PHYSICAL REVIEW VOLUME 76, NUMBER 10 NOVEMBER 15, 1949

# Relative Cross Sections for Pair Production at 17.6 Mev

R. L. WALKER
*Cornell University, Ithaca, New York*

(Received August 5, 1949)

The relative cross sections for production of electron pairs by 17.6 Mev gamma-rays have been measured for Li, C, Al, Cu, Sn, and Pb. A gamma-ray pair spectrometer was used to count the number of pairs emitted from thin radiators of these materials when irradiated by 17.6 Mev gamma-rays produced in the $Li^7(p,\gamma)Be^8$ reaction. For each material, the number of pairs counted per atom of radiator has been extrapolated to a radiator of zero thickness, in order to obtain the relative pair cross section. From these measurements some information can be obtained about the probability of pair production in the field of the atomic electrons, and about the errors in the theoretical pair cross sections of heavy elements, which presumably arise from the use of the Born approximation. It is found that the ratio of the pair cross section of an electron to that of a nucleus of unit charge is $0.8\pm0.3$. The data indicate that the pair cross sections of heavier elements are lower than predicted by the Born approximation, in qualitative agreement with measurements of the total absorption cross sections.

## I. INTRODUCTION

THE cross sections of different elements for the production of electron pairs by high energy gamma-rays are roughly proportional to $Z^2$.[1,2] This proportionality is not exact, however, for threee different reasons. These are the screening of the nuclear coulomb field by the atomic electrons; production of pairs in the field of the atomic electrons; and an inadequacy of the Born approximation, by means of which the $Z^2$ dependence has been obtained in the Bethe-Heitler theory of pair production.

The effect of screening is small at 17.6 Mev and has been calculated with reasonable accuracy by Bethe and Heitler.[1] It may be taken into account by a correction factor $(1-S(Z))$ in the pair cross section. The screening correction, $S(Z)$, varies from 0.01 for lithium, to 0.05 for lead at 17.6 Mev.

The production of pairs in the field of the atomic electrons contributes a term proportional to $Z$ in the cross section per atom. This means the total pair cross section is proportional to $Z(Z+A)$ instead of $Z^2$, where $A$ is the pair cross section of an electron relative to that of a nucleus of unit charge. For light elements, the effect of the electrons may be rather large. The probability of pair production in the field of an electron has been calculated in a number of papers,[3-6] but its value at 17.6 Mev is not known with certainty. Borsellino[3] gives the (interpolated) value $A=0.68$ at 17.6 Mev, for a free electron. The approximate calculations of Wheeler and Lamb[4] take into account the binding of the electrons in the atom, but are valid for light elements only at higher energies. They give for heavier elements, however, a value $A\approx1.0$.

Finally, the $Z^2$ dependence of the pair cross section for a bare nucleus depends upon the use of the Born approximation in the theory of Bethe and Heitler.[1] The condition for validity of the Born approximation is $Ze^2/\hbar v\ll1$ where $v$ is the velocity of the electron (or positron). In the most favorable case of relativistic velocities, this reduces to $Z/137\ll1$, which is true for light elements, but not for heavy ones. Thus deviations from the Born approximation values may be expected for heavy elements. These deviations will be expressed by a correction factor $(1-B(Z))$.

Combining the three effects discussed above, one may write the total pair cross section in the form:

$$\sigma_{\text{pair}}\sim Z(Z+A)(1-S(Z))(1-B(Z)). \qquad (1)$$

From measurements of the relative pair cross sections of different elements, some information may be obtained about the electron pair cross section, $A$, and about the correction $B(Z)$ to the Born approximation values.

The above remarks apply not only to the integral pair cross section, but also to the differential cross section, $\sigma(E^+)dE^+$, for production of a pair with positron energy between $E^+$ and $E^++dE^+$. However, the magnitude of the three "corrections," $A$, $S(Z)$, and $B(Z)$, may be different in different regions of the differential cross section, i.e., different $E^+$. For example, the symmetry of the differential cross section with respect to the positron and electron energies[1,2] is a result of the use of the Born approximation, whereas the true curve is probably asymmetrical. The correction to the Born approximation value is probably different in the region of small positron energies from that at small electron energies.

