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On Realistic Field Theories and the Polarization of the Vacuum

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The formal prescription for the regularization of divergent expressions in quantum electrodynamics which has been recently suggested by Pauli implies very strongly that these same divergences may be similarly canceled in a realistic theory wherein one has a mixture of fields in the manner of Pais and Sakata. Indeed, it may be readily seen that the Pauli regulator scheme corresponds field-theoretically to a family of spinor fields interacting with a family of neutral vector meson fields; the formalism is mathematically consistent (at least to the second order in the coupling constants) but physically unsound owing to the appearance of imaginary coupling constants. An attempt to remedy this defect by considering the most general mixture which is possible within the framework of ordinary field theory also leads to failure. Specifically, the current density which is induced in the vacuum by an external electromagnetic field is calculated to order e^2 for a variety of situations but it turns out that no combination of charged scalar, spinor and vector fields—with or without anomalous magnetic moments—leads to a full compensation of the divergences although the photon self-energy by itself may be made to vanish. It is concluded that, although the notion of a realistic approach to the theory of elementary particles remains an attractive one, the usual linear field theories do not in themselves seem to be adequate for this purpose; there is also always the possibility that it is the perturbation theory which is at fault.

1. FORMALISTIC *VS.* REALISTIC THEORIES

In a recent critique of some of the current procedures in quantum electrodynamics, Pauli[1] has emphasized that the present theory involves the handling of divergent and conditionally convergent integrals the evaluations of which cannot be inferred in an unambiguous way from the theory but must be defined in each instance. Pending the development of new concepts, he has suggested that one handle the divergences of the theory in a purely formal way by first regularizing all singular expressions and then performing a limiting process. Specifically, if $f(x, m)$ is a singular function depending on the space-time coordinate x_μ and the mass m, the regularized function $f_R(x)$ may be defined by the formula

$$f_R(x)=\Sigma c_i f(x, M_i) \tag{1}$$

with $c_0=1$, $M_0=m$. The coefficients c_i are subjected to the restrictions

$$\Sigma c_i=0, \tag{2}$$

$$\Sigma c_i M_i^2=0, \tag{2'}$$

and the limiting process consists in letting the auxiliary masses M_i $(i=1, 2, \cdots)$ tend to infinity. The conditions (2) and (2′) are sufficient to insure that the functions $\bar{\Delta}_R$ and $\Delta_R^{(1)}$ are free of singularities on the light cone unlike their unregularized counterparts.[2] These conditions are also enough to lead to convergent expressions for the electron self-energy and the polarization of the vacuum by an external electromagnetic field (with a correspondingly vanishing photon self-energy) provided one additional rule is added, *viz.*, these expressions must be regularized as a whole and not in parts. Upon performing the final limiting process, logarithmic divergences reappear in the unobservable charge and mass renormalization factors for the electron (unless new conditions are imposed on the c_i's).

The above procedure constitutes, in the terminology of Pauli and Villars, a "formalistic theory" wherein a purely formal recipe for the performance of calculations is given, in contrast to a "realistic theory" in which the mathematical device which effects convergence—in the case at hand, it is the introduction of the auxiliary masses—is not an *ad hoc* affair but rather a consequence of physical concepts. By a "realistic theory" we shall mean, in particular, one which attempts to take into account **all** of the interactions of the elementary particles with one another. It is the aim of this paper to determine whether or not a realistic (though semi-phenomenological) description of elementary particles can be obtained within the domain of ordinary field theory. It is perhaps not surprising that the result will turn out to be negative.

The term "ordinary field theory" as we use it in this work is meant to denote the canonical formalism of Heisenberg and Pauli as applied to the quantization of the scalar Klein-Gordon equation, the Dirac equation and the Maxwell-Proca equations leading to a description of particles with spin zero, one-half and one respectively and with assorted masses and charges. In treating the interaction of these fields with one another, one must add to the vacuum Lagrangians additional terms which are required to be in accord with the general principles of relativistic and gauge invariance. For simplicity, however, one also imposes on the interaction terms the more special restrictions that they involve no derivatives of spinor field variables, no derivatives higher than the first of Bose field variables and, in either case, that they contain the field variables which characterize any one field no more than bilinearly. It need hardly be stressed that, in view of the divergent nature of these theories, all computations must be carried through in a manifestly covariant way; this is feasible at the present time only if the coupling between fields is assumed to be weak.

2. A FIELD-THEORETICAL INTERPRETATION OF THE REGULARIZATION PROCEDURE

In actual fact, the need for a realistic approach to quantum electrodynamics *per se* is much less acute than it seemed two years ago since the present theory works

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[1] W. Pauli, unpublished letter to J. Schwinger; W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949).

[2] The Δ, $\bar{\Delta}$ and $\Delta^{(1)}$ functions which we use in this paper are as tabulated in the Appendix of reference 5.

well and the modifications to be expected of a future correct theory will surely be of an ultra-relativistic nature. On the other hand, it appears very unlikely that the corresponding problem of the interaction of nucleons and mesons can be solved except within a realistic framework and we assume, with Pais[3] and Sakata,[4] that only in a theory which takes into account the mutual interactions of all the elementary particles can one hope to eliminate the divergences; the essential limitation of the present investigation is contained in the fact that we have restricted ourselves to mixtures of ordinary type fields.

It is perhaps interesting to consider first a field-theoretical model which seems to be the most literal transcription of the Pauli regulator scheme but which contains important difficulties in its physical interpretation and hence cannot be taken too seriously from a realistic point of view. It has, on the other hand, the virtue of being mathematically consistent and completely convergent (at least to the second order in the coupling constants) and does seem to explain the apparent arbitrariness of the regularization procedure.

