The Production of Mesons by Photons and Electrons*

HERMAN FESHBACH
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

AND

MELVIN LAX
Department of Physics, Syracuse University, Syracuse, New York

May 9, 1949

THE recent[1] laboratory production of π-mesons by electromagnetic means makes a comparison with the predictions of various meson theories feasible, permitting then the discovery of the fundamental properties of mesons and the nature of their interaction with nuclei. It seems thus appropriate to collect here some of the results given in a review paper on this subject which we shall submit for publication in the near future.

We have considered the production of pseudoscalar and vector mesons. The nucleon-meson coupling energies assumed are (in the non-relativistic limit)

$$H_p=(4\pi)^{\frac{1}{2}}f'\int \psi_n{}^*\boldsymbol{\sigma}\cdot\nabla\phi^*\psi_p d\tau,$$

$$H_v=(4\pi)^{\frac{1}{2}}f\int \psi_n{}^*\boldsymbol{\sigma}/(\nabla\times\boldsymbol{\phi}^*)\psi_p d\tau-i(4\pi)^{\frac{1}{2}}g\int \psi_n{}^*\phi_0{}^*\psi_p d\tau$$

where we have included only the terms which are relevant when the struck particle is the proton. The units used are such that $\hbar=1$, $c=1$, $\mu c^2=1$, $\mu=$ meson mass. The functions $\psi_n{}^*\psi_p$ are the neutron and proton wave-functions, $\phi=$ pseudoscalar meson wave function, $(\boldsymbol{\phi}, \phi_0)$ the vector meson wave-functions, f', f, g are coupling constants. Both strong[2] and weak coupling have been considered and yield results which are in substantial agreement.

The present calculations should give the predictions of the theory correctly for photon energies $\nu'<M$ where ν' is the photon energy in laboratory system and M is the nucleon mass. We have therefore included more completely (but with some approximations) the effects of nucleon recoil which had been neglected in previous calculations.[3,4] Field reactions have been neglected; qualitative estimates indicate that for $\nu'<M$, these effects will be small. This is verified to some extent by the calculations of Hamilton, Heitler, and Peng[3] of the "resistive" part of field reaction according to the methods of Heitler and Peng. These authors however make the additional assumption that $M=\infty$.

Our results are most easily given graphically. The photon energy ν, is the photon energy in the center of mass system so that $\nu=\nu'[1+(2\nu'/M)]^{-\frac{1}{2}}$. *The differential cross sections are all given for the center of mass coordinate system.* The total cross sections are plotted in Fig. 1. The contributions due to the $(f')^2$, f^2, and g^2 terms are plotted separately. In a Möller-Rosenfeld mixture theory $f^2=(f')^2\simeq 0.13$, $g^2=0.05$. The most noticeable features of these curves are: (1) their similar behavior for $\nu\sim 1\sigma\sim(\nu^2-1)^{\frac{1}{2}}/\nu$; (2) the relatively slow variation of the total cross section for production of pseudoscalar mesons, $\sigma_{\max}=0.44\sigma_0 f'^2$ where

$$\sigma_0=[(4\pi e^2/hc)((h)/\mu c)]^2=1.5\times10^{-27}\text{ cm}^2$$

for $\mu=300$ electron masses; (3) the large and rapid increase of cross section for the vector meson. This latter is mainly due to the production of longitudinal mesons whose cross section for large ν increases as ν^2

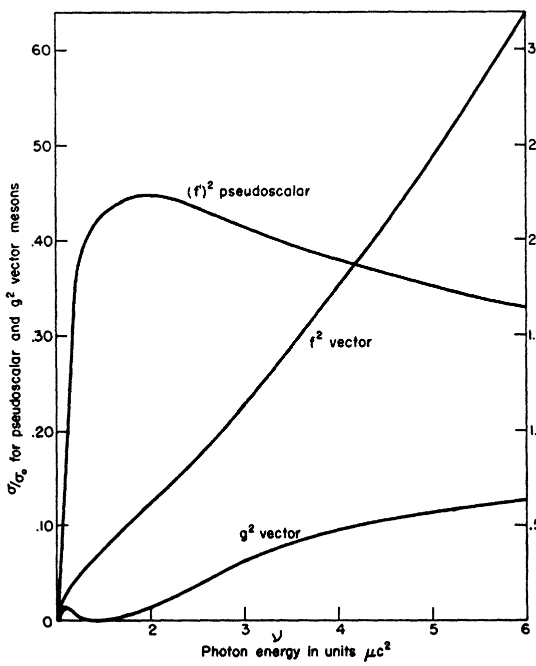

FIG. 1. Total cross section for production of mesons by photons. The photon energy is given in center-of-mass system. Use left hand scale for pseudoscalar mesons and that part of the cross section for vector meson production proportional to g^2. Use the right-hand scale for that part of the cross section for vector mesons which is proportional to f^2. $\sigma_0=4\pi e^2(\hbar/\mu^c)^2$. To obtain cross section multiply ordinate by the appropriate coupling constant, e.g. the cross section for production of pseudoscalar mesons at the maximum is $0.44\sigma_0 f'^2$.

* Assisted by the Joint Program of the ONR and the AEC.

[1] Reported by R. E. Serber, Conference on Theoretical Physics at Oldstone.

[2] J. Schwinger, private communication.

