

Note on the Decay of the π -Meson

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Assuming the symmetric coupling scheme proposed by Wheeler and Tiomno, and others, we have calculated the ratio of the decay rate π -meson \rightarrow electron + neutrino to the decay rate of π -meson \rightarrow μ -meson + neutrino. The electron-neutrino decay proceeds faster, in disagreement with experiment, unless the π -meson is pseudoscalar and the β -decay coupling is pseudovector. Hence if the symmetric coupling scheme is correct and no other direct couplings are introduced, the π -meson must be pseudoscalar and β -decay must be at least partially pseudovector. If symmetric coupling is not assumed, no conclusion of this kind can be drawn.

ALTHOUGH the π -meson appears to decay into a μ -meson and a neutrino at least 100 times faster than into an electron and neutrino, the latter process is to be expected according to certain formulations of meson theory. We have therefore calculated the rate of the decay $\pi \rightarrow e + \nu$, and wish to mention the possible bearing of our results on the nature of the π -meson and nuclear β -decay.

It is customary to assume that the π -meson is coupled directly to nucleons, in order to explain its production in nuclear collisions and to account, at least partially, for nuclear forces. Assuming further that the nucleons are Dirac particles, one then postulates a coupling of the form

$$G(\varphi_P + O\varphi_N)\psi_\pi, \quad (1)$$

or a similar coupling involving first derivatives of ψ_π , where O is some Dirac operator. This interaction leads to the real capture process

$$N + \pi^+ \rightarrow P^+,$$

but it also permits the virtual decomposition of the π :

$$\pi^+ \rightarrow P^+ + N_-, \quad (2)$$

where N_- signifies an anti-neutron.

On the other hand, to account for β -decay along the general lines of the Fermi or Gamow-Teller theories, one postulates the interaction

$$g(\varphi_N^\dagger A \varphi_P)(\varphi_e^\dagger A \varphi_\nu),$$

where A is also a Dirac operator. This interaction leads to the observed β -process

$$N \rightarrow P^+ + e^- + \nu,$$

but it also leads one to expect the reaction

$$P^+ + N_- \rightarrow e^+ + \nu. \quad (3)$$

The virtual decomposition (2) followed by (3) leads to the real decay

$$\pi^+ \rightarrow e^+ + \nu. \quad (4)$$

It can then be concluded that any theory which couples π -mesons to nucleons also predicts the $\pi \rightarrow (e, \nu)$ decay. This argument depends not on the existence of real

anti-neutrons, but only on the role of such particles in virtual processes.

Since (4) has not been observed experimentally, we have compared its rate with that of the observed decay

$$\pi^+ \rightarrow \mu^+ + \nu. \quad (5)$$

To do this, a further hypothesis must be introduced by postulating the nature of the field interaction responsible for (5). One may assume a direct coupling between the π - and μ -fields,¹ or one may assume that (5) goes indirectly. We have tested the symmetrical coupling scheme proposed by Wheeler and others,² according to which (5) goes indirectly. According to this scheme the following three processes

$$\mu\text{-capture} \quad \mu^- + P \rightarrow N + \nu, \quad (6a)$$

$$\beta\text{-decay} \quad N \rightarrow P + e^- + \nu, \quad (6b)$$

$$\mu\text{-decay} \quad \mu^- \rightarrow e^- + \nu + \nu, \quad (6c)$$

result from the direct couplings

$$g_a(\varphi_N^\dagger A \varphi_P)(\varphi_\nu^\dagger A \varphi_\mu), \quad (7a)$$

$$g_b(\varphi_N^\dagger B \varphi_P)(\varphi_\nu^\dagger B \varphi_e), \quad (7b)$$

$$g_c(\varphi_\nu^\dagger C \varphi_\mu)(\varphi_\nu^\dagger C \varphi_e). \quad (7c)$$

All fields in (6) are spinor fields; A , B , and C are Dirac operators.