It is most natural to regard the auxiliary masses which are involved in the regulator functions and which accordingly play a mathematical role only as being associated with real additional fields; we consider, therefore, the case of a family of spinor fields (including the electron field) interacting with a family of neutral vector Bose fields (including the photon field) with vector coupling linking each spinor and Bose field. One then finds, to the second order in the coupling constants, that all the particle self-energies converge as well as the current densities induced in the vacuum by external electromagnetic and mesonic fields provided certain simple relations exist among the masses and coupling constants which bear a close resemblance to conditions (2) and (2′). The photon self-energy is zero.

For simplicity, let us assume that we have, besides the electron field, two additional spinor fields and, besides the photon field, one additional Bose field. Let e_i, g_i and m_i denote the electric charge, mesonic charge and mass of the ith spinor field and let μ be the mass of the Bose particles. It then follows in a trivial way from the results of Schwinger[5] that one has for the current density induced in the vacuum by an external electromagnetic field described by the potential $A_\mu(x)$,[6]

$$\delta j_\mu(x)=i/2\cdot\int d\omega'\cdot\epsilon(x-x')\times\sum_{i=0}^{2}\langle[j_\mu(x,m_i),j_\nu(x',m_i)]\rangle_0 A_\nu(x') \quad (3)$$

with

$$j_\mu(x,m_i)=ie_i/2\cdot[\bar\psi(x,m_i)\gamma_\mu\psi(x,m_i)-\bar\psi'(x,m_i)\gamma_\mu\psi'(x,m_i)];$$

here, $\psi(x,m_i)$, $\bar\psi(x,m_i)$ and $\psi'(x,m_i)$, $\bar\psi'(x,m_i)$ are the field variables which describe the ith spinor and charge conjugate spinor fields respectively, $d\omega'$ is the four-dimensional element of volume and $\epsilon(x-x')$ is, as usual, $+1$ or -1 according as the space-like surface associated with x lies in the future or the past with respect to the corresponding surface associated with x'. It is then a consequence of the sufficiency of the regulator conditions (2) and (2′) that $\delta j_\mu(x)$ will converge and the photon self-energy will vanish provided

$$e_0^2+e_1^2+e_2^2=0,\quad e_0^2m_0^2+e_1^2m_1^2+e_2^2m_2^2=0. \quad (4)$$

In a similar way, for the mesonic current density induced in the vacuum by an external mesonic potential $U_\mu(x)$ one finds

$$\delta J_\mu(x)=i/2\cdot\int d\omega'\cdot\epsilon(x-x')\times\sum_{i=0}^{2}\langle[J_\mu(x,m_i),J_\nu(x',m_i)]\rangle_0 U_\nu(x') \quad (3')$$

with

$$J_\mu(x,m_i)=ig_i/2\cdot[\bar\psi(x,m_i)\gamma_\mu\psi(x,m_i)-\bar\psi'(x,m_i)\gamma_\mu\psi'(x,m_i)].$$

Both $\delta J_\mu(x)$ and correspondingly the self-energy of the neutral vector meson will converge if

$$g_0^2+g_1^2+g_2^2=0,\quad g_0^2m_0^2+g_1^2m_1^2+g_2^2m_2^2=0. \quad (4')$$

Finally, for the self-energy density operator of the ith spinor particle $\mathcal{H}^{(i)}$, one has

$$\mathcal{H}^{(i)}=1/2\cdot[\bar\psi^{(i)}(x)\chi^{(i)}(x)+\text{charge conjugate}], \quad (5)$$

where $(\partial_\lambda\equiv\partial/\partial x_\lambda)$

$$\begin{aligned}\chi^{(i)}(x)&=e_i^2\int d\omega'\{\bar D(x-x')(\gamma_\lambda\partial_\lambda+2m_i)\Delta^{(1)}(x-x',m_i^2)\\&+D^{(1)}(x-x')(\gamma_\lambda\partial_\lambda+2m_i)\bar\Delta(x-x',m_i^2)\}\psi^{(i)}(x')\\&+g_i^2\int d\omega'\{\bar\Delta(x-x',\mu^2)(\gamma_\lambda\partial_\lambda+2m_i)\Delta^{(1)}(x-x',m_i^2)\\&+\Delta^{(1)}(x-x',\mu^2)(\gamma_\lambda\partial_\lambda+2m_i)\times\bar\Delta(x-x',m_i^2)\}\psi^{(i)}(x'). \quad (5')\end{aligned}$$

This converges if

$$e_i^2+g_i^2=0, \quad (6)$$

whence it is clear that one may set

$$g_i=ie_i \quad (6')$$

[3] A. Pais, "On the theory of elementary particles," Verh. Kon. Ac. Amsterdam, Vol. 19 (1947).
[4] S. Sakata, Prog. Theor. Phys. 2, 145 (1947).
[5] J. Schwinger, Phys. Rev. 75, 651 (1949).
[6] We employ natural units throughout with $\hbar=c=1$. The symbol $\langle F\rangle_0$ denotes the vacuum expectation value of F.

which is fully compatible with the earlier restrictions (4) and (4′). We wish also to remark that the self-stress of the ith spinor particle vanishes in this formalism; one gets for $S=\int T_{11}d^3x$ evaluated for a particle at rest the value $m_i/2\pi\cdot(e_i^2+g_i^2)/4\pi$ which eqaals zero by (6).[1,7]

We observe that in order to obtain convergence in this scheme it has been found necessary to employ imaginary coupling constants. This has the consequence that the Hamiltonian is not Hermitian and probability is not conserved. The physical meaning of the present formalism is hence obscure unless a limiting process is performed, in which case the whole affair reduces to a purely formal prescription for performing calculations.

