Theory of the Lithium Two-Alpha-Reactions. II. Angular Distribution of $Li^6(d,\alpha)\alpha$

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An analysis is presented of the variation of the intensity of alphas from $Li^6(d,\alpha)\alpha$ with angle and energy. This variation has been observed to have the form $1+A(E)\cos^2\theta+B(E)\cos^4\theta$, just as in the case of $Li^7(p,\alpha)\alpha$ for which an analysis has already been given. For $Li^6(d,\alpha)\alpha$ it is again the Bose statistics and consequent even parity of the alphas that makes analysis possible. It is assumed that Li^6 has even parity so that only $s,d,\cdots$ deuterons are relevant here, in contrast to $Li^7(p,\alpha)\alpha$ in which $p,f,\cdots$ protons are involved. The two analyses differ also in the spin combinations, but are similar in the assumption of only two contributing states of the compound nucleus, with angular momentum quantum numbers 0 and 2. An expression for the energy variation of the angular distribution in $Li^6(d,\alpha)\alpha$ is derived from the dispersion formula and the treatment with two sharp levels is compared with that for one sharp and one broad level. The latter is found simpler and adequate for obtaining agreement with the experimental $A(E)$ and $B(E)$. After recognizing limitations on the parameters consistent with order-of-magnitude estimates of the matrix elements involved, it is found that the theoretical formulas rather naturally give the general form of the experimental results, including the feature that both $A(E)$ and $B(E)$ rise more slowly in the $Li^6(d,\alpha)\alpha$ reaction than in the $Li^7(p,\alpha)\alpha$ reaction as the bombarding energy is increased from zero, a consequence of the spherical symmetry of the entering s waves which participate in the Li^6 reaction only.

I. INTRODUCTION

THE angular distribution of the $Li^6(d,\alpha)\alpha$ reaction has been investigated by Heydenburg, Hudson, Inglis, and Whitehead.[1] The intensity of the reaction was observed to vary with angle as $1+A(E)\cos^2\theta+B(E)\cos^4\theta$, as expected from the Bose statistics of the product alphas. $A(E)$ rises gradually beginning just below 1 Mev to a broad maximum a little below unity at a bombarding energy E in the neighborhood of 2 Mev, and falls rapidly to zero at 3.5 Mev. The coefficient $B(E)$ remains zero up to almost 1.5 Mev and rises to a positive value of about 0.35 at higher energies. The yield curve observed at right angles to the beam displays a sharp maximum at 0.75 Mev and there is evidence to the approach to another peak just beyond the highest bombarding energy employed, 3.75 Mev.

The reaction $Li^6(d,\alpha)\alpha$ involves the formation of the same compound nucleus Be^8 as is formed in $Li^7(p,\alpha)\alpha$ but the binding energy of a deuteron to Li^6 is 4.96 Mev higher than the binding energy of a proton to Li^7, so the compound nucleus is in a more highly excited state in the Li^6 reaction than in the Li^7 reaction, for comparable bombarding energies. The Li^6 reaction thus makes it possible to explore a different part of the Be^8 spectrum than that explored as yet by the Li^7 reaction.

It is assumed throughout this work that the parity of the ground state of Li^6 is even, since it may be described as an alpha plus two p-nucleons. The results show that the data are compatible with this assumption. This theoretical interpretation of the Li^6 reaction includes the effects of entering s and d deuterons. The s deuterons by themselves give spherical symmetry, and the penetration of d deuterons is necessary for the appearance of the asymmetry. The product of an s wave and a d wave introduces a term $\cos^2\theta$ into the angular distribution, whereas the square of the d wave is responsible for the $\cos^4\theta$ term.

II. CALCULATION OF ANGULAR DISTRIBUTION OF THE Li^6+DEUTERON REACTION

The calculation is based on the Briet-Wigner dispersion formula[2] extended to several compound states, which for a system prepared in a well-defined initial state P may be written

$$\sigma(E,\theta)\sim\lambda^2\left|\sum_r (P/H/r)(r/H/Q)/(E-E_R+(i/2)\Gamma_r)\right|^2. \quad (1)$$

In the paper[3] on $Li^7(p,\alpha)\alpha$ (hereafter referred to as I) it is shown that the cross section for finding an alpha in a small element of solid angle at θ is

$$\sigma(E,\theta)=\sum_P \sigma_P(E,\theta) \quad (2)$$

for an incident unpolarized beam and unpolarized target, where

$$\sigma_P(E,\theta)=4\pi\lambda^2\left|\sum_{lr}\frac{\{(lSom/lSj_rm)[lSj_r/X_{lr}]\phi_l(E)\alpha_{lr}P_{j_r}^m(\cos\theta)\}}{(E-E_r+(i/2)\Gamma_r)}\right|^2. \quad (3)$$

Here, $(lSom/lSj_rm)$ is a transformation coefficient transforming from a representation in which l and S are diagonal to that in which the total angular momentum is diagonal; $[lSj_r/X_{lr}]$ transforms from lSj_r to the in-

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[1] Heydenburg, Hudson, Inglis, and Whitehead, Phys. Rev. **74**, 405 (1948).

