The magnetic moment of La<sup>139</sup> in units of the magnetic moment of the proton is hence:

$$\frac{\mu(\text{La}^{139})}{\mu(\text{H}^1)} = 0.98876 \pm 0.00010.$$

This expression is uncorrected for the slight diamagnetic field at the nucleus of the La<sup>139</sup> due to the Larmor precession of its atomic electrons in the applied field. The best value for this correction can be obtained by using a Hartree atom model calculation for Cs and making a short extrapolation. This gives  $(\Delta H/H)$ La =0.60 percent.

The broadness of the La139 resonance line may be ascribed to a nuclear quadrupole moment Q which, however, has never been measured. Similar broad lines have been observed for I127 and Cs133,3 and also the two Br isotopes,4 all of which are known to have quadrupole moments.

\* This work has been supported in part by the Signal Corps, the Air Materiel Command, and the ONR. <sup>1</sup> W. C. Dickinson and T. F. Wimett, Phys. Rev. **75**, 1769 (1949). <sup>2</sup> W. H. Chambers and D. Williams, Phys. Rev. **76**, 461 (1949).

<sup>3</sup> F. Bitter, private communication. <sup>4</sup> R. V. Pound, Phys. Rev. 72, 1273 (1947).

## Nuclear Quadrupole Moments and Nuclear Shell Structure

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ONSIDERABLY before the recent revival of interest in → nuclear shell structure, Schmidt<sup>1</sup> had found a systematic variation with Z of the deviation of nuclear charge distributions from a sphere (which he derived from nuclear quadrupole moments) and pointed out minima of the magnitudes of nuclear quadrupole moments near the "magic numbers" Z=50 and 82. Schmidt's plot has been extended with more recent data<sup>2,3</sup> and quadrupole moments correlated to some extent with nuclear shell structure.3,4

The following simple model, based on nuclear shell considerations, leads to the proper behavior of known nuclear quadrupole moments, although predictions of the magnitudes of some quadrupole moments are seriously in error.

1. Neutrons and protons fit into single particle levels in a scheme similar to those proposed for correlating spins, thus producing what may be called proton and neutron shells.

2. Proton and neutron shells tend to be oriented or polarized to allow maximum overlap between proton and neutron distributions.

This model leads to the conclusions:

A. For an odd-proton nucleus, the quadrupole moment is primarily dependent on the number of protons P and can be written  $Q_{podd} = Q_p(P)$ , where  $Q_p$  is always positive immediately before, and always negative immediately after a shell is filled.

B. For an odd-neutron nucleus, the magnitude of the quadrupole moment depends on the number of protons, but its sign is determined by the number of neutrons N, being given by  $[Q_p(N)]/[|Q_n(N)|]$ .  $Q_n(N)$  is the electric quadrupole moment which would be produced by the neutrons if they were charged, and  $Q_n$  is very nearly the same function as  $Q_p$ .

C. For odd-odd nuclei, estimation of quadrupole moments is more complex and depends on the way in which the mechanical moments of the odd neutron and odd proton add. If these moments are essentially parallel, the quadrupole moment should be of the same sign and approximately the same magnitude as for a similar odd-proton nucleus, if the neutron and proton mechanical



FIG. 1. Nuclear quadrupole moments divided by the square of the nuclear radius  $(1.5 \times 10^{-13} 4^{1})^2$ . Known moments of odd-proton nuclei and odd-proton odd-neutron nuclei (excepting Li<sup>6</sup> and Cl<sup>36</sup>) are plotted as circles against number of protons, moments of odd-neutron nuclei as crosses against number of neutrons. Arrows indicate closing of major nucleon shells. Solid curve represents regions where quadrupole moment behavior seems established, dashed curve more doubtful regions.

moments are not essentially parallel, the quadrupole moment magnitude should be considerably reduced.

Known quadrupole moments appear to fit these expectations. Figure 1 shows nuclear quadrupole moments<sup>5</sup> as a function of number of nucleons.

The sign of Q near the closing of major shells is that expected in all cases except for Li<sup>7</sup>. In view of uncertainties in the charge distributions in the molecules from which Q for Li<sup>7</sup> was derived, perhaps present indications that Q is positive should not be regarded as conclusive. The negative quadrupole moment of S<sup>33</sup> and the reversal of sign to a positive moment for S<sup>35</sup> is a striking illustration of the above rules since S35 has 19 neutrons, or one less than the filled shell of 20. This model would similarly predict that the quadrupole moments of K<sup>39</sup> and K<sup>41</sup> are positive, that of Sc45 and Zr91 negative.

Se77 seems from microwave measurements to have a quadrupole moment less than  $0.001 \times 10^{-24}$  cm<sup>2</sup>, yet atomic spectra indicate a spin for this nucleus greater than 3/2. Both experiments may be correct, and the Se<sup>77</sup> moment very small because of the spherical distribution of protons in this nucleus due to completion of the 4f sub-shell. Ca<sup>43</sup> is a similar case which should have a complete proton shell and small quadrupole moment.

The negative quadrupole moment of Ge73 with 41 neutrons suggests that a completed sub-shell occurs at 40 neutrons (pointed out as a possibility by Nordheim) corresponding to filling the 3plevels in Nordheim's and Mayer's schemes.<sup>6</sup> If the neutron and proton shells fill in the same manner, the Cb93 quadrupole moment should also be negative and the Zr<sup>91</sup> quadrupole moment small.