It will be seen later that in the present experiments the relative cross section measured for each element is not the integral cross section, but an integral over the central region of the differential cross section.

$$\sigma_{\text{obs}}\approx\int_{k/4}^{3k/4}\sigma(E^+)dE^+. \qquad (2)$$

This should be kept in mind in interpreting the results.

---

[1] H. A. Bethe and W. Heitler, Proc. Roy. Soc. 146, 83 (1934).
[2] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1936).
[3] A. Borsellino, Helv. Phys. Acta 20, 136 (1947).
[4] J. A. Wheeler and W. E. Lamb, Phys. Rev. 55, 858 (1939).
[5] K. M. Watson, Phys. Rev. 72, 1060 (1947).
[6] V. Votruba, Phys. Rev. 73, 1468 (1948).

In addition, pairs produced in the field of an electron are not counted if the "recoil electron" has a high energy (>approx. 1 Mev). Thus these are not included in the observed cross section, but they are probably small in number.[4]

Finally it should be pointed out that information concerning the electron pair cross section and the "Born approximation correction" can also be obtained from measurements of the total gamma-ray absorption cross sections of different elements.[7-9] These measurements have the advantage of being more accurate and less subject to systematic errors than the present experiment. However, at energies below 20 or 30 Mev the effect of pair production in the field of the atomic electrons is much less important in the absorption cross section than in the pair cross section alone. This arises from the fact that Compton scattering contributes more than pair production to the absorption in very light elements. Also, in heavy elements at these energies, nuclear photo-disintegration may contribute a few percent to the total absorption cross section[8, 9] but does not affect the relative pair cross section.

## II. APPARATUS AND PROCEDURE

Pairs produced in radiators of different elements were counted by means of a magnetic pair spectrometer which has been previously described,[10] and which is shown in Fig. 1. Radiators used in these experiments were 3 by 11 cm in area, and their thicknesses are given in Table I. By counting coincidences between any of the four positron counters and any of the four negative electron counters, the spectrometer measures pairs in seven different energy intervals, separated in energy by about 4 percent. In these experiments, the magnetic field of the spectrometer was always adjusted for each radiator so that the center of the observed 17.6 Mev gamma-ray line fell midway between the central energy channel, number 4, and an adjacent channel, number 3. By adding the counts in channels 3 and 4 (after correcting for a difference in statistical weights arising from the fact that channel 4 is fed by coincidences from four counter pairs whereas channel 3 is fed by only three counter pairs), the flat-topped "resolution function" shown in Fig. 2a may be obtained. The flat top is desirable in order to minimize the effects of small variations in the magnetic field, and of energy loss and scattering of the pair electrons in the radiator. A still broader resolution function, shown in Fig. 2b may be obtained by adding the counts from four channels, numbers 2, 3, 4, and 5 (each corrected for statistical weights). Both of these resolution functions have been used in analyzing the data.

The width of the radiator, and its position with respect to the counters determine the region of the differential pair cross section[1, 2] from which pairs may be counted. For example, no pairs in which one electron has a very low energy can be recorded. (The minimum radius of curvature for an electron which is counted is given by one-half the distance from one end of the radiator to the nearest counter.) It may be shown that the counting rate of the spectrometer for a thin radiator is proportional to the integral over the central region of the differential cross section:

$$\sigma_{\text{obs}} \approx \int_{k/4}^{3k/4} \sigma(E^+)dE^+. \tag{2}$$

Actually, the limits of this integral are not exact, and they depend somewhat upon the particular counter pair giving the coincidences. The important point, however, is that the measured cross section, (2), is different from the total integral cross section. This difference would not be significant, of course, if the unscreened differential cross section had the same shape for all elements, as predicted by the theory of Bethe and Heitler. It is important, therefore, only to the extent that the Born approximation gives wrong results.

