occur for small values of m_J so that at the maxima $3m_J^2-J(J+1)$ is negative. The terms in the energy expression which involve m_I for those values of m_J which give rise to lines near the maxima are then:

$$E=m_I g_I \mu_0 H-(\text{const.})(e^2qQ)(m_I^2).$$

If g_I is negative and if w^2qQ is positive, then the high frequency satellite arises from the transition (3/2, 1/2). If e^2qQ is negative, the high frequency satellite arises from the transition (−1/2, −3/2). It follows that qQ for Li^7 in Li_2 and in LiBr is negative while qQ for Na in Na_2 and NaI and for Cl^{35} in KCl is positive.

H. M. Foley[3] has made a calculation of q for the Li_2 molecule using the Bartlett-Furry wave functions. He finds that $q=-0.0062$ atomic units. From this result $Q(Li^7)$ is positive. The actual determination of the magnitude of Q is somewhat more difficult since the observed line shows no evidence of resolution into three peaks. The observed half-widths of the line is 15×10^3 sec.$^{-1}$ and if it is assumed that the two satellite peaks determine the half width of the line, then $Q(Li^7)=+2\times10^{-26}$ cm^2.

I. I. Rabi[4] has proposed a calculation of q from the known force constants of diatomic molecules. He predicts a negative q at each nucleus of a diatomic molecule. The result is in agreement with the results of the present experiments where it has been found that q has the same sign at the Li nucleus in Li_2 and in LiBr and the same sign at the Na nucleus in Na_2 and in NaI. The negative value predicted by Rabi, when applied to the present experimental results also yields a negative value for $Q(Cl^{35})$. The result is in agreement[5] with the known quadrupole moment of Cl^{35}. From these considerations it is found that Q(Na) is negative and that $Q(Li^7)$ is positive.

[1] B. T. Feld and W. E. Lamb, Phys. Rev. **67**, 15 (1945).
[2] H. M. Foley, Phys. Rev. **71**, 747 (1947).

[3] I am indebted to Professor Foley for the results of the calculation made for this purpose. A detailed analysis of the calculation will be published by him.
[4] I am indebted to Professor Rabi for several discussions concerning the calculation of q. The basis of the indicated calculation will be discussed by Professor Rabi in a forthcoming publication.

[5] L. Davis, Jr., B. T. Feld, C. W. Zabel and J. R. Zacharias, Phys. Rev. **73**, 525 (1948).

Relation of Nuclear Quadrupole Moment to Nuclear Shell Structure

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May 23, 1949

As has been pointed out by Mayer,[1] the exceptional stability of nuclei which have proton or neutron numbers of 2, 8, 20, 50, 82, or 126 suggests the existence of shell structures in nuclei. Schemes have been proposed by Nordheim[2] and by Feenberg[3] for assigning orbital quantum numbers to the last odd proton in nuclei. The purpose of the present note is to point out the dependence of the nuclear shape—as evidenced by the nuclear quadrupole moment*—upon the nuclear shell structure. The relationship found provides new evidence for the existence of shell structure in nuclei. Evidence is also given for an inter-relation of the nuclear magnetic moment to the nuclear quadrupole moment, when the nuclei are similar.

Figure 1 is a plot of nuclear quadrupole moment Q as a function of proton number Z. All nuclei with known moments are included except those which are spherically symmetric because they have spins of zero or ½. The available data** suggest the following relationships of quadrupole moment to nuclear shell structure. At the proton numbers 2, 8, 20, 50, and 82 the quadrupole moment is zero or small. When a new shell begins to form the quadrupole moment is negative. As the number of protons in the unfilled shell is increased, Q becomes positive and increases until it reaches a maximum, when the shell is approximately ⅔ filled. It then decreases to zero and changes to a negative sign*** at the "magic proton numbers."

The above mentioned trends are partly masked by the effects of neutron shell structure. The points where a neutron shell is completed are indicated on the chart by arrows. It is seen that there are minima in the curve at these points. The effects of neutron shell structure are shown again in Fig. 2, where absolute magnitude of quadrupole moment is plotted as a function of neutron number. The effect of proton shell structure does not show up in this plot, partly because the data are meager and partly because the negative Q's are plotted as positive.

In an effort to obtain more Q values for comparison I have correlated magnetic moments with quadrupole moments for isotopic pairs having the same spins. The results are shown in Table I. Other things being equal, the magnetic moment appears to decrease in magnitude as the nucleus becomes more elongated along the spin axis—i.e., as its quadrupole moment becomes more positive (or less negative for flattened nuclei). To show this, the quantity $(\mu_1/\mu_2)(Q_1/Q_2)$ when Q_1 and Q_2 are positive, or $(\mu_2/\mu_1)(Q_1/Q_2)$ when Q_1 and Q_2 are negative, has been listed in the last column of Table I. It is seen that this quantity remains approximately constant. Here the subscript 2 refers to the heavier isotope. The fact that there are no marked differences in the mag-

TABLE I. Relation of quadrupole to nuclear magnetic moments.