[2] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

[3] D. R. Inglis, Phys. Rev. **74**, 21 (1948). (See also C. L. Critchfield and E. Teller, Phys. Rev. **60**, 10 (1941).)

coming wave function X_{lr}, having angular properties most nearly matching those of the resonant state r of the compound nucleus, having $j=j_r$. The product $(X_{lr}/H/r)(r/H/Q)$ is factored thus: $\varphi_l(E)\alpha_{lr}P_{jr}{}^m(\cos\theta)$. Here, Q is the final state. α_{lr} is a complex number; $\varphi_l(E)$ is real and arises from penetration of the nuclear barrier; and $P_{jr}{}^m$ is the associated Legendre polynomial introducing the angle factor.

In the reaction $Li^6(d,\alpha)\alpha$ the deuteron spin is $s=1$, the target nucleus has angular momentum $I=1$, and these combine to give spin states $S=0$, 1, 2. The ground state of Li^6 is assumed to be even, so s, d, $g\cdots$ waves may give the even states of the compound nucleus required for the reaction. The successively higher values of l find penetration more and more difficult, so it will suffice to consider only $l=0$ and $l=2$. The Bose statistics of the product alphas also demands that the compound state have even angular momentum j_r. With $l=0$, the spin state $S=0$ gives $j_r=0$, and $S=2$ gives $j_r=2$. With $l=2$, $S=2$ leads to $j_r=0$, 2, 4, $S=1$ to $j_r=2$ and $S=0$ to $j_r=2$.

We are free to assume such disposition of virtual levels of the compound nucleus having $j_r=0$, 2, or 4 as may be necessary to account for the observations. It may be assumed in the interests of simplicity, that the d wave reacts only with states having $j_r=0$ or $j_r=2$ without invoking $j_r=4$. It is found unnecessary to introduce more than two such states to explain the details of the reaction so far as they have been observed.

Our first set of assumptions about the compound nucleus is that it has just two states which contribute appreciably to the reaction in the energy range investigated, a state numbered $r=0$ having $j_0=0$ and a half-width Γ_0 much greater than the range of energies covered by the experiments, so that the energy variation of its resonance denominator may be neglected, and a state numbered $r=2$ having $j_2=2$, and Γ_2 small enough to account for the rapid energy variation of the angular distribution of the product alphas.

With these states, the cross section given by (1), (2), and (3) is

$$\sigma(E,\theta)=\frac{16\pi\lambda^2}{\Gamma_2{}^2(\epsilon^2+1)}\sum_{S,m}\Big|(\Gamma_2/i\Gamma_0)(\epsilon+i)\sum_{l=0,2}(lSom/lSom)[lSo/lo]\alpha_{lo}\phi_l+\sum_{l=0,2}(lSom/lS2m)\times[lS2/l2]\alpha_{l2}\phi_lP_2{}^m\Big|^2. \quad (4)$$

TABLE I. Transformation coefficients $(lSom/lSj_rm)$.

m	(2000/2020)	(210m/212m)	(220m/220m)	(220m/222m)	(020m/022m)	(0000/0000)
2		0	0	$(2/7)^{\frac{1}{2}}$	1	
1		$-(1/2)^{\frac{1}{2}}$	0	$-(1/14)^{\frac{1}{2}}$	1	
0	1	0	$(1/5)^{\frac{1}{2}}$	$-(2/7)^{\frac{1}{2}}$	1	1
-1		$(1/2)^{\frac{1}{2}}$	0	$-(1/14)^{\frac{1}{2}}$	1	
-2		0	0	$(2/7)^{\frac{1}{2}}$	1	

Here we have put

$$(E-E_2)/(\Gamma_2/2)=\epsilon, \quad (5)$$

which then measures the energy deviation from resonance with the state $r=2$ in units of its half-width. The summation indices S and m are implicit in the initial state P. More explicitly we have,

$$\sigma=[16\pi\lambda^2/\Gamma_2{}^2(\epsilon^2+1)]\Big\{\sum_{m\neq0}\Big|\sum_{l=0,2}(l20m/l22m)\times[l22/l2]\alpha_{l2}\phi_l\Big|^2(P_2{}^m)^2+\Big|(\Gamma_2/i\Gamma_0)(\epsilon+i)(2200/2200)[220/20]\alpha_{20}\phi_2+\sum_{l=0,2}(l200/l220)[l22/l2]\alpha_{l2}\phi_lP_2{}^0\Big|^2+\sum_m\Big|(210m/212m)[212/22]\alpha_{22}\phi_2\Big|^2(P_2{}^m)^2+\Big|(\Gamma_2/i\Gamma_0)(\epsilon+i)(0000/0000)\alpha_{00}\phi_0+(2000/2020)[202/22]\alpha_{22}\phi_2(P_2{}^0)\Big|^2\Big\}. \quad (6)$$

