Absorption of Gamma-Rays in Aluminum

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The scattering cross section for gamma-rays in aluminum was measured by two independent methods at the three energies 1.11, 1.71, and 2.76 Mev. The results 0.156, 0.125, and 0.0996 cm^{-1}, respectively, for the three energies are in agreement with the predictions of theory within the experimental error of ± 1 percent. The essential features of the measurements are: (1) Choice of gamma-ray energies and absorbing material such that Compton scattering is the only important process. (2) Use of a geometry which effectively eliminates single and multiple scattering and permits an accurate determination of the background. (3) Elimination of errors due to the other gamma-ray energies emitted by the source. (4) Determination of corrections for absorber impurities, counter deadtime, and random fluctuations in the counters and counter circuits. The two methods of measurement differ primarily in the way in which the errors due to the other gamma-rays emitted by the source are removed.

INTRODUCTION

DURING the course of the last twenty years, many attempts have been made to verify experimentally the Klein-Nishina formula[1] which predicts the angular distribution and total cross section for scattering of gamma-rays. Meitner and Hupfeld,[2] Chao,[3] and Tarrant[4] measured the cross section for carbon using the 2.6-Mev gamma-rays of ThC'' by the straight absorption method. Their results were in agreement with the theory within their experimental accuracy of approximately 5 percent. The cross section in carbon and aluminum in the region from 0.04 to 0.6 Mev was investigated by Hewlett,[5] Allen,[6] and Read and Lauritsen[7] using Coolidge x-ray tubes and crystal spectrometers as a source of monochromatic radiation. Their results were in reasonable agreement with theory although it was necessary to make large corrections for the photoelectric cross section. Numerous measurements[8] were made in the region from 0.6 to 2.6 Mev with results deviating as much as 20 percent from the theory. The large variation in results in the intermediate region led the author to review the methods of measurement with a view toward making a more accurate determination of the validity of the formula.

A check of the Klein-Nishina formula is provided by a measurement either of the angular distribution of the scattered gamma-rays or of the total scattering cross section. In order to determine the total cross section with reasonable accuracy, several sources of error must be considered. First, it is necessary to eliminate or at least take accurate account of the other two of the three principal processes by which gamma-rays are absorbed; namely, the photoelectric effect and pair formation. Since it is extremely difficult experimentally to distinguish reliably between the three, it is essential that the measurements be made in the energy region from one to three Mev, where, for elements of low atomic number, the photoelectric and pair formation cross sections are very small compared to that of the Compton process. An additional factor which undoubtedly contributed to the variations in the results of the earlier investigators is single and multiple scattering. Fortunately, the strong sources now available through the facilities of the Atomic Energy Commission make possible the use of a geometry which effectively eliminates this difficulty, as may be verified both by calculation and by measurement. However, the use of such sources introduces a third source of error which is demonstrated very strikingly in the case of Sb^{124}, for which until the recent work of Kern[9] and co-workers, only two gamma-rays were reported. It is now apparent that gamma-rays of five different energies are emitted. Therefore, it is necessary that the methods employed for the measurement be capable of selecting one energy of the several emitted by the source so that the final measurements can be made with essentially monoenergetic radiation. Other factors which may introduce errors are the impurity of the absorber, bremsstrahlung of the beta-rays and secondary electrons, and instability in the measuring equipment.

METHODS OF MEASUREMENT

Four methods of measurement were investigated but, of these, two were ultimately rejected as being unsuitable for producing accurate results. The first, which utilized the proportional properties of the crystal counter as a means of separating in energy the various gamma-rays emitted by the source, was discarded