[3] W. Heitler and H. W. Peng, Proc. Roy. Irish Acad. 49A, 101 (1943); Hamilton, Heitler, and Peng, Phys. Rev. 54, 78 (1943); J. Hamilton and H. W. Peng, Proc. Roy. Irish Acad. 49A, 197 (1944).

[4] T. S. Chang, Det. Kgl. Danske Vidensk. Slebskab 19, No. 10 (1942).

while the cross section for transverse meson production increases only as $\ln\nu$.

Near threshold, the angular distribution for production of both meson types is spherically symmetrical. Of course only the *sum* of the angular distribution of the transverse and longitudinal mesons is spherical. Each separately is not so that if an experiment could be devised to detect the polarizations of the mesons produced (say by subsequent scattering by a coulomb field) then even near threshold it might be possible to distinguish between meson types.** As the energy of the photons increases the angular distribution increasingly deviates from spherical in a different manner for each meson type. In Fig. 2, we plot angular distributions for $\nu=1.5$. Note the minimum in the pseudoscalar curve at †$\sim 40°$. Note the large value of $\sigma(0)/\sigma(\pi)\sim 2.7$ for vector mesons production whereas for pseudoscalar mesons $\sigma(0)/\sigma(\pi)=1.0$. These features are accentuated as the energy increases. Again the angular distribution of the longitudinal and transverse vector mesons taken separately vary a great deal more with angle than the sum.

These remarks apply to the production of mesons in the collision of a photon with a single nucleon, which may be realized if the target substance is hydrogen. However, if heavier nuclei are involved, certain modifications should occur in the cross sections given above. The threshold energy is lowered inasmuch as a nucleus requires less energy than a proton to absorb a given amount of momentum. The behavior of the cross section near threshold has been given by D. ter Haar,[5] who finds $\sigma\sim(\nu-1)^{5/2}$. At energies sufficiently far above the threshold, the cross section approaches Z (cross section for a single proton). The coherent contribution in this region has been investigated and found to be of order 1 (rather than Z^2). This is in virtue of the essential change of character of the struck nucleus upon emission of a charged meson.

An important special case in which the target nucleus is a deuteron has been investigated. The conclusions drawn in the preceding paragraph have been verified. It is found that the energy region for which $\sigma\sim(\nu-1)^{5/2}$ is small, $O(1/M)$, the cross section taking on a $(\nu-1)^{3/2}$ dependence subsequently. This will be true for all nuclei. The free proton cross-section dependence does not exhibit itself till $\nu\sim 1.5$. There is in addition a special feature of the deuteron case because the final nucleus consists of two neutrons. The Pauli exclusion principle requires that the singlet state of the system have even space dependence while the triplet state must have odd space dependence. Transitions to the singlet state are possible for the pseudoscalar and vector meson cases. However only that part of the

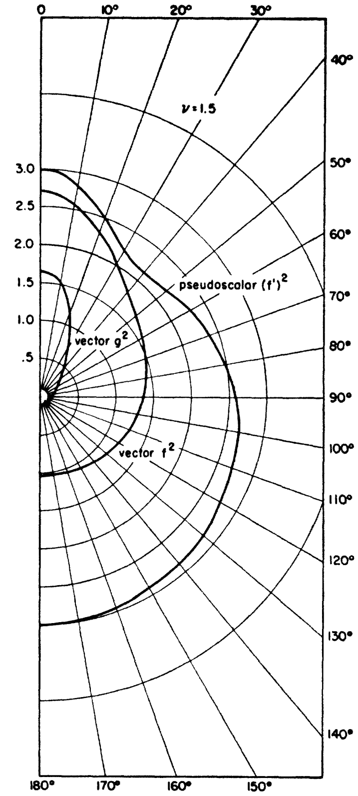

FIG. 2. Angular distribution for production of mesons by photons. The scale is arbitrarily chosen for each type if interaction. The total integrated cross sections for each meson type may be obtained from Fig. 1.

interaction which will change the spin of the nucleon will operate, leading to a numerical reduction of the cross section from the free-proton value.

Finally, the production of mesons by electrons has been calculated by the Weizsäcker and Williams[6] method. The method may be easily applied to the production of pseudoscalar mesons, regarding the struck nucleon as being infinitely heavy. For production of vector mesons the rapid increase of the cross section for production by photons with increasing photon energy make the close collisions between electron and nucleon important. The Weizsäcker and Williams method in its usual form is then inapplicable and must be suitably modified.[7] We then obtain a lower bound to the cross section which increases logarithmically with electron energy for pseudoscalar meson production, while for the vector meson, the energy dependence is considerably stronger than quadratic. In addition, the magnitude of the cross section for electron production of mesons is reduced relative to the cross section for photon production by the factor $2e^2/\pi\sim 1/215$.

** Note the angular distribution for scalar mesons is $\sin^2\theta$ near threshold and the energy dependence of total cross section is $(\nu^2-1)^{\frac{3}{2}}$. See L. W. Nordheim and G. Nordheim, Phys. Rev. **54**, 254 (1938).

[5] D. ter Haar, Science **108**, 57 (1948).

[6] C. F. v. Weizsäcker, Zeits. f. Physik **88**, 612 (1934); E. J. Williams, Det. Kgl. Danske Vidensk. Selbskab **13**, No. 4 (1935). See also G. C. Wick, Ricerca Scientifica Jan. 1940, p. 49.

[7] M. Lax and H. Feshbach, Phys. Rev. **73**, 1271A (1948).