It is interesting to see what this formalism (which is suggested by the use of regulators in quantum electrodynamics in problems of order e^2) implies for the kind of regularization to be used for transitions of higher order. One infers that all internal photon lines are to be regulated separately; also, that each closed electron loop is to be regulated as a single unit (the terminology "internal photon line" and "closed electron loop" is that associated with the Feynman diagram).[8] It is clear that one will lose the gauge invariance of the theory by regulating independently the individual segments which constitute a closed electron loop for charge would be no longer conserved in detail. The simplest case of a closed electron loop occurs in the calculation of the polarization of the vacuum where one must regularize the expression as a whole.[1,9]

3. THE POLARIZATION OF THE VACUUM

We ask finally the question: Can one obtain a mathematically and indeed physically consistent picture of elementary particles by the expedient of mixing ordinary fields? To answer this question, it turns out to be sufficient to consider the polarization of the vacuum by an external electromagnetic field. The requirements of gauge invariance somewhat restrict the number of possible situations which need to be investigated and we have accordingly considered the cases of an electromagnetic field interacting with a scalar field, with a spinor field (with vector and tensor coupling) and with a vector field (with vector and tensor coupling). (The first two of these five possibilities have already been extensively treated in the literature,[1,5,10] but we include them here for the sake of comparison. We must also remark that we have not considered the cases of particles with spin higher than one or the case of charged massless vector mesons—these remain as real though perhaps academic possibilities.)

The basic theory which has been used is that of Schwinger[5] except as modifications have become necessary whenever one has had to transcribe an expression containing time derivatives from the Schrödinger to the interaction representation.[11] The equation of motion in the interaction representation assumes the form

$$i\delta\Psi[\sigma]/\delta\sigma=\mathcal{H}(x,\sigma)\Psi[\sigma], \tag{7}$$

where $\Psi[\sigma]$ is the state vector of the system and $\mathcal{H}(x,\sigma)$ is the Hamiltonian density for the interaction between one of the fields in question and the external electromagnetic field. To the first order in the coupling constant, one may then write for the solution of (7)

$$\Psi[\sigma]=\left(1-i\int_{-\infty}^{\sigma}\mathcal{H}(x',\sigma')d\omega'\right)\Psi[-\infty],$$

where $\Psi[-\infty]$ is the state vector which characterizes the initially undisturbed vacuum state of the system. For the induced current density $\delta j_\mu(x)$ one then finds

$$\begin{aligned}\delta j_\mu(x)&=(\Psi[\sigma],j_\mu(x,\sigma)\Psi[\sigma])-\langle(j_\mu(x,\sigma))_{A=0}\rangle_0\\&=\langle j_\mu(x,\sigma)-(j_\mu(x,\sigma))_{A=0}\rangle_0\\&\quad-i\int_{-\infty}^{\sigma}\langle[j_\mu(x,\sigma),\mathcal{H}(x',\sigma')]\rangle_0 d\omega'.\end{aligned}\tag{8}$$

We have used the notation $j_\mu(x,\sigma)$ in the right-hand side to emphasize that the operator which enters into (8) must be appropriate to the interaction representation. We remark, however, that, since we are interested in corrections to the current density of order e^2, it is only the linear part of the Hamiltonian $\mathcal{H}_1(x')$, say, which is relevant, and indeed it is only the linear part of the current density operator, say, $j_\mu(x)$ which enters into the commutator. Furthermore, we discard that part of $\langle j_\mu(x,\sigma)\rangle_0$ which does not involve the external field. Assuming, finally, that the external electromagnetic field cannot create pairs, we may rewrite (8) somewhat more conveniently, *viz.*,

$$\begin{aligned}\delta j_\mu(x)&=\langle j_\mu(x,\sigma)-(j_\mu(x,\sigma))_{A=0}\rangle_0\\&\quad-i/2\cdot\int\langle[j_\mu(x),\mathcal{H}_1(x')]\rangle_0\epsilon(x-x')d\omega'.\end{aligned}\tag{8′}$$

We consider the application of (8′) to the several individual cases.

(*a*) *Charged scalar field.* Let $U(x)$ and $U^*(x)$ be the field variables which characterize the charged scalar field and let $A_\mu(x)$ denote the unquantized potential in

[7] A. Pais and S. T. Epstein, Rev. Mod. Phys. **21**, 445 (1949).
[8] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[9] J. Rayski, Phys. Rev. **75**, 1961 (1949).
[10] A. Pais and G. E. Uhlenbeck, Phys. Rev. **75**, 1321 (1949); Umezawa, Yukawa, and Yamada, Prog. Theor. Phys. **3**, 317 (1948), **4**, 25 (1949); R. Jost and J. Rayski, Helv. Phys. Acta, in press. We have recently learned from Professor H. Yukawa that H. Umezawa and R. Kawabe (Prog. Theor. Phys., in press) have also examined the case of vector mesons with vector coupling using basically non-covariant methods with results that are essentially in agreement with ours; in particular, they are led to impose the same conditions that we do (Eqs. (29) and (29′)) to insure the vanishing of the photon self-energy.

[11] S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3**, **1**, 101 (1948).

terms of which the external electromagnetic field is described. The external current $J_\mu(x)$ is then given by $J_\mu(x) = -(\delta_{\mu\nu}\Box - \partial_\mu\partial_\nu)A_\nu(x)$. The Lagrangian density for a charged scalar field interacting with an external electromagnetic field is[12]

$$\mathcal{L} = -D_\mu^* U^* \cdot D_\mu U - m^2 U^* U,$$

where $D_\mu = \partial_\mu - ieA_\mu$, $D_\mu^* = \partial_\mu + ieA_\mu$; one then finds for the charge-current four vector s_μ which is associated with the above Lagrangian

$$s_\mu = \delta \int \mathcal{L} d\omega / \delta A_\mu = ie(D_\mu^* U^* \cdot U - U^* D_\mu U).$$