[1] O. Klein and Y. Nishina, Zeits. f. Physik **52**, 853 (1929).
[2] L. Meitner and H. Hupfeld, Zeits. f. Physik **67**, 147 (1930).
[3] C. Y. Chao, Phys. Rev. **36**, 1519 (1930).
[4] G. T. B. Tarrant, Proc. Roy. Soc. **128**, 345 (1930).
[5] C. W. Hewlett, Phys. Rev. **17**, 284 (1921).
[6] S. J. M. Allen, Phys. Rev. **27**, 266 (1926).
[7] J. Read and C. C. Lauritsen, Phys. Rev. **45**, 433 (1934).
[8] W. Genter lists the references for many of the earlier papers in J. de phys. et rad. **6**, 274 (1935). Some of the later papers are: C. C. Lauritsen and J. R. Oppenheimer, Phys. Rev. **46**, 80 (1934); G. Groetzinger and L. Smith, Phys. Rev. **67**, 53 (1945); J. M. Cork and R. W. Pidd, Phys. Rev. **66**, 227 (1944); J. M. Cork, Phys. Rev. **67**, 53 (1945); C. L. Cowan, Phys. Rev. **74**, 1841 (1948); and, C. L. Davisson and R. D. Evans, Phys. Rev. **73**, 1239 (1948). The results of the last two papers are in reasonably good agreement with the predictions of theory.
[9] Kern, Zaffarano, and Mitchell, Phys. Rev. **73**, 1142 (1948).

because of polarization effects in the crystal. The second, in which a beta-ray spectrometer was used to distinguish between the gamma-rays by selecting the secondary electrons ejected from a thin lead target, was rejected primarily because of the low counting rate obtained with a geometry suitable for eliminating errors due to scattering. The two methods actually used are termed the "filter" and the "coincidence" methods.

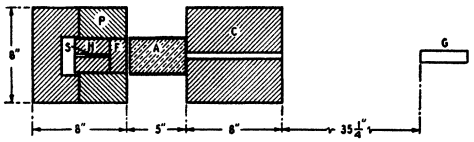

FIG. 1. Geometrical arrangement of the source S, absorber A, collimator C, and Geiger gamma-ray counter G, for the filter method. F=filter, H=source holder, P=lead shield.

The Filter Method

This method, which for the ideal case of a monoenergetic source would be the most straightforward, is the one employed by the majority of the earlier investigators. The geometrical arrangement is shown in Fig. 1. The measured absorption coefficient is defined through the relation $\tau_m=(1/x)\log N_0/N$ where x is the thickness of the absorbing material and N and N_0 are the true counting rates, proportional to the number of quanta transmitted unscattered, with and without the absorber in the beam, respectively. For a monoenergetic source τ_m is the true value of the absorption coefficient for the material of the absorber. In the actual case in which more than one energy of gamma-rays is emitted, it may be shown that by filtering the radiation, say through lead, the measured coefficient approaches the true value for that energy which has the least absorption in the filter. A check on the theory is obtained by plotting τ_m as a function of the filter thickness and comparing these data with the curve calculated using the theoretical values of the coefficients.

The principal features of the geometry are, first, the small solid angle subtended by the counter and, second, the method of determining the background counting rate. The background was measured for each filter thickness both with and without the absorber in the path by filling the axial hole in the collimator with lead. It was shown that this eliminated all the radiation reaching the counter through the collimator. Hence, the background includes all scattered radiation reaching the counter by paths other than through the collimator, and this is independent of whether a measurement is being made of the actual counting rate or of the background associated with it. Therefore, the only error in the measurement due to background is that due to single and multiple scattering occurring within the slit of the collimator. This was shown to be less than one percent by measuring the absorption coefficient as a function of the distance of the counter from the collimator. The results are in good agreement with calculations which show that the total error due to scattering[10] should be less than 0.2 percent.

The Coincidence Method

The coincidence method was essentially that used by Groetzinger and Smith.[11] The geometrical arrangement was the same as that in Fig. 1, with the exception that the filter was omitted and the single counter was replaced by two beta-counters with their thin mica windows facing each other. An aluminum filter placed between the counters was just thick enough to stop completely the beta-rays ejected from the first counter by the lower energy gamma-rays. The coincidence counting rate was then determined with and without the absorber in the collimated beam. The background, due mainly to accidental coincidences, was determined with an aluminum absorber between the counters just thick enough to stop completely the electrons ejected from the first counter by the highest energy gamma-rays. These data then permitted a determination of the absorption coefficient for the highest energy line emitted by the source.