The factor $(\Gamma_2/i\Gamma_0)$ appears here twice since $j_r=0$ arises from both $l=0$, $S=0$ and $l=2$, $S=2$. The sums over m appearing here are easily evaluated by use of the values[4] for $(lSom/lSj_rm)$ given in Table I. In order to simplify the terms in the brackets, we divide through by $a=[202/22]\alpha_{22}$ and we put

$$\begin{aligned}\Gamma_2[000/00]\alpha_{00}/i\Gamma_0a&=R_0+iI_0\\ \Gamma_2[220/20]\alpha_{20}/i\Gamma_0a&=R_1+iI_1\\ [022/02]\alpha_{02}/a&=R_2+iI_2\\ [222/22]/[202/22]&=R_3+iI_3\\ [212/22]/[202/22]&=R_4+iI_4.\end{aligned} \quad (7)$$

Here R is the real part and I the imaginary part. In Table II are listed sums used in the Li^6 reaction with $j_r=0$ and $j_r=2$. Using Tables I and II with definitions (7), we find

$$\sigma=[16\pi\lambda^2\phi_0{}^2/\Gamma_2{}^2(\epsilon^2+1)][202/22]\alpha_{22}{}^2R^2\times\{[(\epsilon^2+1)+b]+\phi[c+d\epsilon+\cos^2\theta(e\epsilon+f)]+\phi^2[(\epsilon^2+1)g+h\epsilon+i+(j+k'\epsilon)\cos^2\theta+l\cos^4\theta]\}. \quad (8)$$

TABLE II. Sums used in Li^6 reaction with $j_r=0$ and 2.

l_1	l_2	$\sum_{m=-2}^{2}(l_120m/l_122m)\times(l_220m/l_222m)(P_2{}^m)^2$	$\sum_{m=-2}^{2}(l_110m/l_112m)\times(l_210m/l_212m)(P_2{}^m)^2$
0	0	5	
0	2	$(5/14)^{\frac{1}{2}}(1-3\cos^2\theta)$	
2	2	$5/14(4-9\cos^2\theta+9\cos^4\theta)$	$15/2(\cos^2\theta-\cos^4\theta)$

[4] See E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Oxford University Press, New York, 1935), pp. 76–77.

TABLE III. Barrier penetrabilities of the s and d waves.

$R/(e^2/mc^2)$	E/mc^2	1	1.5	2	3	4	6	8
1.50	φ_0	0.500	0.820	1.207	1.127	0.887	0.725	0.648
	φ_2	0.021	0.046	0.076	0.146	0.225	0.399	0.591
	φ	0.042	0.056	0.063	0.130	0.254	0.550	0.912
2.25	φ_0	0.936		1.190	0.904	0.795	0.687	0.626
	φ_2	0.076		0.258	0.479	0.734		0.800
	φ	0.081		0.217	0.530	0.923		1.278
3.00	φ_0		1.253	1.025	0.851	0.766	0.672	
	φ_2	0.187	0.381	0.604		1.126	0.786	
	φ		0.304	0.589		1.470	1.170	

In Eq. (8) we have put

$$
\begin{aligned}
R^2&=(R_0^2+I_0^2)\\
b&=5(R_2^2+I_2^2)/R^2\\
c&=\{(50/7)^{\frac{1}{2}}(R_2R_3+I_2I_3)-(R_1I_2-I_1R_2)+(5)^{\frac{1}{2}}I_0\}/R^2\\
d&=\{-(R_1R_2+I_1I_2)-(5)^{\frac{1}{2}}R_0\}/R^2\\
e&=\{3(R_1R_2+I_1I_2)+(9/5)^{\frac{1}{2}}R_0\}/R^2\\
f&=\{[-30/(14)^{\frac{1}{2}}](R_2R_3+I_2I_3)\\
&\qquad+3(R_1I_2-I_1R_2)-3(5)^{\frac{1}{2}}I_0\}/R^2\\
g&=(1/5)(R_1^2+I_1^2)/R^2\\
h&=(10/35)^{\frac{1}{2}}(R_3R_1+I_3I_1)/R^2\\
i&=\{(10/7)(R_3^2+I_3^2)+5/4\\
&\qquad+(10/35)^{\frac{1}{2}}(I_3R_1-I_1R_3)\}/R^2\\
j&=\{-(45/14)(R_3^2+I_3^2)+(15/2)(R_4^2+I_4^2)\\
&\qquad-(30/4)-(18/7)^{\frac{1}{2}}(I_3R_1-I_1R_3)\}/R^2\\
k'&=-(18/7)^{\frac{1}{2}}(R_3R_1+I_3I_1)/R^2\\
l&=\{(45/14)(R_3^2+I_3^2)\\
&\qquad-(15/2)(R_4^2+I_4^2)+45/4\}/R^2. \qquad (9)
\end{aligned}
$$

Also, $\phi=\varphi_2/\varphi_0$ is the ratio of the penetration amplitudes of the incoming d and s waves.