X_1/X_2	μ_1/μ_2	Q_1/Q_2	Sign of Q	$(\mu_1/\mu_2)(Q_1/Q_2)$ when Q is positive or $(\mu_2/\mu_1)(Q_1/Q_2)$ when Q is negative
Cl^{35}/Cl^{37}	1.20	1.28	−	1.07
Cu^{63}/Cu^{65}	0.94	1	−	1.12
Ga^{69}/Ga^{71}	0.79	1.58	+	1.24
Br^{79}/Br^{81}	0.93	1.20	+	1.12
Eu^{151}/Eu^{153}	2.24	0.48	+	1.08
Re^{185}/Re^{187}	0.99	1.08	+	1.07
			Average	1.12

TABLE II. Some estimated quadrupole moments.

	Q in 10^{-24} cm^2
In^{113}	~1.3
Cs^{137}	≤0.2
K^{39}	~−0.03
K^{41}	~−0.02
Rb^{87}	~0.17
Sb^{121}	~−0.9
La^{139}	~0.2

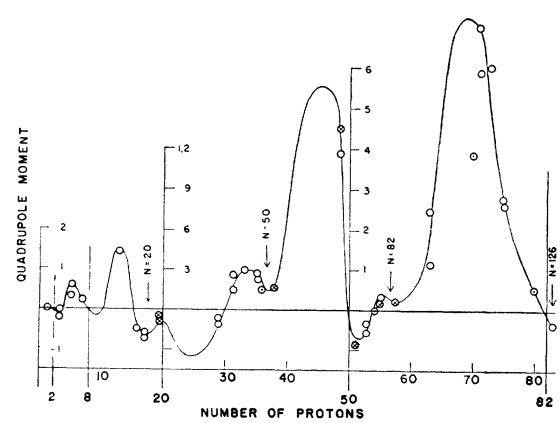

FIG. 1. A plot of nuclear quadrupole moment as a function of the number of protons in the nucleus. Q is in units of 10^{-24} cm². O's represent nuclei with an odd proton. ⊙'s represent nuclei with an odd neutron. ⊗'s represent quadrupole moments which have not been measured but have been estimated by the method described here.

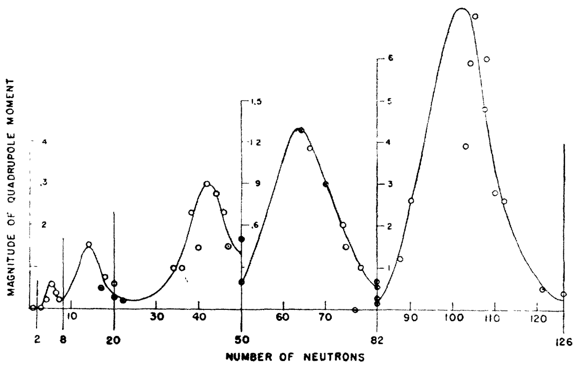

FIG. 2. A plot of the magnitude of quadrupole moment as a function of the number of neutrons in the nucleus. The units and symbols have the same significance as in Fig. 1.

netic moments of isotopic pairs with spins of $\frac{1}{2}$ also supports this relation since there is no difference in shape for these pairs.

The quadrupole moment of In^{113} is estimated as $\sim 1.3\times 10^{-24}$ cm^2 and that of Cs^{137} as $\sim +0.2\times 10^{-24}$ cm^2 from the relation $(\mu_1/\mu_2)(Q_1/Q_2)=1.12$, using the parameters of the corresponding isotopes In^{115} and Cs^{133}. Correctly or incorrectly, the same relation is applied to nuclei of identical spin and orbital momentum, which do not have the same atomic number but which have very nearly the same number of particles. In this way the quadrupole moments of K^{39} and K^{41} have been estimated from Cl^{37}, that of Rb^{87} from Br^{81}, that of Sb^{121} from I^{127}, and that of La^{139} from Cs^{137}. The values obtained are listed in Table II. Figure 1 can be used to obtain similar rough estimates of unknown nuclear moments.

* A nucleus with a positive quadrupole moment is elongated along the spin axis, and one with a negative quadrupole moment is flattened along the spin axis.