INSTRUMENTATION

Although the only instrumentation involved in the final measurements was usual counting equipment, a detailed knowledge of the operating characteristics was necessary for accurate measurements. The regulated high voltage supply for the Geiger counters was stable to better than 1 percent for large fluctuations in the line voltage. A scale of 64 counting circuit was constructed having a resolving time of about 20 microseconds, sufficiently fast considering the deadtime of the counters. The coincidence circuit was a modified Rossi type, with a resolving time of 1.6 microseconds, again sufficient for the relatively low counting rates used. The counting time was determined by two synchronous electric clocks connected in parallel, a precaution found to be necessary for the particular type of clock used.

The Geiger counters were constructed especially for the measurements. The gamma-ray counters were made from brass tubing four inches long and one inch in diameter fitted with an eight-mil central wire. The beta-counters were the end-fire type, one inch in diameter with mica windows approximately one mil thick. One counter was two inches long but the other, placed nearest the collimator was only $1\frac{1}{8}$ inches long. The reason for such small length to width ratio is clear on consideration of the angle of scattering *versus* energy of the secondary Compton electrons.

All the counters were subjected to periodic tests. Whenever the slope of the plateau curve became greater than 3 percent per 100 volts for the gamma-counters and 5 percent per 100 volts for the beta-counters, or whenever the length of the plateau became

[10] I am indebted to Dr. E. S. Lennox for making this calculation.
[11] G. Groetzinger and L. Smith, Phys. Rev. **67**, 53 (1945).

less than 150 volts, the counters were refilled. An alcohol-argon mixture at 11 cm pressure was used. Because of their short length and large mica window, the beta-counters showed considerable hysteresis, thus care was taken not to reduce the applied voltage during the period of measurement. Statistical checks were made occasionally to be sure that multiple counts were not occurring. Because of the difference in counting rates with and without the absorber in the beam, it was necessary to correct for the counting losses due to the counter deadtime. For low counting rates it is difficult to measure the counting losses with accuracy. Therefore, the corrections were based on the measured value of the deadtime. Because the deadtime is a sensitive function of the voltage above the threshold, the counters were always operated 100 volts above the threshold.

EXPERIMENTAL PROCEDURE AND RESULTS

In order to reduce the errors due to random variations, a definite procedure was followed in taking the data. The equipment was allowed to warm up for approximately twenty minutes. The four counting rates necessary to calculate τ_m were determined in the following time sequence: (1) without absorber in the beam, 100 seconds, (2) with the absorber, 300 seconds, (3) background with absorber, 100 seconds, and (4) background without absorber, 100 seconds. This sequence was repeated many times so that in one set of data variations of the counters, circuits, natural background, or decay of the source would average out. The thickness of the aluminum absorber was adjusted so that the counting rate with the absorber in the beam was approximately one-third that without the absorber. The optimum thickness (to obtain the greatest statistical accuracy in a given time) is a function of the ratio of the counting rate to the background.[12] With the low counting rates sometimes prevailing in these measurements, the ratio was relatively small; hence, the absorber thickness was optimized for this condition. For the filter method, each point plotted on the filter curve is an average of two or three such sets, while at least ten sets were obtained at each energy by the coincidence method.

The three sources employed in these measurements were Zn^{65}, Sb^{124}, and Na^{24}. The energies of the gamma-rays emitted were taken from the latest published data.[13] The first two sources were obtained from Oak Ridge, while the Na^{24} was made in the Michigan cyclotron by bombarding sodium meta-borate with deuterons. The source was allowed to stand 24 hours before being used for the filter method to allow decay of the short half-life impurities which could exist. Tests were made which indicated there were no impurities present of energies comparable with those of Na^{24} at the time of measurement.