The gamma-rays used in these experiments were produced by bombarding thick lithium targets with 0.46 Mev protons from the Cornell cyclotron. The gamma-rays thus produced from the 440-kev resonance in the $\text{Li}^7(p,\gamma)\text{Be}^8$ reaction have a spectrum which consists of a sharp line at 17.6 Mev, and an apparently broad line near 14.8 Mev.[10] The possible effect on the measurements of the lower energy component (which has a relative intensity about half that of the 17.6 Mev line) will be mentioned later.

The relative number of pairs emitted from each radiator was measured by comparing the counting rate of the spectrometer using this radiator with that obtained from a "standard" 0.006-in. Pb radiator. For

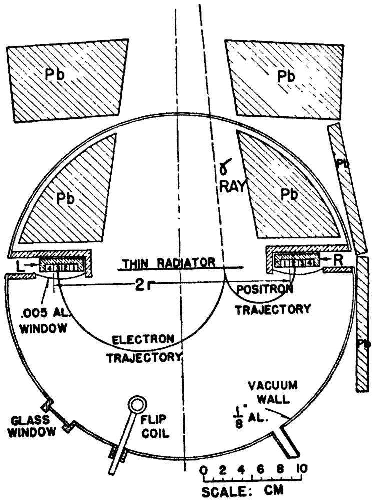

FIG. 1. Diagram of the gamma-ray pair spectrometer. (Horizontal section through the four-inch magnet gap.) The distance between cyclotron target and radiator was about 58 cm in these experiments.

---

[7] J. L. Lawson, Phys. Rev. **75**, 433 (1949).

[8] R. L. Walker, Phys. Rev. **76**, 527 (1949).

[9] G. D. Adams, Phys. Rev. **74**, 1707 (1948).

[10] R. L. Walker and B. D. McDaniel, Phys. Rev. **74**, 315 (1948).

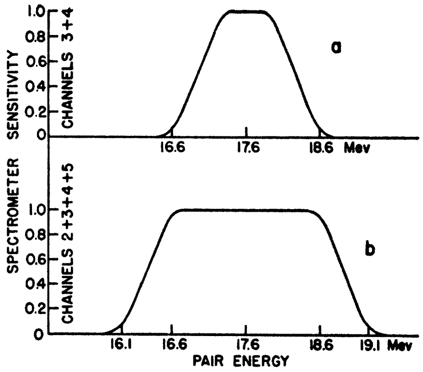

FIG. 2. Resolution functions of the spectrometer under the conditions used in the present experiments: (a) Counting rates of channels 3 and 4 added together, (b) counting rates of channels 2, 3, 4, and 5 added.

this comparison the gamma-ray intensity was monitored by two small Geiger counters mounted in 4.5 cm lead shields. To avoid systematic counting errors, the counting rates of the two radiators being compared were measured alternately, until 7 to 12 readings had been obtained for each one. Enough counts were observed to obtain a statistical accuracy of about 2.0 percent in the ratio of the counting rate of each radiator to that of the standard.

From this ratio has been subtracted a background, which ranged from one to ten percent, depending on the radiator. This background was the counting rate obtained with no radiator in the spectrometer (relative to the counting rate with the standard radiator). Some of the background arises from pairs produced in the mounting frame and Scotch Tape used to support the radiators.

## III. EFFECTS OF SCATTERING AND ENERGY LOSS OF THE PAIR ELECTRONS IN IN THE RADIATION

In order to obtain the ratio of the pair cross sections of different materials from the ratio of counting rates with radiators of these materials in the spectrometer, it is necessary that the spectrometer efficiency in counting the pairs actually produced be the same for the different materials. Since this efficiency is affected by energy loss and scattering of the pair electrons in the radiator, an extrapolation was made for each element to a radiator of zero thickness. In making this extrapolation, it is of interest to investigate the effects of energy loss and scattering.