With these definitions we can write finally,

$$\sigma=C(\epsilon)[1+A(\epsilon)\cos^2\theta+B(\epsilon)\cos^4\theta] \qquad (10)$$

where

$$
\begin{aligned}
C(\epsilon)&=\pi\{4\lambda\phi_0[202/22]\alpha_{22}\}^2D(\epsilon)/\Gamma_2^2(\epsilon^2+1)\\
D(\epsilon)&=a+(\epsilon^2+1)+\varphi(c+d\epsilon)+\varphi^2\{(\epsilon^2+1)g+h\epsilon+i\}\\
A(\epsilon)&=\phi[e\epsilon+f+\phi(j+k'\epsilon)]/D(\epsilon)\\
B(\epsilon)&=l\phi^2/D(\epsilon).
\end{aligned} \qquad (11)
$$

Here $C(\epsilon)$ represents the yield at $\theta=90°$. It need not be considered however in a study of the angular distribution. Its form illustrates our assumption of a broad state with $j_r=0$ and a sharp state with $j_r=2$, since ϵ is related to the resonant energy E_2 and the bombarding energy E in center-of-mass coordinates by Eq. (5).

If there were no entering d wave, φ would become zero and $A(\epsilon)$ and $B(\epsilon)$ would vanish, the d wave being necessary for the appearance of the asymmetry. Also since φ is more important than φ^2 at low energies, $B(\epsilon)$ should rise from zero at a higher energy than $A(\epsilon)$.

The explicit form of the parameters is given here so that a comparison of their values in terms of the integrals appearing in the theory with the values required to fit the data can be made.

III. PENETRABILITY OF THE NUCLEAR POTENTIAL BARRIER

In both of the lithium two-alpha-reactions the "outgoing" matrix element $(r/H/Q)$ can be assumed to be independent of the energy in the ranges considered, since these reactions are highly exoergic. The energy dependence of $\phi_l(E)$ arises almost entirely from the penetration of the incoming wave.

Tabulated values of the amplitudes of the regular and irregular solutions of the wave equation with Coulomb field at the edge of the nucleus are not available as yet for the required values of the parameters and the WKB approximation method is used for the calculation of penetrability. In the case Li^6+deuteron we deal with $l=0$ and $l=2$. For a given l the barrier for the Li^6 case is lower than that for Li^7 due to the greater reduced mass of the system Li^6+H^2 and the larger radius, R, of the top of the barrier one must assume to allow for the extension of the deuteron. This brings the barrier height into the region of experimental energies for $l=0$, and also for $l=2$ if we take $R<1.8\ e^2/mc^2$. φ_l is put equal to $|B_l-E|^{-\frac{1}{4}}e^{-C_l}$ with B_l the barrier height and C_l defined as $[(2M)^{\frac{1}{2}}/\hbar]\int_R^{r_l}(U_l-E)^{\frac{1}{2}}dr$, the usual integral across the barrier.[5] The use of $(l+\frac{1}{2})^2\hbar^2/2Mr^2$ as the l-barrier in $U_l(r)$ keeps the WKB approximation a good one for energies closer to the barrier height than the use of $l(l+1)\hbar^2/2Mr^2$ would permit.[6]

Calculations were made for three values of R, the effective nuclear radius. For $R=1.50e^2/mc^2$, $B_0=2.38\ mc^2$ and $B_2=11.51mc^2$. For $R=2.25e^2/mc^2$, $B_0=1.50mc^2$ and $B_2=5.56mc^2$. For $R=3.00e^2/mc^2$, $B_0=1.10mc^2$ and $B_2=3.38mc^2$. Values of φ_l and φ are tabulated in Table III for the three radii assumed and for some of the values of E used.

Values for $\varphi_l(E)$ are calculated on both sides of the barrier height and a smooth curve is drawn through this region.[7] The arbitrariness of the curve near the peak does not greatly affect our final results because the nuclear radius chosen avoids the use of this region.

In Fig. 1, $\varphi=\varphi_2/\varphi_0$ is plotted against the energy for

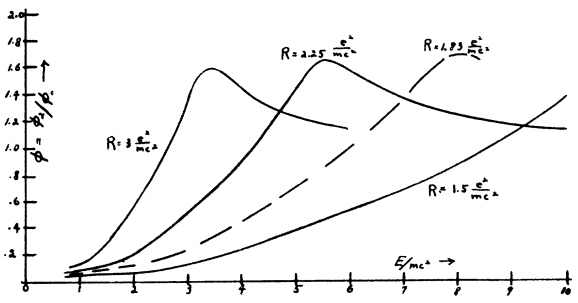

FIG. 1. Ratio of penetrabilities of $l=2$ wave to $l=0$ wave as function of incoming energy for several assumed radii in $d+Li^6$.