It has been found³ that $g_a \cong g_b \cong g_c$. We have assumed

$$g_a = g_b = g_c$$

and

$$A = B = C. \quad (8)$$

These three couplings are thus assumed to be of the same nature and strength. According to the sym-

¹ R. E. Marshak and H. A. Bethe, *Phys. Rev.* **72**, 506 (1947); R. Latter and R. F. Christy, *Phys. Rev.* **75**, 1459 (1949); Taketani, Nakamura, Ono, and Sasaki, *Phys. Rev.* **76**, 60 (1949).

² J. Tiomno and J. A. Wheeler, *Rev. Mod. Phys.* **21**, 153 (1949); O. Klein, *Nature* **161**, 897 (1948); Lee, Rosenbluth, and Yang, *Phys. Rev.* **75**, 905 (1948).

³ Pseudoscalar β -decay does not give the agreement in coupling constants which makes the symmetric coupling scheme so attractive.

TABLE I. Ratio of $\pi \rightarrow (e, \nu)$ to $\pi \rightarrow (\mu, \nu)$ -decay for couplings (1) and (7).

	Type of β -decay				
	Scalar	P-scalar*	Vector	P-vector	Tensor
Meson					
Scalar	5.1	f	f	f	f
P-scalar	f	5.1	f	1.0×10^{-4}	f
Vector	f	f	4.0	f	2.4
P-vector	f	f	f	4.0	f

metrical scheme (5) is a second-order process

$$\pi^+ \rightarrow P^+ + N_-, \quad (9a)$$

$$P^+ + N_- \rightarrow \mu^+ + \nu. \quad (9b)$$

The matrix element for (9b) is contributed by (7a). We have compared the rates of the two second-order decays (4) and (5) for various operators in the fundamental interaction (7) and several meson field couplings in (1). The decay may progress through either of two intermediate states: (a) A π -meson disappears with the production of a pair of virtual nucleons which in turn are annihilated with the creation of a (μ, ν) -pair. (b) A vacuum fluctuation produces a (μ, ν) -pair and a nucleon pair. The latter disappear with the absorption of the π -meson. The transition probability is given by:

$$\begin{aligned} \tau^{-1} = 2\pi |H|^2 \rho(E) = \sum_{\sigma_P \sigma_N} 2\pi g^2 G^2 \left| \int d\mathbf{p} \sum_{\sigma_P \sigma_N} \left(\frac{1}{2\mu_\pi} \right)^{\frac{1}{2}} \right. \\ \times \left(\frac{\varphi_\pi \cdot \langle u_P^\dagger O u_N \rangle \langle u_N^\dagger A u_P \rangle \cdot \langle u_\mu^\dagger A u_\nu \rangle}{2E_P - \mu_\pi} \right. \\ \left. + \frac{\langle u_P^\dagger A u_N \rangle \cdot \langle u_\mu^\dagger A u_\nu \rangle \varphi_\pi \cdot \langle u_N^\dagger O u_P \rangle}{2E_P + \mu_\pi} \right)^2 \rho(E), \quad (10) \end{aligned}$$

$\hbar = c = 1$. μ_π is the mass of the π -meson. u_P, u_N, u_μ, u_ν are the spinors for free protons, anti-neutrons, mesons, and anti-neutrinos, respectively. φ_π is the amplitude of the π -meson wave function. $2E_P$ is the energy of the virtual nucleon pair and the integration is over all momenta of the nucleons. This integration leads to a divergent answer for the two rates of decay $\pi \rightarrow e + \nu$ and $\pi \rightarrow \mu + \nu$. The same divergent integral appears in both. If this integral can be made finite the ratio of decay rates is independent of how this is accomplished.

Taking $\mu_P = \mu_N$ and using the masses $\mu_\pi = 286$, $\mu_\mu = 215$, $\mu_e = 1$, $\mu_\nu = 0$, the ratio of density of final states for the competing decays is

$$\frac{\rho_e(E) \text{ decay into } (e, \nu)}{\rho_\mu(E) \text{ decay into } (\mu, \nu)} = 3.3$$

Table I gives the ratio of $\pi \rightarrow (e, \nu)$ to $\pi \rightarrow (\mu, \nu)$ -decay for couplings (1) and (7). Table II gives this ratio when the coupling of (1) involves first derivatives of ψ_π . f signifies a forbidden transition for both processes. In most instances the decay of the π -meson is forbidden. The symmetric coupling scheme is in agreement with

 TABLE II. Ratio of $\pi \rightarrow (e, \nu)$ to $\pi \rightarrow (\mu, \nu)$ -decay when the coupling of (1) involves first derivatives of ψ_π .