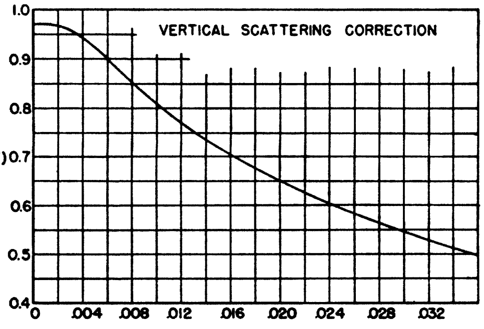

FIG. 3. Vertical scattering correction. $f(T)$ is the fraction of pairs produced which is *not* lost because of vertical scattering.

Scattering of the pair electrons in the radiator has two effects on the efficiency. One of these is that a pair will not be counted if either electron is scattered so far in the vertical direction that it strikes a magnet pole and thus does not enter the counter window. The fraction $f(T)$ of pairs *not* lost because of this vertical scattering has been calculated as a function of the radiator thickness, $T$, and is shown in Fig. 3. This calculation has been made from the known geometry of the spectrometer, and from the angular distribution of electrons multiply scattered in emerging from the radiator.[11] The angles of emission in the pair production process itself[12] are smaller than those arising from multiple scattering except for very thin radiators. They have been included in a rough way in the curve of Fig. 3. As an aid in extrapolating the data to zero thickness of radiator, corrections have been made for the fraction of pairs lost by vertical scattering. This might appear to introduce significant errors, since the calculated curve of Fig. 3 is certainly not exact, especially at large radiator thicknesses. However, it is felt that the errors in the relative cross sections introduced by this procedure are unimportant since the same correction curve, $f(T)$, has been applied to all elements.

The second effect of scattering of the pair electrons is an apparent lowering of the energy of the pair. This results from the properties of 180 degrees focusing, by means of which the spectrometer measures the momentum component of an electron normal to the radiator. This apparent energy loss, combined with the actual energy loss by inelastic collisions, means that the apparent spectrum of pairs produced by 17.6 Mev gamma-rays is not a sharp line at 17.6 Mev, but a continuous spectrum in a small energy interval just below 17.6 Mev. If this energy interval is larger than the flat region of the resolution functions shown in Fig. 2, then a loss in efficiency will result from this "line broadening." Sample corrections for these effects have been calculated, but no corrections have been applied to the data. The calculations indicate a sizable loss of efficiency for thicker radiators of light elements if the narrow resolution function 2a is used. No significant losses (except perhaps for the two or three heaviest radiators) are expected if the wider resolution function 2b is used.

## IV. RESULTS

The relative pair counting rate per atom for each radiator measured is shown in Table I. The column $R_{3-4}/N$ gives this data as obtained from channels 3 and

---

[11] See, for example, B. Rossi and K. Greisen, Rev. Mod. Phys. 13, 263–265 (1941).

[12] M. Stearns, Phys. Rev. 76, 836 (1949).

4 of the spectrometer, corresponding to the narrow resolution function 2a. The column headed $R_{2-5}/N$ gives the data obtained from channels 2, 3, 4, and 5, corresponding to the wide resolution function 2b. Also shown in Table I are the vertical scattering corrections $f(T)$.

The relative counting rates, corrected for vertical scattering losses, have been plotted in Fig. 4 as a function of the radiator thickness for each material. These curves have then been extrapolated to zero thickness to obtain the relative pair cross sections. The slopes of the curves for light elements using the narrow resolution function 2a are probably a result of energy loss in the radiator. As discussed above, the effect of this energy loss is unimportant if the broad resolution function 2b is used, and the slopes of the corresponding curves are seen to be greatly reduced. In fact, the curves for heavy elements have a slight positive slope. This may reflect errors in the vertical scattering correction $f(T)$, but it may also be a result of the broad 14.8 Mev lithium gamma-ray line. Because of the way in which the magnetic field was adjusted for each radiator the fraction of counts arising from the tail of the 14.8 Mev line would be expected to increase with the radiator thickness.