On going over to the interaction representation one has

$$\left.\begin{aligned} \mathcal{H}_1 &= -ie(\partial_\mu U^* \cdot U - U^* \partial_\mu U) A_\mu, \\ j_\mu &= ie(\partial_\mu U^* \cdot U - U^* \partial_\mu U) - 2e^2 U^* U (A_\mu + n_\mu n_\nu A_\nu), \end{aligned}\right\} \quad (9)$$

where $n_\mu(x)$ is the unit normal to the spacelike surface σ at the point x $(n_\mu^2 = -1)$. After a brief calculation wherein one employs the relations

$$\left.\begin{aligned} [U(x), U^*(x')] = [U^*(x), U(x')] = i\Delta(x-x'), \\ \langle U^*(x)U(x') + U^*(x')U(x)\rangle_0 = \Delta^{(1)}(x-x'), \end{aligned}\right\} \quad (10)$$

one finds from (8′) and (9) that

$$\delta j_\mu(x) = -e^2 \int K_{\mu\nu}(x-x') A_\nu(x') d\omega', \quad (11)$$

where

$$\begin{aligned} K_{\mu\nu} = &-\partial_\mu \Delta^{(1)} \cdot \partial_\nu \bar{\Delta} - \partial_\nu \Delta^{(1)} \cdot \partial_\mu \bar{\Delta} \\ &+ \partial_\mu \partial_\nu \Delta^{(1)} \cdot \bar{\Delta} + \Delta^{(1)} \cdot \partial_\mu \partial_\nu \bar{\Delta} \\ &+ \delta_{\mu\nu}(-\Delta^{(1)} \cdot \Box \bar{\Delta} - \Box \Delta^{(1)} \cdot \bar{\Delta} + 2m^2 \Delta^{(1)} \bar{\Delta}). \quad (11') \end{aligned}$$

The induced current density no longer involves the timelike normals $n_\mu(x)$ which is as it should be, cancellation of the surface-dependent terms having been effected by use of the identity

$$-1/2 \cdot \int d\omega' F(x, x') \partial_\mu \epsilon(x-x') \cdot \partial_\nu \Delta(x-x') = n_\mu(x) n_\nu(x) F(x, x). \quad (12)$$

As has been emphasized by Jost and Rayski,[10] the induced current $\delta j_\mu(x)$ as given by (11) is by no means clearly gauge invariant; indeed, for gauge invariance one should have $\partial K_{\mu\nu}/\partial x_\nu = 0$ but instead one finds $\partial K_{\mu\nu}/\partial x_\nu = 2\delta(x)\partial_\mu \Delta^{(1)}$ which is undetermined.

A detailed evaluation of (11) wherein the method of integration is along the lines described in references 1* and 5 leads ultimately to the following expression $(\alpha = 1/137)$:

$$\begin{aligned} \delta j_\mu(x) = &-i\alpha/4\pi \\ &\int_{-1}^{1} dy [\exp\{im^2 z - iz/4 \cdot (1-y^2)\Box\}/z]_{z=0} A_\mu(x) \\ &- \alpha/24\pi \cdot \mathcal{I}_2 J_\mu(x) - \alpha/120\pi \cdot (\Box/m^2) J_\mu(x) \\ &- \alpha/1680\pi \cdot (\Box/m^2)^2 J_\mu(x) + \cdots. \quad (13) \end{aligned}$$

In tabulating these and succeeding results, it has been convenient to employ the notations

$$\left.\begin{aligned} \mathcal{I}_1 &= 1/im^2 \cdot \int_{-\infty}^{\infty} \exp(im^2 z)/z^2 \cdot \epsilon(z) dz; \\ \mathcal{I}_2 &= \int_{-\infty}^{\infty} \exp(im^2 z)/z \cdot \epsilon(z) dz, \end{aligned}\right\} \quad (14)$$

where $\epsilon(z) = z/|z|$ and z is a scalar invariant with the dimensionality of a coordinate squared; it is then clear that I_1 diverges as $(1/z)_{z=0}$ or as the square of a momentum while I_2 goes as $(\log z)_{z=0}$. We shall reserve a discussion of (13) until later.

(*b*) *Charged spinor field with magnetic moment* $e/2m - e\delta$. Let $\psi(x)$, $\bar{\psi}(x)$ and $\psi'(x)$, $\bar{\psi}'(x)$ designate the spinor field and its charge conjugate field, respectively. It is then well known that the addition to the Lagrangian density

$$\begin{aligned} \mathcal{L} = -1/2 \cdot \bar{\psi}(\gamma_\mu \partial_\mu + m)\psi - 1/2 \cdot \bar{\psi}'(\gamma_\mu \partial_\mu + m)\psi' \\ + ie/2 \cdot (\bar{\psi}\gamma_\mu \psi - \bar{\psi}'\gamma_\mu \psi') A_\mu \end{aligned}$$

of a term of the form

$$\mathcal{L}' = ie\delta/4 \cdot (\bar{\psi}\sigma_{\mu\nu}\psi - \bar{\psi}'\sigma_{\mu\nu}\psi') F_{\mu\nu},$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the external electromagnetic field, leads to an anomalous magnetic moment for the spinor particle amounting to $-e\delta$.[13] Indeed, the current density is now properly given by

$$\begin{aligned} s_\mu = \delta \int (\mathcal{L} + \mathcal{L}') d\omega / \delta A_\mu = ie/2 \cdot (\bar{\psi}\gamma_\mu \psi - \bar{\psi}'\gamma_\mu \psi') \\ - ie\delta/2 \cdot \partial_\nu (\bar{\psi}\sigma_{\nu\mu}\psi - \bar{\psi}'\sigma_{\nu\mu}\psi'). \end{aligned}$$

It seems to be of some importance to consider this possibility as a real one for there is no *a priori* reason why spin one-half particles (apart from the electron and positron) should be described without an anomalous magnetic moment term. It is a simple matter to determine the Hamiltonian density and the current density

[12] G. Wentzel, *Quantentheorie der Wellenfelder* (Deuticke, Wien, 1943).