[5] H. A. Bethe, Rev. Mod. Phys. **9**, 178 (1937).
[6] Breit, Wheeler, and Yost, Phys. Rev. **49**, 174 (1936).
[7] N. H. Frank and L. A. Young, Phys. Rev. **38**, 80 (1931).

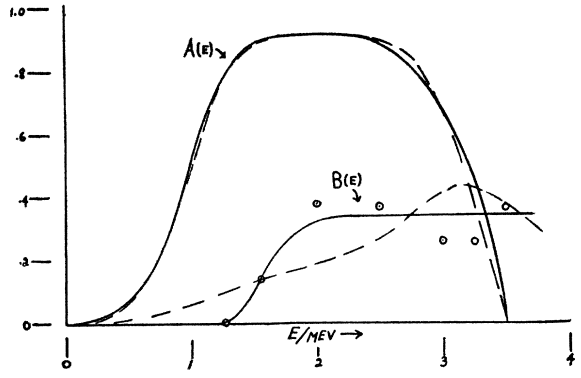

FIG. 2. Theoretical (dashes) and experimental (straight line) curves for $A(E)$ and $B(E)$ in $Li^6(d,\alpha)\alpha$ with $\Phi=kE$.

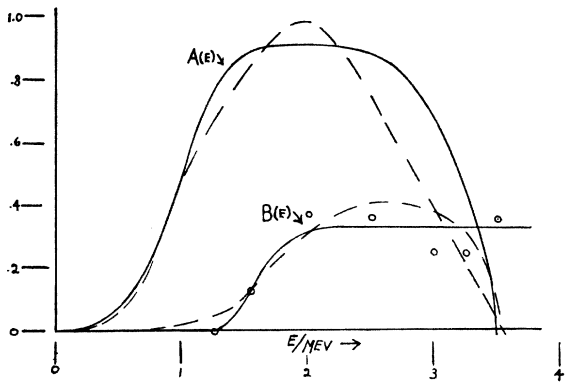

FIG. 3. Theoretical (dashes) and experimental (straight line) curves for $A(E)$ and $B(E)$ in $Li^6(d,\alpha)\alpha$ with $\Phi=kE^2$, rough choice of coefficients.

each assumed radius. The ratio for $R=1.83e^2/mc^2$ interpolated from the calculated values is shown also in Fig. 1. The choice of this radius will be discussed below. It is worth noting here that for $R=1.83e^2/mc^2$, $B_4=22.36mc^2$ and φ_4 is negligible compared to φ_0 and φ_2 in the energy range considered, so that $l=4$ can be omitted in the analysis.

IV. COMPARISON WITH EXPERIMENTAL RESULTS

We test the assumption that a broad state with $j_r=0$ and a sharp state with $j_r=2$ will account for the experimental results, the first peak in the observed excitation curve at 0.75 Mev being the $j_r=2$ resonance where $\epsilon=0$, and the subsequent approach to a second peak being due to increasing penetrability of the $l=2$ wave.

Under this assumption we first choose R for Li^6+H^2 such that $l=2$ has its maximum penetrability "at the second peak," or more specifically, at $E=4$ Mev. This corresponds to $R=1.83e^2/mc^2$. This radius would put the maximum penetrability for $l=0$ between the peak at 0.75 Mev and 1 Mev and would account for the broadness of the peak on the high energy side and the more gradual descent to a higher minimum there. This value of R is larger than the corresponding value for Li^7+H^1 ($R=1.5e^2/mc^2$) as taken in I, and by Christy and Latter,[8] and is easily accounted for as the distance out to the center-of-mass of the deuteron when the two nuclei begin to interact strongly.

Under these assumptions we obtained (10) and (11). Since the maximum for φ occurs in the observed energy range, we cannot neglect φ^2 in our determination of $A(\epsilon)$ and $B(\epsilon)$. This, of course, introduces a large number of undetermined constants. In order to cut these parameters to a minimum we set $\varphi=kE$ for the range considered and we choose k so that this will be a fair approximation in the region from 1.5 to 3.5 Mev (e.g., $k=0.3$ (Mev)$^{-1}$).

Putting in $E=(\Gamma_2/2)\epsilon+E_2$ [9] and $\varphi=kE$ we obtain

$$A(E)=(HE^3+JE^2+KE)/(E^4+BE^3+CE^2+DE+F) \quad (12)$$

and

$$B(E)=LE^2/(E^4+BE^3+CE^2+DE+F)$$

where the coefficients are expressible in terms of the constants in (9) and E_2, Γ_2 and k.