TABLE I. Relative counting rate per atom for each radiator.*

| Radiator material | Thickness (g/cm²) | Thickness (Radiation lengths) | $f(T)$ | $N$ ($10^{24}$ atoms | $R_{3-4}/N$ | $R_{2-5}/N$ | Statistical standard error |
|---|---|---|---|---|---|---|---|
| Li | 0.2061 | 0.00181 | 0.969 | 0.590 | 0.194 | 0.181 | 1.4% |
| C | 0.1138 | 0.00220 | 0.967 | 0.1883 | 0.703 | 0.620 | 1.7% |
| C | 0.1937 | 0.00374 | 0.947 | 0.3206 | 0.644 | 0.591 | 1.9% |
| C | 0.3462 | 0.00669 | 0.882 | 0.5731 | 0.548 | 0.567 | 2.1% |
| C | 0.5902 | 0.01141 | 0.781 | 0.9768 | 0.371 | 0.442 | 2.0% |
| Al | 0.1099 | 0.00419 | 0.940 | 0.0810 | 2.95 | 2.68 | 1.8% |
| Al | 0.2897 | 0.01104 | 0.787 | 0.2135 | 2.26 | 2.24 | 1.5% |
| Al | 0.4729 | 0.01803 | 0.673 | 0.3485 | 1.71 | 1.94 | 1.5% |
| Cu | 0.0489 | 0.00369 | 0.947 | 0.01528 | 14.29 | 12.54 | 2.0% |
| Cu | 0.1383 | 0.01044 | 0.800 | 0.04326 | 12.08 | 11.16 | 1.8% |
| Sn | 0.0368 | 0.00423 | 0.939 | 0.00617 | 38.8 | 34.5 | 1.7% |
| Sn | 0.0962 | 0.01106 | 0.787 | 0.01610 | 34.0 | 31.2 | 1.9% |
| Pb | 0.0315 | 0.00534 | 0.917 | 0.003023 | 95.3 | 84.4 | 1.7% |
| Pb | 0.0599 | 0.01015 | 0.807 | 0.005747 | 83.0 | 75.7 | 2.1% |
| Pb | 0.1233 | 0.02090 | 0.637 | 0.01183 | 64.6 | 62.1 | 2.0% |
| Pb | 0.1861 | 0.03154 | 0.532 | 0.01785 | 55.5 | 55.3 | |

* $N$ is the total number of atoms in the radiator.

$R_{3-4}$ and $R_{2-5}$ are ratios, corrected for background, of the spectrometer counting rate when the listed radiator is in position, to the counting rate with the "standard" Pb radiator, (0.1861 g/cm²).

$R_{3-4}$ is this ratio measured by channels 3 and 4 of the spectrometer, corresponding to the narrow resolution function 2a.

$R_{2-5}$ is this ratio obtained from channels 2, 3, 4, and 5 of the spectrometer, corresponding to the wide resolution function 2b.

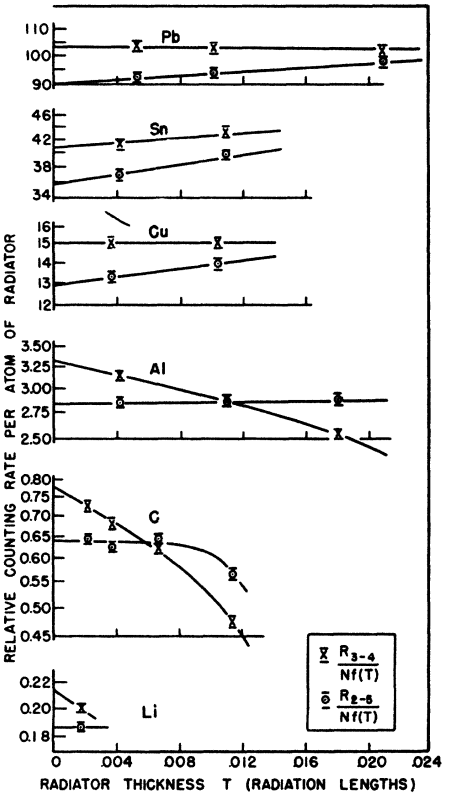

FIG. 4. Relative counting rates per atom for each radiator, corrected for vertical scattering. The crosses represent data obtained from channels 3 and 4, corresponding to the narrow resolution function of Fig. 2a. The circles are data obtained from channels 2, 3, 4, and 5, corresponding to the wide resolution function 2b. For each of these sets of data the relative pair cross section of each element is obtained from the extrapolation to zero radiator thickness.