The odd-odd nuclei with known quadrupole moments consist of Li<sup>6</sup>, B<sup>10</sup>, N<sup>14</sup>, Cl<sup>36</sup>, and Lu<sup>176</sup>. Their moments correspond to expectations from the model described above, and those cases for which the moments of neutron and proton shells are parallel (B<sup>10</sup>, N<sup>14</sup>, Lu<sup>176</sup>) are plotted in Fig. 1.

Gordy's statement<sup>3</sup> that completion of a neutron shell tends to make quadrupole moments more negative does not follow from the above considerations. This is perhaps not serious because the evidence for this empirical observation is questionable. A number of quadrupole moments plotted by Gordy are values he predicts on the basis of an empirical relation between nuclear electric quadrupole moments and magnetic dipole moments. The most significant applications of this relation are to cases of nuclei of the same spin and differing by two protons. In the only two known nuclear pairs of this type (Cu<sup>65</sup>-Ga<sup>69</sup>, Ga<sup>71</sup>-As<sup>75</sup>), Gordy's relation is in considerable error and in one case it gives the wrong sign for the quadrupole moment. Gordy has plotted the Cs133 quadrupole moment as  $+0.3 \times 10^{-24}$  cm<sup>2</sup> and from this value derived other moments. However, known information on Cs133 allows only the conclusion that for this nucleus  $|Q| \leq 0.3 \times 10^{-24}$ cm<sup>2</sup>. If the Cs<sup>133</sup> moment is taken as zero or negative and if the predicted quadrupole moments are questioned, then no evidence appears to remain that completion of a neutron shell makes nuclear quadrupole moments more negative.

For the nuclei which contain closed proton shells plus or minus only one proton, the nuclear shell model gives not only a definite prediction of the sign of Q, but also, if the state of the odd proton is determined from the nuclear spin and magnetic moment, it gives a fairly definite value for the quadrupole moment magnitude. In the cases Li<sup>7</sup>, N<sup>14</sup>, and Bi<sup>209</sup>, the calculated magnitudes are in substantial agreement with those observed. This model also gives roughly the proper ratios between the quadrupole moments of In<sup>113</sup>, In<sup>115</sup>, Sb<sup>121</sup>, and Sb<sup>123</sup> (protons differing by one from a closed shell of 50). However, in spite of the rather extensive success of this model, it appears difficult to reconcile the magnitudes of the In<sup>113</sup>, In<sup>115</sup>, and Sb<sup>123</sup> quadrupole moments with a nuclear shell model including a closed shell at 50 nucleons. They are larger by a factor of four than can reasonably be produced by a single particle. In addition the very large quadrupole moments in the vicinity of Z=71 present difficulties to a nuclear shell model. To give  $Lu^{176}$  its quadrupole moment of  $7 \times 10^{-24}$  cm<sup>2</sup>, approximately 35 protons in the most favorable orbits must contribute to Q. Since the first fifty protons are presumably in a closed shell and contribute nothing to Q, only 21 protons are available and even all of them could hardly be put in the few orbits which contribute most to a positive quadrupole moment.

Failure of the nuclear shell model to give correct quadrupole moments is in contrast to the situation with nuclear magnetic moments, which can all be accounted for by a suitable admixture of states of a single nucleon. In the shell model approximation, these large quadrupole moments must represent a considerable contribution from the protons in the closed shells. The polarization of this core would presumably require a sharing of angular momentum between the protons of the incomplete shell and those of the closed shells. The magnitude of the polarization, however, and the resulting large asymmetry of the nucleon distribution is hardly consistent with the single particle-central field quantization which is the basis of the shell structure model.

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<sup>6</sup> E. Feenberg and K. C. Hammack, Phys. Rev. 75, 1877 (1949); R. D. Hill, Phys. Rev. 76, 998 (1949).
<sup>6</sup> Data taken from table of nuclear moments by H. L. Poss (Brookhaven National Laboratory Report to be published).
<sup>8</sup> Feenberg, Hammack, and Nordheim, Phys. Rev. 75, 1968 (1949).

## The Sm<sup>151</sup> Beta-Ray Spectrum\*

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LONG-LIVED samarium beta-emitter was first identified A LUNG-LIVED samarium occus of R. J. Hayden and L. G. Lewis.<sup>1</sup> The half-life has been given recently by M. G. Inghram<sup>2</sup> to be about 200 years and independently by J. A. Marinsky<sup>3</sup> to be 1000±350 years. G. W. Parker and P. M. Lantz<sup>4</sup> have prepared sources of the activity and characterized its radiation by means of aluminum absorption curves. They report that maximum betaenergy to be approximately 65 kev.



FIG. 1. Distribution of beta-rays of Sm<sup>151</sup>. The counting rate per unit momentum is plotted against the coil current in amperes.

The source material used in this study was fission product samarium. Its chemical separation from the other fission products has been described elsehwere4 in detail. The method consists of a gross separation of the fission products on an Amberlite IR-1 ion exchange column. This column effectively separates the trivalent cations from other cationic and anionic fission products and also separates the yttrium and cerium quite well from the other rare earths. After an intermediate separation of the cerium earths the fraction of eluate recognized by its gamma-activity to contain europium also contains the samarium and some element 61. Since the concentrations of other rare earth activities relative to the europium and samarium had been reduced, it was possible to achieve good separation of samarium from europium and element 61 by a final ion exchange column separation using optimum conditions for elution. The purified samarium was converted to the chloride and the aqueous solution was evaporated on a 50



FIG. 2. Kurie plot of Sm<sup>151</sup> source which contained Eu impurity.

