$$

In starting this result, a limit of error is given which is four times larger than the precision measure obtained by calculating $(\langle \epsilon^2 \rangle_{\text{Av}} / N)^{\frac{1}{2}}$ where ϵ^2 is the mean square deviation from the mean, and N is the number of observations. The limit is more than twice as large as the uncertainty in the frequency measurements, and more than 70 percent of all the observations fell within it.

The systematic error which arises from the fact that an oscillating rather than a rotating r-f field is used¹² was, because of the small value of H_1 used here, too small to be significant.

The paramagnetism of the oxygen present in the sample alters the value of the magnetic field at a given atom of He³. The time average value of the local field is increased by the fraction $4\pi\chi/3$ because the time-averaged situation of the He³ atom is that it is in a spherical cavity surrounded by a paramegnetic medium of volume susceptibility χ . If the sample vessel had a spherical shape, it would cause a *decrease* in the value of the local field by an amount $4\pi\chi/3$ so that the net result would be zero, at least in the first order in χ . For the non-spherical vessel used here, the fractional *decrease* in local field was about $\pi\chi$. Thus, the net increase in the local field for 10 atmospheres of oxygen at 20°C was about 0.5 part in 10⁶. However, since the situation

¹² F. Bloch and A. Siegert, Phys. Rev. 57, 522 (1940).

for the H_2 molecules was almost exactly the same, no correction was needed for this cause.

DIAMAGNETIC CORRECTION

The above frequency ratio is not strictly equal to the ratio of the nuclear gyromagnetic ratios of He³ and H¹, but refers rather to the gyromagnetic ratio of He³ atoms and H_2 molecules.

To obtain the nuclear gyromagnetic ratio, a correction has to be made for the shielding effect of the atomic electrons. Such corrections have been discussed by Lamb¹³ and can be made reliably, provided the electronic wave functions are sufficiently well known. Lamb's formula, derived for a spherically symmetric electronic distribution, may be written

$$\frac{H'}{H} = -\frac{1}{3}\alpha^2 Z \langle 1/r \rangle_{Av}, \qquad (10)$$

where H' is the magnetic field in the z direction produced by the electron at the nucleus, H is the external magnetic field (taken in the z direction), α is the fine structure constant $(1/\alpha = 137.02)$, Z is the number of electrons, and where the radial distance r is measured in units of the Bohr radius. In both H2 and He3, the shielding is due to two electrons so that the shielding in the two cases differs only in the value of $\langle 1/r \rangle_{Av}$. Although the electron distribution in H₂ is not spherically symmetric, the above formula should hold approximately, since, in the present case, it is required to average the correction over all orientations of the hydrogen molecule. Further consideration of this point is in progress.

For H_2 , the formula for the electron density given by Nordsieck¹⁴ was used to calculate $\langle 1/r \rangle_{AV}$. For He³, several of the simpler variational wave functions given by Hylleraas¹⁵ gave fairly concordant results. The reduction of the magnetic field at the nucleus produced by the electrons was found^{**} to be 32 parts in 10⁶ for H₂ and 70 parts in 10⁶ for He³. Accordingly, the ratio of the nuclear gyromagnetic ratios is increased to,

$|\gamma_{\rm He^3}|/|\gamma_p| = 0.761815.$

MEASUREMENT AT HIGHER FIELD

The earlier measurements in He³ were made at a higher value of the magnetic field, namely, 8641 gauss. At this time, the hydrogen had not yet been added to the He³ sample. Measurements were made in the same manner as described above except that only $He^3 - H_2O$ comparisons were made. Comparisons were made with tiny water samples placed at three different positions around the He³ sample and at a distance of about 1 cm from its center. A survey of the magnetic field was made using the probe so that differences in the value of the magnetic field at the positions of the samples could be taken into account. The result obtained was

$$\gamma(\text{He}^3)/\gamma(\text{H}_2\text{O}) = 0.761779 \pm 0.000014.$$

The uncertainty given here is due to the uncertainty in the knowledge of the magnetic field at the samples. This result is of interest because it shows that within the limits of error which are given, the ratio of the gyromagnetic ratios is independent of the magnitude of the magnetic field which is used in its measurement.

REMARKS

Villars¹⁶ has interpreted the anomalous moments of H³ and He³ as being due to the existence of an exchange moment in these nuclei. In an earlier note to this journal,¹⁷ the value of this exchange moment was calculated by correcting the observed moments for the effect of the orbital motions using the theory developed by Sachs and Schwinger¹⁸ and of Sachs.¹⁹ At that time, there was a possibility that the anomaly might be due to orbital motions. The moment of He³ is now known accurately enough so that this possibility can be definitely excluded.

It is to be hoped that this addition of precise knowledge about the magnetic moment of He³ will provide a clue to the nature of the nuclear field.

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¹³ W. E. Lamb, Phys. Rev. 60, 817 (1941).

¹⁴ A. Nordsieck, Phys. Rev. 58, 310 (1940).
¹⁵ E. A. Hylleraas, Zeits. f. Physik 54, 347 (1929).

^{**} I am indebted to Mr. Herbert Kanner for checking these calculations for me.

¹⁶ F. Villars, Phys. Rev. 72, 256 (1947); Helv. Phys. Acta 20, 476 (1947).

 ¹⁰ H. L. Anderson, Phys. Rev. **73**, 919 (1948).
 ¹⁸ R. G. Sachs and J. Schwinger, Phys. Rev. **70**, 41 (1946).
 ¹⁹ R. G. Sachs, Phys. Rev. **72**, 312 (1947).



FIG. 4. Photograph of the experimental arrangement: The magnet has a 5-in. diameter pole and a 1-in. gap, and is shown with the He^3 sample in place.





FIG. 6. Typical records of simultaneous observation of the He³ and H₂O magnetic resonance signals with reactive unbalance of the bridge. The 30-cycle modulation amplitude was $H_m = 0.064$ gauss. The resonance was traversed four times in the above portion of the record, twice from above and twice from below. The jogs which appear in the pattern are due to an unsteadiness in the magnetic field of about 1 part in 10⁵. Other irregularities were due to noise. The H₂O receiver was more noisy than usual when this particular record was taken.



Fig. 7. Typical records of simultaneous observations of the H_2 and H_2O magnetic resonance signals with reactive unbalance of the bridge.