The quickest and best fit of these expressions to the experimental curves is obtained by solving simultaneously for the seven coefficients in $A(E)$. A choice of L in $B(E)$ then normalizes this curve. This has been done and with the following values for the coefficients the curves have been plotted in Fig. 2 with the experimental curves.

$H=-0.4577$ (Mev)	$C=26.584$ (Mev)^2
$J=1.6020$ (Mev)^2	$D=-33.483$ (Mev)^3
$K=0$ (Mev)^3	$F=16.879$ (Mev)^4
$B=-8.6540$ (Mev)	$L=0.1375$ (Mev)^2.

From Fig. 2 it is evident that $A(E)$ is an excellent fit to experiment, lying everywhere within the experimental uncertainty. $B(E)$, however, is a very poor fit indeed, and attempts to improve the fit to $B(E)$ with Eqs. (12) resulted in a much poorer fit for $A(E)$. A more exact expression for φ appears to be necessary in order to provide an $A(E)$ and $B(E)$ with sufficient parameters to obtain a good fit with experiment everywhere. This indicates that the results are sensitive to the exact assumptions made, thus making our assumptions more significant.

It is clear from Fig. 1 that $\varphi=kE^2$ will represent the curve for $R=1.83e^2/mc^2$ better than $\varphi=kE$. In fact, if we take $k=0.114$ (Mev)$^{-2}$, $\varphi=kE^2$ is almost an exact fit over the whole range $0<E<3.75$ Mev. Putting in kE^2 for φ and $E=(\Gamma_2/2)\epsilon+E_2$ in (11) we obtain

$$A(E)=(AE^5+BE^4+CE^3+DE^2)/(E^6+FE^5+GE^4+HE^3+JE^2+KE+M) \quad (13)$$

and

$$B(E)=LE^4/(E^6+FE^5+GE^4+HE^3+JE^2+KE+M)$$

[8] R. F. Christy and R. Latter, Rev. Mod. Phys. **20**, 185 (1948).

[9] Defined in laboratory system, which introduces factor $(\frac{2}{3})^2$ in $C(\epsilon)$. See footnote 12 of I.

where

$$\begin{aligned}
A&=(\Gamma_2/2)k'\\
B&=(\Gamma_2^2/4)j-E_2\Gamma_2k'/2\\
C&=\Gamma_2e/2k\\
D&=(\Gamma_2^2f/4k-E_2\Gamma_2e/2k)\\
F&=h\Gamma_2/2-2gE_2\\
G&=i\Gamma_2^2/4-hE_2\Gamma_2/2+g(\Gamma_2^2/4+E_2^2)\\
H&=\Gamma_2d/2k\\
J&=1/k^2+\Gamma_2^2c/4k-\Gamma_2dE_2/2k\\
K&=-2E_2/k^2\\
M&=\Gamma_2^2(a+1)/4k^2+E_2^2/k^2\\
L&=\Gamma_2^2l/4
\end{aligned}\qquad(14)$$

defined in terms of the constants in (9), and E_2, Γ_2, and k.

Before fixing our coefficients to give the best fit to experiment, we inquire whether or not the values of the coefficients determined from their definition in terms of the matrix elements, etc., in the theoretical treatment can give us the general form of the experimental curves. The exact values of the matrix elements cannot be calculated but one may conclude that the coefficients $a, b, c, \cdots$ are probably of the order of magnitude 1, varying perhaps from $\frac{1}{5}$ to 5, and that some (a, g) are required to be positive. It is possible that some of these coefficients may be zero since they involve differences of matrix elements, but the probability of such an occurrence seems small. Furthermore $k\cong\frac{1}{9}$, $\Gamma_2\cong(\frac{1}{4})$ Mev, and $E_2\cong(\frac{3}{4})$ Mev. The "expected" range of the coefficients in (14) is not sufficiently narrow to yield a curve of any definite shape, but with a small amount of trial adjustment of the coefficients within this range a rough fit was easily obtained (Fig. 3). Here

$$\begin{aligned}
A&=-0.008\text{ Mev} & M&=48.0\text{ (Mev)}^6\\
B&=0.80\text{ (Mev)}^2 & K&=-91.0\text{ (Mev)}^5\\
C&=-2.92\text{ (Mev)}^3 & J&=55.0\text{ (Mev)}^4\\
D&=4.64\text{ (Mev)}^4 & H&=-5.98\text{ (Mev)}^3\\
L&=0.11\text{ (Mev)}^2 & G&=2.00\text{ (Mev)}^2\\
& & F&=-4.02\text{ Mev.}
\end{aligned}\qquad(15)$$

The appearance of the late and sharp rise of the $B(E)$ curve is a feature of these equations obtained even with only rough values of the coefficients which the previous Eqs. (12) could not yield while still retaining a good fit to $A(E)$. Hence the improved fit of equations (13) with coefficients (14), obtained by use of the more nearly correct expression for the penetration ratio $\varphi=kE^2$, rather than $\varphi=kE$, seems quite significant even though the improvement involved an increase in the number of arbitrary parameters. By more careful curve-fitting still better coefficients could be obtained, but as Fig. 3 implies, these would compare favorably with the ranges of values allowed by order-of-magnitude estimates in the theory.