Meson	Type of β -decay				
	Scalar	P-scalar	Vector	P-vector	Tensor
Scalar	f	f	5.1	f	f
P-scalar	f	f	f	1.0×10^{-4}	f
Vector	f	f	f	f	2.4
P-vector	f	f	f	f	2.4

experimental facts only if the π -meson is pseudoscalar (with either pseudoscalar or pseudovector coupling to nucleons) and the β -decay coupling contains a pseudovector term. According to the table the β -decay coupling may also contain arbitrary admixtures of scalar, vector, and tensor terms, since these do not contribute to the $\pi - \mu$ or $\pi - e$ decays. This result is in agreement with Wigner's conclusion⁴ that the evidence from nuclear β -decay requires a pseudovector coupling and does not exclude scalar or pseudoscalar terms.

The transition probability for the pseudoscalar meson decay is

$$\begin{aligned} \tau^{-1} = \left(\frac{G^2}{\hbar c} \right) \left(g^2 \frac{\mu_\pi^2 \mu_P^2 c^8}{\hbar^6 c^6} \right) \left(\frac{\mu_\pi c^2}{\hbar} \right) \left(\frac{\mu_\pi^4 - \mu_\mu^4}{\mu_\pi^4} \right) \\ \times \left(\frac{\mu_\pi^2 - \mu_\mu^2}{64\pi^5 \mu_\pi^2} \right) \left(1 - \frac{c p_\mu}{E_\mu} \right) \\ \times \left[\ln(\theta + \{\theta^2 + 1\}^{\frac{1}{2}}) - \frac{\theta}{(\theta^2 + 1)^{\frac{1}{2}}} \right]^2, \quad (11) \end{aligned}$$

where θ is the cut-off momentum in units of $\mu_P C$. A covariant calculation using an invariant cut-off procedure of Feynman⁵ leads to the same expression with the bracket replaced by

$$\left[\ln \left(\frac{\lambda}{\mu_P} \right) + 1 - \frac{(4\mu_P^2 - \mu_\pi^2)^{\frac{1}{2}}}{\mu_\pi} \sin^{-1} \left(\frac{\mu_\pi}{2\mu_P} \right) \right]^2, \quad (11a)$$

where terms of order μ_P/λ and higher have been dropped. λ is a cut-off with the dimensions of mass. For large cut-offs both expressions are essentially equal. Although the expressions (11) are only logarithmically divergent, they are sensitive to the choice of cut-off. Choosing $G^2/\hbar c = \frac{1}{3}$ and $g = 2 \times 10^{-49}$ erg-cm³, if $\theta = 1$, $\tau = 9 \times 10^{-6}$ sec.; $\theta = 10$, $\tau = 6 \times 10^{-8}$ sec.; $\theta = 100$, $\tau = 1 \times 10^{-8}$ sec. Steinberger⁶ has also calculated the lifetime for this decay, and after cutting off with Pauli regulators, he finds 2×10^{-2} sec. (pseudoscalar coupling). In view of the sensitivity of the result to the cut-off procedure, and in the absence of any reliable theory, we believe that no conclusion, either for or against symmetric coupling, can be drawn from a consideration of the *absolute* rate. We emphasize, however, that the *ratios* in the table are independent of the divergent integrals. The above calculation is also open to the

⁴ E. P. Wigner, Phys. Rev., to be published.

⁵ R. P. Feynman, Phys. Rev. **74**, 1430 (1948).

⁶ J. Steinberger, Phys. Rev., to be published.

objections (1) that it is based on the Dirac theory for the nucleons, including, in particular, the virtual role of the anti-nucleons and (2) that it employs the usual perturbation methods of field theory. If these objections are tentatively ignored, the calculation suggests that the symmetric coupling scheme is tenable only if the π -meson is pseudoscalar and β -decay coupling is at least partially pseudovector. On the other hand, it is possible that a correct calculation of the divergent integrals will not agree with experiment even in the pseudoscalar case. In that event it would seem natural

to reject the symmetric coupling scheme in favor of direct π - μ -coupling.⁷

We wish to thank Dr. R. F. Christy for helpful discussion.