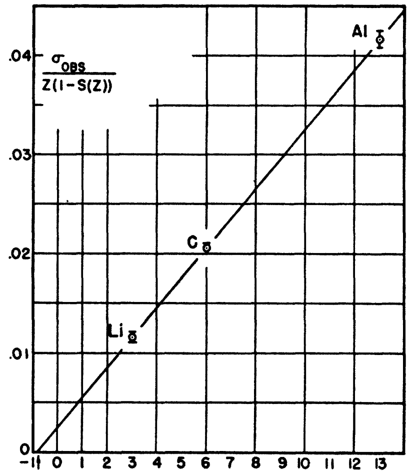

FIG. 5. Graph of $\sigma_{\text{obs}}/Z[1-S(Z)]$ vs. $Z$. The intercept on the $Z$-axis gives the electron pair cross section, $A=0.8$, relative to the pair cross section of a nucleus of unit charge.

The relative pair cross sections obtained from data taken with the two resolution functions of Fig. 2 are listed in Table II. Both sets of values have been arbitrarily normalized so that the measured (relative) cross sections of the lightest elements will agree numerically with the theoretical Born approximation values for the integral cross sections (see Fig. 6). This normalization is convenient even though the measured cross section (2) differs from the total integral cross section. (In the "theoretical" values, the electron cross section $A=0.8$ has been used.) Also shown in Table II are the screening corrections, $S(Z)$ calculated from the

TABLE II. Observed relative pair cross sections and their comparison with the theoretical Born approximation values. The value $A=0.8$ for the electron pair cross section has been used in the "theoretical" cross sections.

| Element | Screening correction $S(Z)$ | $\sigma_{3-4}$ normalized ($10^{-24}$ cm²) | $\sigma_{2-5}$ normalized ($10^{-24}$ cm²) | Average $=\sigma_{\text{obs}}$ ($10^{-24}$ cm²) | $\sigma_{\text{obs}}/\sigma_{\text{Born}}$ |
|---|---|---|---|---|---|
| Li | 0.010 | 0.0342 ±4% | 0.0347 ±2% | 0.0344 ±4% | 0.997 |
| C | 0.014 | 0.1248 ±3% | 0.1194 ±2% | 0.1221 ±3% | 0.991 |
| Al | 0.021 | 0.531 ±2% | 0.530 ±2% | 0.530 ±2% | 0.987 |
| Cu | 0.031 | 2.416 ±3% | 2.406 ±3% | 2.411 ±2% | 0.942 |
| Sn | 0.040 | 6.54 ±3% | 6.70 ±3% | 6.62 ±3% | 0.890 |
| Pb | 0.050 | 16.48 ±2% | 16.80 ±2% | 16.64 ±2% | 0.845 |

low energy formula of Bethe and Heitler[1] involving the function $C(\gamma)$.

Assuming that the Born approximation is valid for light elements (i.e., $B(Z)\approx 0$), if one plots $\sigma_{\text{obs}}/Z[1-S(Z)]$ against $Z$, the negative intercept on the $Z$ axis should give the electron pair cross section, $A$ (see Eq. (1)). This has been done in Fig. 5 and the value $A=0.8\pm 0.3$ obtained. The rather large estimate of error arises from the fact that the measured cross section of lithium (for which the electron effect is greatest) is less reliable than those of the other elements. This is due in part to the fact that only one thickness of lithium was measured, since thicker lithium radiators would have given too great an energy loss. Also important, however, is the large effect of any impurities which might be present in the lithium, resulting from the fact that the number of pairs produced per gram in different elements is roughly proportional to $Z$.[*] The error in $A$ quoted above is estimated not only from Fig. 5, but from a consideration of the relation of the cross sections of the three lightest elements measured to those of the heavier elements, as illustrated in Fig. 6.