* In performing the integrations, we have followed the convention of Pauli and Villars in assigning to $\int d^4k \cdot k_\mu k_\nu \exp(izk_\lambda^2)$ the value $-\delta_{\mu\nu}(\pi^2/2z^3)\epsilon(z)$ although it might seem just as appropriate to have written $-\delta_{\mu\nu}(\pi^2/2z^3)(\epsilon(z) - z\delta(z))$. In any case, it may be readily seen that the only essential effect of the inclusion of the $\delta(z)$ term is to modify all quadratic divergences in the same way so that none of our conclusions with respect to the compensation of the divergences are in any way affected.

[13] W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).

in the interaction representation, *viz.*,

$$\mathcal{H}=\mathcal{H}_1=-ie/2\cdot(\bar{\psi}\gamma_\mu\psi-\bar{\psi}'\gamma_\mu\psi')A_\mu -ie\delta/4\cdot(\bar{\psi}\sigma_{\mu\nu}\psi-\bar{\psi}'\sigma_{\mu\nu}\psi')F_{\mu\nu}, \quad (15)$$

$$\begin{aligned}j_\mu=&ie/2\cdot(\bar{\psi}\gamma_\mu\psi-\bar{\psi}'\gamma_\mu\psi')-ie\delta/2\cdot\partial_\nu(\bar{\psi}\sigma_{\nu\mu}\psi-\bar{\psi}'\sigma_{\nu\mu}\psi')\\ &-e^2\delta(\bar{\psi}\psi+\bar{\psi}'\psi')(A_\mu+n_\mu n_\nu A_\nu)\\ &-e^2\delta^2/4\cdot\{\bar{\psi}[\sigma_{\rho\lambda}(\gamma_\mu+n_\mu n_\nu\gamma_\nu)+(\gamma_\mu+n_\mu n_\nu\gamma_\nu)\sigma_{\rho\lambda}]\psi\\ &+\bar{\psi}'[\sigma_{\rho\lambda}(\gamma_\mu+n_\mu n_\nu\gamma_\nu)+(\gamma_\mu+n_\mu n_\nu\gamma_\nu)\sigma_{\rho\lambda}]\psi'\}F_{\rho\lambda}. \quad (15')\end{aligned}$$

Equations (8′), (15), and (15′) together with the relations

$$\left.\begin{aligned}&[\psi_\alpha(x),\bar{\psi}_\beta(x')]_+=-iS_{\alpha\beta}(x-x'),\\ &\langle\bar{\psi}_\alpha(x)\psi_\beta(x')\rangle_0=(S^{(1)}(x'-x)-iS(x'-x))_{\beta\alpha}/2\end{aligned}\right\}(16)$$

lead ultimately to the following expressions for the induced current density. It is convenient to write $\delta j_\mu(x)$ as the sum of three parts, *viz.*,

$$\delta j_\mu(x)=\delta j_\mu^{(v)}(x)+\delta j_\mu^{(vt)}(x)+\delta j_\mu^{(t)}(x); \quad (17)$$

$\delta j_\mu^{(v)}(x)$ is the current density one would get if one had the vector interaction only between the spinor particle and the electromagnetic field (corresponding to an electric charge but no anomalous magnetic moment), $\delta j_\mu^{(t)}(x)$ is the current density one would get if one had the tensor interaction only (corresponding to an anomalous magnetic moment but no electric charge) and $\delta j_\mu^{(vt)}(x)$ is the cross term. Then

$$\delta j_\mu^{(v)}(x)=-e^2\int K_{\mu\nu}(x-x')A_\nu(x')d\omega', \quad (18)$$

where

$$K_{\mu\nu}=4[\partial_\mu\Delta^{(1)}\cdot\partial_\nu\bar{\Delta}+\partial_\nu\Delta^{(1)}\cdot\partial_\mu\bar{\Delta}-\delta_{\mu\nu}(\partial_\lambda\Delta^{(1)}\cdot\partial_\lambda\bar{\Delta}+m^2\Delta^{(1)}\bar{\Delta})]; \quad (18')$$

$$\delta j_\mu^{(vt)}(x)=8e^2\delta m\int\Delta^{(1)}(x-x')\bar{\Delta}(x-x')J_\mu(x')d\omega'; \quad (19)$$

$$\delta j_\mu^{(t)}(x)=4e^2\delta^2\int M_{\mu\rho\sigma}(x-x')F_{\rho\sigma}(x')d\omega', \quad (20)$$

where

$$\begin{aligned}M_{\mu\rho\sigma}=\partial_\nu[&\delta_{\sigma\mu}(\partial_\nu\Delta^{(1)}\cdot\partial_\rho\bar{\Delta}+\partial_\rho\Delta^{(1)}\cdot\partial_\nu\bar{\Delta})\\ &+\delta_{\nu\rho}(\partial_\sigma\Delta^{(1)}\cdot\partial_\mu\bar{\Delta}+\partial_\mu\Delta^{(1)}\cdot\partial_\sigma\bar{\Delta})\\ &-\delta_{\nu\rho}\delta_{\mu\sigma}(\partial_\lambda\Delta^{(1)}\cdot\partial_\lambda\bar{\Delta}-m^2\Delta^{(1)}\bar{\Delta})]. \quad (20')\end{aligned}$$

We observe once more that $\delta j_\mu(x)$ no longer involves surface-dependent terms, these having canceled in view of (12). We also note that, while $\delta j_\mu^{(v)}(x)$ is in fact not gauge invariant—indeed, as Pauli has pointed out,[1] $\partial K_{\mu\nu}/\partial x_\nu=-4\delta(x)\partial_\mu\Delta^{(1)}$—the introduction of tensor coupling does not lead to any new non-gauge invariant terms. This means that, insofar as the question of the photon self-energy is concerned, it is irrelevant whether or not we have tensor coupling.