⁷ According to Lopes, Phys. Rev. **74**, 1722 (1948) the hypothesis of direct π - μ -coupling demands a pseudoscalar meson with pseudo-vector coupling if one wishes to retain the Yukawa scheme for β -decay. Yukawa (Rev. Mod. Phys. **21**, 474 (1949)) has pointed out that this peculiarity of the pseudoscalar case can also be inferred from the earlier work of Sakata. However, according to more detailed calculations the direct coupling scheme does not give satisfactory lifetimes for β -decay, even if the π -meson is pseudoscalar.

Precise Measurement of the Gyromagnetic Ratio of He³

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The gyromagnetic ratio of He³ was compared to that of H₂ gas by the magnetic resonance method with the result

$$|\gamma(\text{He}^3)|/|\gamma(\text{H}_2)| = 0.7617866 \pm 0.0000012.$$

To obtain the nuclear gyromagnetic ratios, a diamagnetic correction must be made which raises this by approximately 38 parts in 10⁶ to

$$|\gamma_{\text{He}^3}|/|\gamma_p| = 0.761815.$$

Intensity measurements were consistent with a spin of $\frac{1}{2}$. In nuclear magnetons, the magnetic moment $\mu_{\text{He}^3} = (-)2.12815$ based on the value $\mu_p = 2.79353$ obtained by Taub and Kusch. The negative sign has not yet been verified.

A PRELIMINARY measurement of the gyromagnetic ratio of He³ has already been reported in this journal.¹ In the present paper, a new series of measurements also based on the magnetic resonance method^{2,3} is reported using the same sample of He³, but in which the capabilities of the method were more fully exploited.

MAGNETIC RESONANCE METHOD

The technique used was a refinement of the one which was used in a measurement of H³,⁴ being quite similar to that developed by Bloembergen, Purcell, and Pound.⁵ The sample formed the core of a small coil which was one element of a radiofrequency bridge. The coil and sample were placed in the magnetic field between the poles of an electromagnet, with the axis of the coil at right angles to that of the poles of the magnet. Under these circumstances, a nucleus with spin I may orient itself in $2I+1$ ways with respect to the direction of the magnetic field, each having a different energy value. Transitions between neighboring states will be induced when the frequency of the signal applied to the coil

satisfies the resonance condition,

$$2\pi\nu = |\gamma|H. \quad (1)$$

Here, $\gamma = \mu/\hbar I$ is the ratio of the magnetic moment to the angular momentum of the nucleus, ν is the frequency in cycles per second, and H is the magnetic field intensity in gauss at the nucleus.

The magnetic field is modulated about the resonance value by a small amount at 30 cycles per second. Under suitable conditions,* the transitions which occur each time the resonance value of the field is traversed produce a change in the impedance of the coil and an unbalance of the bridge. Thus, the magnetic resonance effect produces a 30-cycle modulation in the radiofrequency signal which feeds the bridge, and this may be detected by means of an ordinary radio receiver. The extreme sharpness of the resonance which may be obtained makes possible the great accuracy of the method.

It is seen that a measurement of γ requires a measurement of both the frequency ν and the magnetic field H . Since ν can be measured with much greater accuracy than H , it is preferable and also convenient to compare the value of ν with that of some standard nucleus in

¹ H. L. Anderson and A. Novick, Phys. Rev. **73**, 919 (1948).

² Purcell, Torrey, and Pound, Phys. Rev. **69**, 37 (1946).

³ Bloch, Hansen, and Packard, Phys. Rev. **69**, 127 (1946); **70**, 474 (1946); F. Bloch, Phys. Rev. **70**, 460 (1946).

⁴ H. L. Anderson and A. Novick, Phys. Rev. **71**, 372 (1947).

⁵ Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).

* A detailed exposition of the theory of the method and the conditions for observing the magnetic resonance effect may be found in the pioneer papers, references 2 and 5.