A filter curve was obtained for two of the three sources. It was not feasible in the case of Zn^{65} because of the low intensity of the annihilation radiation compared to the intensity of 1.11-Mev line. To obtain a curve required such thickness of filter that the counting rates were reduced below practical values. Plots of the experimental data for Sb^{124} and Na^{24} are given in Figs. 2 and 3, respectively, along with the calculated filter curves. The short vertical lines represent the standard deviation for each point based on the total number of counts.

The results of the coincidence measurements for the three sources are given in Table I. The data include certain corrections which are discussed below. The theoretical values of the absorption coefficients were calculated from the formulas given in Heitler,[14] using the latest values of the fundamental constants.

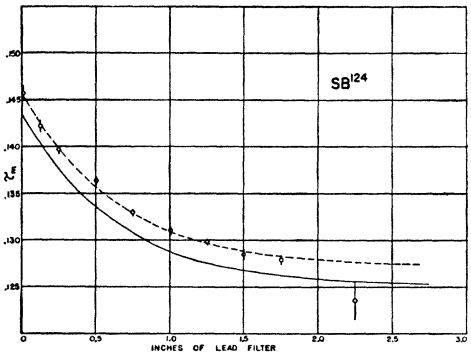

FIG. 2. The measured absorption coefficient *versus* thickness of lead filter for Sb^{124}. The vertical lines through each point represent the standard deviation based on the total number of counts. The solid curve was calculated using the theoretical values of the absorption coefficients. The dotted curve was obtained by adjusting slightly the values of k and τ in lead, holding fixed the value of τ in aluminum.

[12] This calculation was an extension of one of the cases treated by R. Peierls, Proc. Roy. Soc. **A149**, 467 (1935). More recently M. E. Rose and M. M. Shapiro, Phys. Rev. **74**, 1853 (1948), have treated this and additional cases in some detail.

DISCUSSION OF RESULTS

The filter curves (solid curves in Figs. 2 and 3) were calculated using the theoretical values of the absorption coefficients by the relation

$$\tau_m = \tau_2{}^a + \frac{1}{x_2}\log\frac{[1+\sum k_i e^{(\tau_1{}^a-\tau_1{}^i)x_1}]}{[1+\sum k_i e^{(\tau_1{}^a-\tau_1{}^i)x_1} e^{(\tau_2{}^a-\tau_2{}^i)x_2}]},$$

where the subscripts 1 and 2 refer to lead and aluminum, respectively, and the letter superscripts refer to a particular gamma-ray energy. Thus, $\tau_2{}^a$ indicates the total absorption coefficient in aluminum for the gamma-ray of energy a. $k^i = \epsilon_i N_0{}^i/\epsilon_a N_0{}^a$, where ϵ is the counter efficiency for energy i, and $N_0{}^i$ is the number of gamma-rays emitted by the source per

[13] Kern, Zaffarano, and Mitchell, reference 9; C. Sharp Cook and L. M. Langer, Phys. Rev. **73**, 1149 (1948); Jensen, Laslett, and Pratt, Phys. Rev. **73**, 529 (1948); K. Siegbahn, Phys. Rev. **70**, 127 (1946).

[14] W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1944).

second of energy i. Thus, to obtain the curve, it is necessary to know the energies and the intensities of all the gamma-rays emitted by the source, as well as the efficiency of the counter for the various rays. For the sources used in these measurements, the energies are believed known to about 1 percent and the intensities to perhaps 5 percent. The efficiency of a Geiger counter for the various cathode materials as a function of energy was determined by Bradt and co-workers.[15] A check on the consistency of these data was made for Na^{24}, which emits two lines of equal intensity, by plotting the usual absorption curve. From this curve the ratio k may be determined and compared with the calculated value.