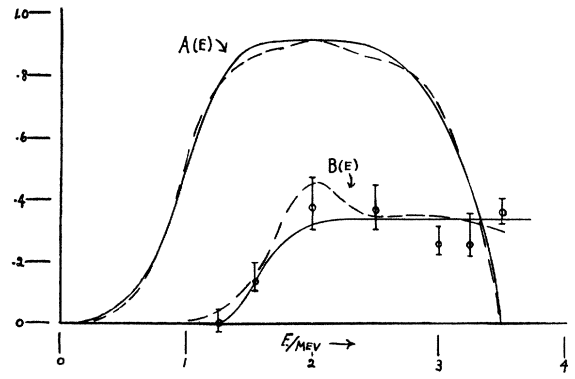

FIG. 4. Theoretical (dashes) and experimental (straight line) curves for $A(E)$ and $B(E)$ in $Li^6(d,\alpha)\alpha$ with $\Phi=kE^2$.

It would be gratifying if it were evident from comparison of the theoretical expressions for $A(E)$ and $B(E)$ in the $Li^7(p,\alpha)\alpha$ reaction with those of the $Li^6(d,\alpha)\alpha$ reaction, that the $A(E)$ and $B(E)$ curves rise from zero at a higher energy in the Li^6 case. This would correspond to the physical situation that the easy entrance of s-waves makes the $Li^6(d,\alpha)\alpha$ reaction symmetric at very low energies and that the p-waves responsible for the Li^7 reaction at low energies may introduce asymmetry in that reaction. For $Li^7(p,\alpha)\alpha$ the result[3] comparable to (13) is

$$\begin{aligned}
A(E)&=(C_0E^2+C_1E+C_2)/(E^2+C_3E+C_4)\\
B(E)&=(C_5E^2+C_6E)/(E^2+C_3E+C_4).
\end{aligned}\qquad(16)$$

For both $A(E)$ and $B(E)$ the form of the numerator in $Li^6(d,\alpha)\alpha$ requires a later rise with energy than the numerators in $Li^7(p,\alpha)\alpha$. In (13) the lowest power of E is 2 in $A(E)$ and 4 in $B(E)$, whereas in (16) the lowest power of E is 0 in $A(E)$ and 1 in $B(E)$. The qualitative differences being considered in these reactions appear in the energy region $0<E<1$ Mev, and in this region one can claim that in effect, the denominators in (13) and (16) have the same form. Not only are high powers of E less important here, but (14) and (15) show also that the coefficients of the high powers of E in the denominators of (13) are relatively very small. Hence, the nature of the numerator predominates and the results give rise somewhat naturally to the later rise in the angular distribution curves of $Li^6(d,\alpha)\alpha$ observed experimentally. The observed vanishing of $A(E)$ near $E=0$ for $Li^7(p,\alpha)\alpha$ is caused[3] by a pure coincidence in the placing of the resonant state of the compound nucleus, and there is no consideration of barrier penetration to oppose a rapid rise.

In order to demonstrate that an improved fit can be obtained with (14) by more exact curve-fitting, we choose values for our parameters by solving simultaneously fixing four coefficients by the $B(E)$ experimental curve and the remaining seven by the $A(E)$ experimental curve. The resulting curves are plotted in Fig. 4 with the experimental curves for comparison. The coefficients are assigned the following values

$$\begin{aligned}
A&=-0.2648\text{ (Mev)} & G&=63.248\text{ (Mev)}^2\\
B&=1.8966\text{ (Mev)}^2 & H&=-164.215\text{ (Mev)}^3\\
C&=-4.3292\text{ (Mev)}^3 & J&=232.222\text{ (Mev)}^4\\
D&=3.2706\text{ (Mev)}^4 & K&=-171.094\text{ (Mev)}^5\\
F&=-12.531\text{ (Mev)} & L&=0.0110\text{ (Mev)}^2\\
& & M&=52.379\text{ (Mev)}^6.
\end{aligned}\qquad(17)$$

The localized hump in the $B(E)$ curve and the dips in the $A(E)$ curve both derive from the curve fitting condition $[dB(E)/dE]_{E=2.5}=0$. Had another condition been imposed in its place the fit could be better than shown. However, the already good fit obtained and the uncertainty in the experimental values make further fitting seem superfluous.