** With few exceptions the measured Q values are those listed by H. H. Goldsmith and D. R. Inglis (Brookhaven National Laboratory Report, B N L-1-5, Oct. 1, 1948). The Q values for B^{10} and B^{11}, 0.06×10^{-24} cm^2 and 0.03×10^{-24} cm^2 respectively, are those recently determined by Gordy, Ring and Burg (to be published). That for In^{115}, 1.17×10^{-24} cm^2 is the value recalculated by B. T. Feld (Nuclear Science Series, National Research Council, Preliminary Report No. 2, Sept., 1948). That for Li^6, ~ 0, and that for Li^7, $\sim -0.02\times 10^{-24}$ cm^2, are from a recent note by P. Kusch (Phys. Rev. **75**, 887 (1949)). That for I^{129}, -0.43×10^{-24} cm^2, is from Livingston, Gilliam, and Gordy (to be published).

*** In one case, $Z=20$, the sign becomes negative before the "magic proton number" is reached. This anomoly appears to arise from the filling of the neutron shell.

[1] M. G. Mayer, Phys. Rev. **74**, 235 (1948).
[2] L. W. Nordheim, Phys. Rev. **75**, 1894A (1949).
[3] Eugene Feenberg, Phys. Rev. **75**, 320 (1949).

Yields of Manganese in Spallation Reactions

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May 11, 1949

ONE approach previously used in the study of high energy spallation reactions involves the bombardment of a single target isotope and the determination of the relative yields of many products.[1] In the present work a complementary approach has been used: the bombardment of a variety of elements and the determination of the yields of two particular product isotopes. This method has the advantage that yields can be compared more exactly, since the same radiation is always measured. It also has the disadvantage that most of the target elements have several stable isotopes, so that the reacting nucleus cannot be uniquely specified. All of the elements (excepting only rubidium and krypton) from atomic number 24 (chromium) to 38 (strontium) were bombarded with 190-Mev deuterons in the 184-inch cyclotron. The yields of 5.8-day Mn^{52} and 2.59-hr. Mn^{56} were determined.

After each bombardment the target was dissolved and a known amount of inactive manganese was added, together with carriers for the other elements possibly produced. The manganese was separated chemically and the decay of its activity was followed. The chemical procedures used were varied depending on the target element, more complicated procedures being required for those far from manganese because of the low yields. The purest available materials (usually Hilger's "spectroscopically pure" grade) were used for targets.

In order to compare the yields from one bombardment with those from another, a monitor target was included in each experiment. A piece of 10 mil copper foil cut to the exact shape and area of the target was placed immediately behind the target during exposure to the deuteron beam. Since all targets used were thin, it was assumed that each copper monitor foil received the same irradiation as the accompanying target. The yield of Cu^{64} in the monitor was taken to be a measure of the relative intensity of the deuteron beam.

The measured activities were corrected to infinite bombardment time and 100 percent chemical yield. In addition, the counting efficiency of Mn^{52} was assumed to be 40 percent relative to that of Mn^{56} and corrected accordingly. (Mn^{52} decays 35 percent by positron emission, 65 percent by K capture;[2] the 40 percent figure includes the contribution of electromagnetic radiation to the counting rate.)

The data are presented in Fig. 1. The relative cross section, σ, is equal to the corrected manganese activity per mole of target element divided by the corrected Cu^{64} activity per mole of copper monitor. The uncertainty attached to each determination is indicated by the size of the plotted point.

It should be noted that the yield of the 21-minute isomer of Mn^{52} which decays[3] directly to Cr^{52} has not been included. The total yields of Mn^{52} are therefore somewhat higher than the data indicate.

The absolute cross section corresponding to $\sigma=1$ is approximately 0.03 barn, according to a recent direct determination[4] of the cross section for Cu^{64}.

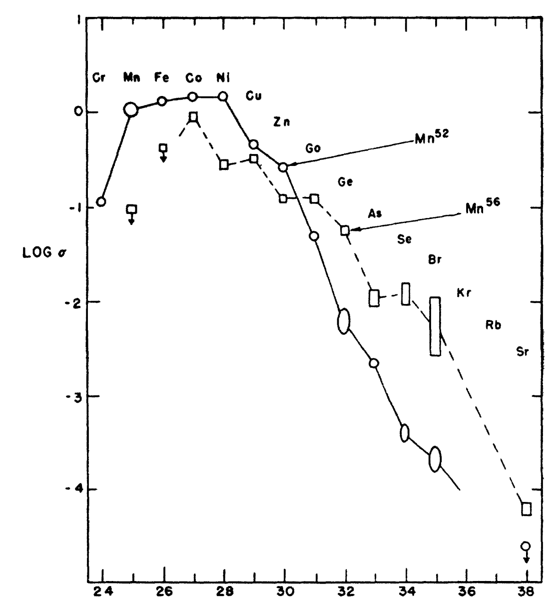

FIG. 1. Relative cross sections for the production of two manganese isotopes by bombardment of various elements with 190-Mev deuterons. Points with arrows attached represent upper limits.