A comparison of the data with the calculated filter curve for Sb^{124} might be considered as indicating a discrepancy of approximately 2 percent in view of the fact that the shapes of the curves agree very well. By increasing the value of $\tau_2{}^a$ by 2 percent, the curves may be brought into very good agreement. However, this indicates clearly the difficulty of the method, particularly when so many energies are emitted by the source that it is not feasible to check the consistency of the data as indicated above. The value of k depends on the efficiency of the counter and the intensities of the lines. The efficiency depends somewhat on the geometry of the counter and thus the values taken from Bradt should not be considered accurate unless his geometry is reproduced.[16] Likewise, the intensities are not known with certainty. In addition, although the energies of the gamma-rays are known to 1 percent, the values of the total absorption coefficients in lead for the various energies are not known with that accuracy. The dotted curve which fits the data very well was obtained by adjusting slightly the values of k and τ in lead, holding fixed the value of τ in aluminum, which depends only on the Compton cross section. It might be pointed out that for the data available[17] prior to Kern's, no reasonable fit was possible.

The case for Na^{24} is less open to question as only two lines are believed to exist. The calculated curve is about 0.5 percent low. A slight change in the coefficients for lead and in k, well within their uncertainty, gives very good agreement as is indicated by the dotted curve. There is apparently some very low energy radiation present which is rapidly filtered out.

Although a filter curve was not obtained for Zn^{65}, a measurement was made of the apparent coefficient without any filter. When corrected for the three to five percent annihilation radiation emitted,[18] the measured value becomes 0.1567 cm^{-1}, which is in good agreement with the theoretical value of 0.1572 cm^{-1}.

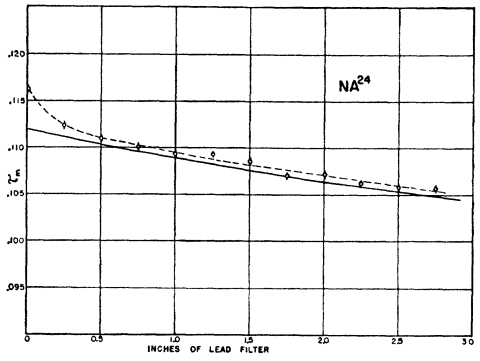

FIG. 3. The filter curve for Na^{24}.

The results of the coincidence method are less susceptible to these uncertain errors, and the measured values show agreement with the theoretical values within the standard deviation of the data. Of the three energy values for which the coefficient was measured in aluminum, perhaps the most reliable is that at 1.11 Mev. The photoelectric and pair cross sections are completely negligible at this energy, and the source is the most ideal of the three, the only other energy emitted being the annihilation radiation which is sufficiently far removed to allow good filtering of the secondary electrons. Perhaps the least reliable is antimony. At the time of measurement, it was believed that the highest energy emitted was 1.71 Mev. The subsequent values listed by Kern indicated a weak gamma-ray at 2.04 Mev, with an intensity of 3 percent. It would, therefore, be expected that the data would be slightly lower than the theoretical value for 1.71 Mev.

As was indicated previously, certain corrections were applied to the original data. The solid angle subtended by the counter was such that radiation scattered through slightly less than 1° would reach the counter. On the basis of the differential cross section, this amounts to an error of about 0.2 percent in the counting rate with the absorber in the beam. A second correction is the loss of true counts due to the deadtime of the counter. For the counting rates used, the correction to τ amounted to less than 0.5 percent for the filter method and approximately 0.9 percent for the coincidence method. An additional correction of 0.3 percent was made for the impurity of the absorber. The aluminum was analyzed by quantitative spectroscopic means and was found to contain iron as the major impurity. The error due to bremsstrahlung of the beta-rays emitted by the source was completely negligible.

A check on the stability of the circuits was furnished by comparing the standard deviation calculated from the number of counts with the standard deviation calculated from the deviation of the individual sets from the mean. These were in close agreement for zinc and sodium indicating no large random fluctuations. However, for antimony, the first energy measured, there was an appreciable difference. This was later found to

[15] H. Bradt *et al.*, Helv. Phys. Acta **19**, 77 (1946).
[16] K. Siegbahn also indicates a discrepancy, Phys. Rev. **70**, 127 (1946).
[17] W. E. Meyerhof and G. Scharff-Goldhaber, Phys. Rev. **72**, 273 (1947).
[18] W. M. Good and W. C. Peacock, Phys. Rev. **69**, 680 (1946); E. N. Jensen (private communication).

be due to fluctuations in the counter supply and the coincidence circuit, and were subsequently eliminated. The standard deviation given in Table I for antimony was calculated from the deviation of the individual sets from the mean.