In this case, with $\varphi=kE^2$, both $A(E)$ and $B(E)$ can be considered good fits. The $B(E)$ curve shows the late and rapid rise characteristic of this reaction. The experimental uncertainty in the points is also shown in Fig. 4, and $B(E)$ is seen to be a good fit, in spite of the localized hump which is not considered significant. $A(E)$ again lies everywhere within the experimental uncertainty in the curve. The values of this set of coefficients are also fairly reasonable from the point of view discussed in connection with (14) and (15). For example, C in (17) makes $|e|=0.9$ after Γ_2 from the observed resonance and k from penetration calculations have been used, and similarly D makes $f\approx 1$. The value of $|H|$ in (17) is unreasonably large, requiring $|d|\approx 30$, but seems to be associated with the unexpected localized hump in $B(E)$, and this somewhat unlikely value would probably disappear with more refined curve fitting.

V. ALTERNATIVE SET OF COMPOUND STATES

The assumption investigated above that the angular distribution is to be attributed to a broad state of the compound nucleus with $j_r=0$ and a sharp state with $j_r=2$, seems to be the simplest basis for interpretation of the experimental results. The sharp state $j_r=2$ gives rise to the observed resonance, and the great breadth of the state with $j_r=0$ is naturally associated with the ease with which such a state is expected to break up into two alphas.

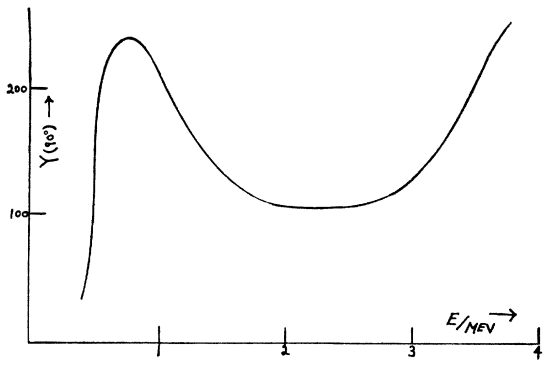

FIG. 5. Relative cross section for $Li^6(d,\alpha)\alpha$ at 90°.

The fitting of the data with the above assumptions does not guarantee their validity, however, and, in fact, a simple modification of them suggests itself when one investigates the $\sigma(E, 90°)$ curve, Fig. 5. The rise to a second peak near 3.5 Mev can also be attributed to a sharp s-state, rather than due only to increasing penetrability of d-wave deuterons. Under the assumption of two sharp states, with $j_r=0$ and $j_r=2$, one can carry out the calculations, closely paralleling those already carried out, and one finds

$$A(\epsilon_0,\epsilon_2)=\frac{\phi(p+q\epsilon_0+r\epsilon_2+s\epsilon_0\epsilon_2+t\epsilon_0^2)+\phi^2(u+v\epsilon_0^2+w\epsilon_0+y\epsilon_2+z\epsilon_0\epsilon_2)}{(a\epsilon_0^2+b\epsilon_2^2+c)+\phi(d+e\epsilon_0+f\epsilon_2+g\epsilon_0\epsilon_2+h\epsilon_0^2)+\phi^2(i+j\epsilon_0^2+k\epsilon_2^2+l\epsilon_0+m\epsilon_2+h\epsilon_0\epsilon_2)}. \quad (18)$$

$B(\epsilon_0,\epsilon_2)$ has the same denominator as $A(\epsilon_0,\epsilon_2)$ but its numerator is $l'\varphi^2(1+\epsilon_0^2)$.

Here,

$$\epsilon_0=(E-E_0)/(\Gamma_0/2) \quad \text{and} \quad \epsilon_2=(E-E_2)/(\Gamma_2/2) \quad (19)$$

and the coefficients a, b, $\cdots z$ are defined in terms of matrix elements as in (9) but they are more involved and more numerous, and interpretation of their roles becomes quite complex.

Substitution of (19) in (18) and taking $\varphi=kE^2$ gives

$$A(E)=(C_1E^6+C_2E^5+C_3E^4+C_4E^3+C^5E^2)/(E^6+C_7E^5+C_8E^4+C_9E^3+C_{10}E^2+C_{11}E+C_{12}).$$

$B(E)$ has the same denominator as $A(E)$, but its numerator is $C_6E^6+C_{13}E^5+C_{14}E^4$.

Hence, we arrive at expressions giving one additional curve-fitting parameter in $A(E)$ and two additional parameters in $B(E)$. Of course, the number of implicit parameters has increased and the relation of the explicit ones to the matrix elements has become more complex. Also, both numerator and denominator are now the same order in E.

Since the simpler assumptions fit the experimental results, it is clear that these assumptions give more parameters than are necessary. A fit in this case would be less significant than with fewer parameters and its interpretation would be more involved. These assumptions may have to be invoked only if experiments obtaining $A(E)$ and $B(E)$ and $\sigma(E, 90°)$ are performed for $E>3.75$ Mev and the added complexity requires the greater freedom allowed by (18). If extension of the experiments demonstrates that a resonance does in fact exist near 4 Mev, then these assumptions will be the simplest valid ones. However, we may than have to consider the small effect of the $l=4$ wave, and the effect of the fact that φ may not behave as E^2 beyond the peak.