Upon carrying through a more explicit evaluation, one finds eventually that

$$\begin{aligned}\delta j_\mu^{(v)}(x)=&i\alpha/2\pi\cdot\int_{-1}^{1}dy[\exp\{im^2z-iz/4\cdot(1-y^2)\Box\}/z]_{z=0}A_\mu(x)\\ &-\alpha/6\pi\cdot\mathcal{I}_2J_\mu(x)-\alpha/15\pi\cdot(\Box/m^2)J_\mu(x)\\ &-\alpha/140\pi\cdot(\Box/m^2)^2J_\mu(x)+\cdots, \quad (21)\end{aligned}$$

$$\delta j_\mu^{(vt)}(x)=\alpha\delta m/\pi\cdot\mathcal{I}_2J_\mu(x)+\alpha\delta m/3\pi\cdot(\Box/m^2)J_\mu(x)+\alpha\delta m/30\pi\cdot(\Box/m^2)^2J_\mu(x)+\cdots, \quad (21')$$

$$\begin{aligned}\delta j_\mu^{(t)}(x)=&-\alpha\delta^2m^2/2\pi\cdot{}_2J_\mu(x)\\ &-\alpha\delta^2m^2/12\pi\cdot(\mathcal{I}_2+2)(\Box/m^2)J_\mu(x)\\ &-\alpha\delta^2m^2/20\pi\cdot(\Box/m^2)^2J_\mu(x)+\cdots. \quad (21'')\end{aligned}$$

(*c*) *Charged vector field with magnetic moment* $(1-\gamma)e/2m$. Let $U_\mu(x)$ and $U_\mu^*(x)$ be the field variables which characterize the charged vector field. Here, too, it is possible to add to the Lagrangian density[12]

$$\mathcal{L}=-1/2\cdot(D_\mu^*U_\nu^*-D_\nu^*U_\mu^*)(D_\mu U_\nu-D_\nu U_\mu)-m^2U_\nu^*U_\nu$$

a term of the form

$$\mathcal{L}'=ie\gamma/2\cdot(U_\mu^*U_\nu-U_\nu^*U_\mu)F_{\mu\nu};$$

this extra term leads to an alteration of the magnetic moment of the vector meson from $e/2m$ to $(1-\gamma)e/2m$.[13] There is again no obvious reason why the vector form of interaction should be favored over the tensor and we consider both possibilities. The current density which is associated with the total Lagrangian is given by

$$\begin{aligned}s_\mu=&\delta\int(\mathcal{L}+\mathcal{L}')d\omega/\delta A_\mu\\ =&ie[U_\nu^*(D_\nu U_\mu-D_\mu U_\nu)-(D_\nu^*U_\mu^*-D_\mu^*U_\nu^*)U_\nu]\\ &-ie\gamma\partial_\nu(U_\nu^*U_\mu-U_\mu^*U_\nu).\end{aligned}$$

On carrying through the transition to the interaction representation, one finds for the linear terms in the Hamiltonian density

$$\mathcal{H}_1=-ie(U_\nu^*f_{\nu\mu}-f_{\nu\mu}^*U_\nu)A_\mu-ie\gamma/2\cdot(U_\mu^*U_\nu-U_\nu^*U_\mu)F_{\mu\nu}, \quad (22)$$

where $f_{\nu\mu}=\partial_\nu U_\mu-\partial_\mu U_\nu$. The current density operator assumes much more formidable proportions (we keep terms to e^2):

$$
\begin{aligned}
&j_\mu \\
&=\{ie(U_\nu{}^*f_{\nu\mu}-f_{\nu\mu}{}^*U_\nu) \\
&-e^2[(A_\mu U_\sigma{}^*-A_\sigma U_\mu{}^*)U_\sigma+U_\sigma{}^*(A_\mu U_\sigma-A_\sigma U_\mu)] \\
&-e^2n_\mu n_\nu[(A_\nu U_\sigma{}^*-A_\sigma U_\nu{}^*)U_\sigma+U_\sigma{}^*(A_\nu U_\sigma-A_\sigma U_\nu)] \\
&-e^2n_\sigma n_\nu[(A_\mu U_\nu{}^*-A_\nu U_\mu{}^*)U_\sigma+U_\sigma{}^*(A_\mu U_\nu-A_\nu U_\mu)] \\
&-e^2/m^2\cdot n_\lambda n_\nu(f_{\lambda\mu}{}^*f_{\nu\sigma}+f_{\nu\sigma}{}^*f_{\lambda\mu})A_\sigma \\
&-e^2\gamma/m^2\cdot n_\lambda n_\nu(f_{\nu\mu}{}^*U_\sigma+U_\sigma{}^*f_{\nu\mu})F_{\lambda\sigma}\} \\
&+\{-ie\gamma\partial_\nu(U_\nu{}^*U_\mu-U_\mu{}^*U_\nu) \\
&-e^2\gamma/m^2\cdot\partial_\lambda'[\{f_{\lambda\nu}{}^*(x')U_\mu(x)+U_\mu{}^*(x)f_{\lambda\nu}(x')\}A_\nu(x')]_{x=x'} \\
&-e^2\gamma n_\nu n_\lambda[U_\nu{}^*(A_\lambda U_\mu-A_\mu U_\lambda)+(A_\lambda U_\mu{}^*-A_\mu U_\lambda{}^*)U_\nu] \\
&-e^2\gamma/m^2\cdot n_\nu n_\lambda(f_{\sigma\lambda}{}^*\partial_\nu U_\mu+\partial_\nu U_\mu{}^*\cdot f_{\sigma\lambda})A_\sigma \\
&+e^2\gamma/m^2\cdot[(n_\mu n_\nu\partial_\rho'+n_\nu n_\rho\partial_\mu'+n_\rho n_\mu\partial_\nu'+2n_\mu n_\nu n_\rho n_\lambda\partial_\lambda') \\
&\cdot(\{U_\nu{}^*(x)f_{\sigma\rho}(x')+f_{\sigma\rho}{}^*(x')U_\nu(x)\}A_\sigma(x'))]_{x=x'} \\
&-e^2\gamma^2/m^2\cdot\partial_\lambda'[\{U_\nu{}^*(x')U(_\mu x)+U_\mu{}^*(x)U_\nu(x')\}F_{\lambda\nu}(x')]_{x=x'} \\
&-e^2\gamma^2/m^2\cdot n_\nu n_\rho(U_\sigma{}^*\partial_\nu U_\mu+\partial_\nu U_\mu{}^*\cdot U_\sigma)F_{\sigma\rho} \\
&-e^2\gamma^2/m^2\cdot[(n_\mu n_\nu\partial_\sigma'+n_\nu n_\sigma\partial_\mu'+n_\sigma n_\mu\partial_\nu'+2n_\mu n_\nu n_\sigma n_\lambda\partial_\lambda') \\
&\cdot(\{U_\nu{}^*(x)U_\rho(x')+U_\rho{}^*(x')U_\nu(x)\}F_{\sigma\rho}(x'))]_{x=x'}\};
\end{aligned}
\tag{22'}
$$