The measured values for the total cross section are in very good agreement (approximately 1 percent) with the predictions of theory (Table I and Figs. 2 and 3). Any attempt to obtain a closer check at the present time would have little meaning inasmuch as the energies of the gamma-rays are known only to about 1 percent. The results of this investigation indicate, therefore, that the Klein-Nishina formula is in agreement with experiment in the energy range one to three Mev for aluminum.

TABLE I. Absorption coefficients for aluminum (cm^{-1}).

Energy Mev	Theoretical Compton	Photo-electric	Pair	Total	Measured value
1.11	0.1572	0.0001	——	0.157	0.156±0.001
1.708	0.1253	——	0.0009	0.126	0.125±0.002
2.76	0.0949	——	0.0042	0.0991	0.0996±0.0008

ACKNOWLEDGMENTS

The work described in this paper has proceeded at intervals over a period of the last two years. During the course of this time, many people have contributed to the end result. The author would like to express his appreciation to Professors H. R. Crane and J. M. Cork, who originally suggested the investigation, and to Professor D. M. Dennison, all of whom offered valuable advice during the early part of the work. I am indebted to Messrs. Roger Grismore, J. S. King, and J. W. Teener for the many hours they have spent in taking and analyzing data, and to Messrs. H. Westrick and O. Haas for their invaluable aid in building and altering equipment as needed. This work was supported in part by the Bureau of Ordnance, U. S. Navy, under Contract NOrd-7924.

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Theory of Complex Spectra. IV

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The calculation of the coefficients of fractional parentage and of the energy matrices for the configurations f^n is simplified very much by the use of the theory of groups. Tables of results are given.

1. INTRODUCTION

IT was shown in two previous papers[1] that the calculations on complex spectra may be simplified by the introduction of tensor operators and coefficients of fractional parentage. These coefficients may be calculated by Eqs. (9) of III and (11) of III, but it appears that for the configurations f^n Eqs. (11) of III are too cumbersome for practical use.

By considering the meaning and the properties of the coefficients of fractional parentage from the standpoint of the theory of groups, we shall see that these calculations may be somewhat simplified and that a very fortunate and important simplification takes place exactly for the configurations f^n.

In Section 4 we shall classify the states of f^n as the basis of some group representations and in Section 5 we shall find some properties of the coefficients of fractional parentage which will avoid the use of Eqs. (11) of III; the results of the calculations will be given in Tables III and IV. The energy matrices will be calculated in Section 6, and also these calculations will be simplified by group-theoretical considerations.

Before treating the very argument of this paper, we shall give in Section 2 a formula which should have its natural place in Section 5 of III, but was unfortunately obtained only after the publication of that paper, and we shall prove in Section 3 a corollary of Schur's lemma, which will be very useful in the following calculations.

2. THE MATRIX OF SYMMETRIC SCALAR OPERATORS

The matrix components between two states of l^n of the scalar operator (30) of III were calculated in (33a) of III by taking only the last term of the summation and then multiplying the result by $\frac{1}{2}n(n-1)$. It appears, on the contrary, more convenient to limit the sum of (30) of III to the first $n-1$ electrons and then to multiply by $n/(n-2)$. Thus, we obtain easily

$$(l^n\alpha SL|G|l^n\alpha' SL) = [n/(n-2)] \sum_{\alpha_1\alpha_1' S_1L_1} (l^n\alpha SL\{|l^{n-1}(\alpha_1 S_1L_1)lSL) \times (l^{n-1}\alpha_1 S_1L_1|G|l^{n-1}\alpha_1' S_1L_1) \times (l^{n-1}(\alpha_1' S_1L_1)lSL|\}l^n\alpha' SL). \quad (1)$$

[1] G. Racah, Phys. Rev. **62**, 438 (1942) and **63**, 367 (1943) (which will be referred to as II and III. We refer to these papers for definitions and notations.