As discussed in the introduction, the theoretical pair cross sections calculated with the Born approximation, are proportional to $Z^2$, except for corrections for screening and for the production of pairs in the field of the atomic electrons; that is,

$$\sigma_{\text{Born}}\sim Z(Z+A)(1-S(Z)). \tag{3}$$

In order to find the deviations of the measured cross sections from the Born approximation values (i.e., the correction $B(Z)$ of Eq. (1)), the ratio $\sigma_{\text{obs}}/\sigma_{\text{Born}}$ has been plotted as a function of $Z$, in Fig. 6. In this graph, the relative cross sections, $\sigma_{\text{obs}}$, have been normalized to make the ratio $\sigma_{\text{obs}}/\sigma_{\text{Born}}$ approach 1 at small $Z$, where the Born approximation should be valid. The values of $\sigma_{\text{Born}}$ are not purely theoretical, since the experimental electron cross section, $A=0.8$, has been used in (3). The curve of Fig. 6 indicates that the correction to the Born approximation is linear in $Z$,

$$B(Z)=(0.0019\pm 0.0005)Z,$$

but the data are not accurate enough to actually determine the $Z$-dependence of this correction. For example, the data are not greatly inconsistent with a $Z^2$-dependence, as suggested by Lawson,[7] and as might seem more reasonable from consideration of the total absorption cross sections.[8]

According to Fig. 6, the correction to the Born approximation is about $0.16\pm 0.4$ for lead, whereas a measurement of the total absorption cross section[8] indicates a correstion of 0.10. The latter figure is based on the assumption that the absorption of 17.6 Mev gamma-rays takes place only by pair production, Compton scattering, and atomic photoelectric effect, and that the theoretical cross sections for the latter two processes are correct. It will be too low if any other processes contribute to the absorption. Actually the nuclear $(\gamma,n)$ process may make a contribution of a few percent, and if a correction for this effect is made, it is possible that the value of $B(Z)$ obtained from the absorption data would agree within the experimental error with that found from Fig. 6.

Exact agreement between the two values of the Born approximation correction might not be expected since the absorption involves the total, integral pair cross section, whereas only the central half of the differential cross section, (2), is measured in the present experiments. As discussed in the introduction, the error in the Born approximation result is undoubtedly different in different regions of the differential cross section curve. It is interesting that the error in the central region seems to be as large as the error in the total, integral cross section.

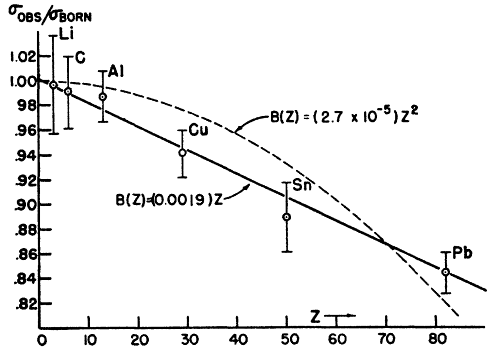

FIG. 6. Comparison of the observed relative pair cross sections $\sigma_{\text{obs}}$, with the theoretical, Born approximation results, $\sigma_{\text{Born}}=(3.055\times 10^{-27}\text{ cm}^2)Z(Z+A)(1-S(Z))$. (In the above curve, the value $A=0.8$ has been used.) The measured cross sections have been normalized in such a way as to make the ratio $\sigma_{\text{obs}}/\sigma_{\text{Born}}$ approach unity for small $Z$, where the Born approximation should be valid.

## ACKNOWLEDGMENTS

I am indebted to Professor B. D. McDaniel for many discussions concerning these experiments, and to Bruce Dayton for help in the operation of the cyclotron.

This work was supported in part by the Office of Naval Research.

---

[*] A spectrographic analysis of the lithium radiator, made by the New England Spectrochemical Laboratories of Ipswich, Massachusetts, showed only small amounts of impurities (0.1 percent ± a factor 10). However, the qualitative nature of this analysis, together with the fact that the likely surface impurities, $C$, $N$, and $O$ were not included in it, make it impossible to rule out the effects of impurities.