the first grouping of terms on the right-hand side of (22′) is the interaction representation transcription of the "vectorial" part of the current density with the second grouping corresponding to the "tensorial" part.

One may next calculate the induced current density $\delta j_\mu(x)$ using (8′), (22) and (22′) together with the relations

$$
\left.
\begin{aligned}
&[U_\mu(x),\,U_\nu{}^*(x')]=[U_\mu{}^*(x),\,U_\nu(x')] \\
&\qquad=i(\delta_{\mu\nu}-1/m^2\cdot\partial_\mu\partial_\nu)\Delta(x-x'), \\
&\langle U_\mu{}^*(x)U_\nu(x')+U_\nu{}^*(x')U_\mu(x)\rangle_0 \\
&\qquad=(\delta_{\mu\nu}-1/m^2\cdot\partial_\mu\partial_\nu)\Delta^{(1)}(x-x').
\end{aligned}
\right\}
\tag{23}
$$

It is again convenient to express $\delta j_\mu(x)$ as the sum of a pure vector part $\delta j_\mu{}^{(v)}(x)$, a pure tensor part $\delta j_\mu{}^{(t)}(x)$ and a cross term $\delta j_\mu{}^{(vt)}(x)$, i. e.,

$$\delta j_\mu(x)=\delta j_\mu{}^{(v)}(x)+\delta j_\mu{}^{(vt)}(x)+\delta j_\mu{}^{(t)}(x), \tag{24}$$

where

$$\delta j_\mu{}^{(v)}(x)=-e^2\int K_{\mu\nu}(x-x')A_\nu(x')d\omega' \tag{25}$$

with

$$
\begin{aligned}
K_{\mu\nu}=3[&-\partial_\mu\Delta^{(1)}\cdot\partial_\nu\bar{\Delta}-\partial_\nu\Delta^{(1)}\cdot\partial_\mu\bar{\Delta} \\
&+\partial_\mu\partial_\nu\Delta^{(1)}\cdot\bar{\Delta}+\Delta^{(1)}\cdot\partial_\mu\partial_\nu\bar{\Delta} \\
&+\delta_{\mu\nu}(-\Delta^{(1)}\cdot\Box\bar{\Delta}-\Box\Delta^{(1)}\cdot\bar{\Delta}+2m^2\Delta^{(1)}\bar{\Delta})] \\
-2/m^2\cdot[&\partial_\mu\partial_\nu\Delta^{(1)}\cdot\Box\bar{\Delta}+\Box\Delta^{(1)}\cdot\partial_\mu\partial_\nu\bar{\Delta} \\
&-\partial_\mu\partial_\rho\Delta^{(1)}\cdot\partial_\nu\partial_\rho\bar{\Delta}-\partial_\nu\partial_\rho\Delta^{(1)}\cdot\partial_\mu\partial_\rho\bar{\Delta} \\
&+\delta_{\mu\nu}(\partial_\rho\partial_\sigma\Delta^{(1)}\cdot\partial_\rho\partial_\sigma\bar{\Delta}-\Box\Delta^{(1)}\cdot\Box\bar{\Delta})];
\end{aligned}
\tag{25'}
$$

$$
\begin{aligned}
\delta j_\mu{}^{(vt)}(x)=-2e^2\gamma\int d\omega'[&\Delta^{(1)}(x-x')\bar{\Delta}(x-x') \\
&-1/m^2\cdot\partial_\sigma\Delta^{(1)}(x-x')\cdot\partial_\sigma\bar{\Delta}(x-x')]J_\mu(x');
\end{aligned}
\tag{26}
$$

$$\delta j_\mu{}^{(t)}(x)=-e^2\gamma^2\int M_{\mu\rho\sigma}(x-x')F_{\rho\sigma}(x')d\omega' \tag{27}$$

with

$$
\begin{aligned}
M_{\mu\rho\sigma}=\partial_\nu[&2\delta_{\mu\sigma}\delta_{\nu\rho}\Delta^{(1)}\bar{\Delta} \\
&+1/m^2\cdot\delta_{\nu\rho}(\partial_\mu\Delta^{(1)}\cdot\partial_\sigma\bar{\Delta}+\partial_\sigma\Delta^{(1)}\cdot\partial_\mu\bar{\Delta}) \\
&-1/m^2\cdot\delta_{\mu\sigma}(\Delta^{(1)}\cdot\partial_\rho\partial_\nu\bar{\Delta}+\partial_\rho\partial_\nu\Delta^{(1)}\cdot\bar{\Delta}) \\
&-1/m^4\cdot\delta_{\nu\rho}(\partial_\sigma\partial_\lambda\Delta^{(1)}\cdot\partial_\mu\partial_\lambda\bar{\Delta}+\partial_\mu\partial_\lambda\Delta^{(1)}\cdot\partial_\sigma\partial_\lambda\bar{\Delta} \\
&\qquad-\partial_\mu\partial_\sigma\Delta^{(1)}\cdot\Box\bar{\Delta}-\Box\Delta^{(1)}\cdot\partial_\mu\partial_\sigma\bar{\Delta})].
\end{aligned}
\tag{27'}
$$

To bring about the cancellation of the surface-dependent terms in $\delta j_\mu(x)$, use was made of (12) plus the additional identity

$$
\begin{aligned}
-1/2\cdot\int d\omega'F(x,x')\{&\partial_\nu\partial_\mu\partial_\rho[\epsilon(x-x')\Delta(x-x')] \\
&-\epsilon(x-x')\partial_\nu\partial_\mu\partial_\rho\Delta(x-x')\} \\
=[(n_\mu n_\nu\partial_\rho'&+n_\nu n_\rho\partial_\mu'+n_\rho n_\mu\partial_\nu' \\
&+2n_\mu n_\nu n_\rho n_\sigma\partial_\sigma')F(x,x')]_{x=x'}.
\end{aligned}
\tag{12'}
$$

We note that, as in the spinor case, it is $\delta j_\mu{}^{(v)}(x)$ which is not gauge invariant and the introduction of the tensor coupling does not lead to any new non-gauge invariant terms which means, as before, that the amount of tensor coupling will not play a role in the discussion of the photon self-energy. Indeed, we observe that the first part of $K_{\mu\nu}$ (25′) is exactly three times the corresponding quantity for the case of a charged scalar field (11′) whereas the second part leads to a gauge invariant current; one may readily verify that for the charged vector meson field $\partial K_{\mu\nu}/\partial x_\nu=6\delta(x)\partial_\mu\Delta^{(1)}$.

The ultimate evaluation of $\delta j_\mu(x)$ leads to the following results:

$$
\begin{aligned}
\delta j_\mu{}^{(v)}(x)=&-3i\alpha/4\pi \\
&\cdot\int_{-1}^{1}dy[\exp\{im^2z-iz/4\cdot(1-y^2)\Box\}/z]_{z=0}A_\mu(x) \\
&+\alpha/4\pi\cdot(\mathcal{J}_1-\mathcal{J}_2/2)J_\mu(x) \\
&-\alpha/24\pi\cdot(\mathcal{J}_2+3/5)(\Box/m^2)J_\mu(x) \\
&\qquad-17\alpha/1680\pi\cdot(\Box/m^2)^2J_\mu(x)+\cdots,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\delta j_\mu{}^{(vt)}(x)=&-\alpha\gamma/2\pi\cdot(\mathcal{J}_1+\mathcal{J}_2/2)J_\mu(x) \\
&+\alpha\gamma/8\pi\cdot(\mathcal{J}_2-2/3)(\Box/m^2)J_\mu(x) \\
&+\alpha\gamma/40\pi\cdot(\Box/m^2)^2J_\mu(x)+\cdots,
\end{aligned}
\tag{28'}
$$

$$
\begin{aligned}
\delta j_\mu{}^{(t)}(x)=&-\alpha\gamma^2/4\pi\cdot(\mathcal{J}_1-\mathcal{J}_2)J_\mu(x) \\
&+\alpha\gamma^2/16\pi\cdot(\mathcal{J}_1-2\mathcal{J}_2/3+4/3)(\Box/m^2)J_\mu(x) \\
&-\alpha\gamma^2/96\pi\cdot(\mathcal{J}_2+4/5) \\
&\qquad\times(\Box/m^2)^2J_\mu(x)+\cdots.
\end{aligned}
\tag{28''}
$$

4. DISCUSSION

The essential content of the preceding section is contained in Eqs. (13), (21–21″) and (28–28″). We have already remarked that the amount of tensor coupling has no bearing on the photon self-energy whether we are considering spinor or vector mesons. If we let N_{sc}, N_{sp} and N_v denote the number of charged scalar, spinor and vector fields which interact simultaneously with the external electromagnetic field and $m_{sc}^{(i)}$, $m_{sp}^{(i)}$ and $m_v^{(i)}$ the corresponding masses, then it is evident that the photon self-energy will vanish provided

$$2N_{sp}=N_{sc}+3N_v, \tag{29}$$

$$2\Sigma(m_{sp}^{(i)})^2=\Sigma(m_{sc}^{(i)})^2+3\Sigma(m_v^{(i)})^2; \tag{29'}$$

these equations are the analogs of the regulator conditions (2) and (2′) and clearly allow many possibilities.

A corresponding cancellation of the divergences which arise in the gauge invariant terms in the current density is not possible however. As has been observed by many authors,[10] the charge renormalization factors in the scalar and spinor cases (the latter with vector coupling only) diverge logarithmically and have the same sign. The introduction of an anomalous magnetic moment for the spinor field intensifies the difficulty in that it leads to a logarithmically divergent $\Box J_\mu$ term. Unfortunately, this last infinity cannot be canceled by the corresponding one which occurs in the case where we have a charged vector meson field (with vector coupling only) for here, again, the signs in the two situations are the same. Indeed, in the vector meson case we obtain also a quadratic divergence in the charge renormalization term. Finally, the introduction of tensor coupling for the vector meson field leads to a $(\Box)^2J_\mu$ divergence.

The inference to be drawn from all this is certainly not that the realistic approach is an incorrect one. Indeed, the very closedness of quantum electrodynamics on the one hand together with the complete failure of all meson theories on the other are fully compatible with and emphasize the necessity for a realistic description of elementary particles. We can only conclude that the usual linear field theories do not seem to be adequate for the job although the possibility that it is actually the perturbation theory which is at fault cannot be completely ignored.

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