

molecular forces. In this note a theoretical formula for the absorption of the latter kind is given.

When a molecule is under an electrostatic field \mathbf{E} and a radiation field $\mathbf{E}_r \cos \omega t$, the perturbation Hamiltonian which makes the molecule absorb light quanta is $\mathbf{u}(\mathbf{E} + \mathbf{E}_r \cos \omega t)$, where \mathbf{u} is the dipole moment of this molecule. In our problem, the external field \mathbf{E} is due to the molecules surrounding the one under consideration. Thus, if the i th molecule has dipole moment \mathbf{u}_i and quadrupole moment \mathbf{Q}_i (which is represented by $Q_{zz} = 2P$ and $Q_{xx} = Q_{yy} = -P$ for the above type molecules), and is at a distance \mathbf{R}_i from the central molecule, then

$$\mathbf{E} = \sum_i \mathbf{E}_i = \sum_i \left\{ \frac{2\mu_i \cos \theta_i}{R_i^4} + \frac{3P_i(3 \cos^2 \theta_i - 1)}{2R_i^6} + \dots \right\} \mathbf{R}_i, \quad (1)$$

where θ_i is the angle between \mathbf{R}_i and the axis of i th molecule.

If ψ_i is the wave function of i th molecule, the absorption coefficient of the transition $\psi_0 \rightarrow \psi_0^*$ at the resonance frequency ν is

$$\begin{aligned} & \left\{ \frac{\sum_{**} \left(\int \psi_0 \dots \psi_N \mathbf{u}_0 \mathbf{E} \psi_0^{**} \dots \psi_N d\tau \right) \times \left(\int \psi_0^{**} \dots \psi_N \mathbf{u}_0 \psi_0^* \dots \psi_N d\tau \right)}{W_0 - W_0^{**}} \right\}^2 n \\ &= \frac{8\pi^3 \nu}{3hc} \left\{ \sum_{**} \frac{\int \psi_0 \mathbf{u}_0 \psi_0^{**} d\tau' \int \psi_0^{**} \mathbf{u}_0 \psi_0^* d\tau'}{W_0 - W_0^{**}} \right\}^2 \\ & \quad \times \left(\int \psi_1 \dots \psi_N \mathbf{E} \psi_1 \dots \psi_N d\tau'' \right)^2 n \\ &= \frac{8\pi^3 \nu}{3hc} \left(\int \psi_0 \alpha \psi_0^* d\tau' \right)^2 \left\{ \sum_i \left[\frac{2\bar{\mu}_i \cos \theta_i}{R_i^4} + \frac{3\bar{P}_i(3 \cos^2 \theta_i - 1)}{2R_i^6} + \dots \right] \mathbf{R}_i \right\}^2 n, \quad (2) \end{aligned}$$

where α is the polarizability tensor, and n is the number of molecules in unit volume. It must be remembered that only the diagonal elements (permanent moments) $\bar{\mu}_i$ and \bar{P}_i appear in (2). The matrix elements of α have already been obtained.³

We will calculate the sum of \mathbf{E}_i , assuming the so-called statistical theory.⁴ When the molecules are randomly distributed, the \mathbf{E}_i 's are also spherically distributed. To sum these electric field vectors is equivalent to searching for the position of a Brownian particle whose unit displacement is spherically distributed. The solution of the latter problem is obtained by Markoff's method;⁵ thus, the probability that the summed field has an absolute value between E and $E + dE$ is given by the formula

$$W(E)dE = \frac{4\pi \exp(-3E^2/2 \langle \sum_i E_i^2 \rangle)}{(2\pi \langle \sum_i E_i^2 \rangle/3)^{3/2}} E^2 dE, \quad (3)$$

where $\langle \sum_i E_i^2 \rangle$ is the average of $\sum_i E_i^2$, which is

$$\begin{aligned} \langle \sum_i E_i^2 \rangle &= \int_{R_0}^{\infty} \int_0^{\pi} \left\{ \frac{2\bar{\mu} \cos \theta}{R^3} + \frac{3\bar{P}(3 \cos^2 \theta - 1)}{2R^4} + \dots \right\}^2 \\ & \quad \times 2\pi \sin \theta d\theta R^2 dR n \\ &= 4\pi n \left\{ \frac{4\bar{\mu}^2}{9R_0^6} + \frac{9\bar{P}^2}{25R_0^6} + \dots \right\}. \quad (4) \end{aligned}$$

The absorption coefficient of gas is

$$\begin{aligned} & \frac{8\pi^3 \nu}{3hc} \left(\int \psi_0 \alpha \psi_0^* d\tau' \right)^2 \int_0^{\infty} E^2 W(E) dE n \\ &= \frac{16\pi^3 \nu}{3hc} \left(\int \psi_0 \alpha \psi_0^* d\tau' \right)^2 \langle \sum_i E_i^2 \rangle n. \quad (5) \end{aligned}$$

For the Q -branch of symmetric diatomic molecules at room temperature this formula is reduced to

$$\frac{192\pi^4 \nu n^2}{25hcR_0^6} \bar{P}^2 \left(\beta^2 + \frac{11}{30} \gamma^2 \right), \quad (6)$$

where

$$\beta = \int \psi^{\frac{1}{2}} (\alpha_{xx} + \alpha_{yy} + \alpha_{zz}) \psi^* d\tau,$$

$$\gamma = \int \psi (\alpha_{zz} - \alpha_{xx}) \psi^* d\tau.$$

In the paper of Crawford, Welsh, and Locke,² the intensity curve for the vibration line of O_2 near 1556 cm^{-1} is given, from which the absorption coefficient of the Q -branch is calculated to be $4.0 \times 10^8 \text{ cm}^{-1} \text{ sec}^{-1}$ at 1 atmos.

In this case, n is $2.72 \times 10^{19} \text{ cm}^{-3}$, R_0 may be considered to be equal to twice the interatomic distance of O_2 , thus $R_0 = 2.4\text{A}$. For \bar{P} , there are no experimental data, but, from the curve of Lassette and Dean,⁶ we can estimate it to be $3.4 \times 10^{-26} \text{ cm}^2$. In order to explain the experimental result with these values, $\beta^2 + (11/30)\gamma^2$ must be $2.1 \times 10^{-50} \text{ cm}^4$. Although no direct measurements of the latter exist, at least the order of this value is certainly valid. (An estimation with classical model yields $6.6 \times 10^{-50} \text{ cm}^4$ for it.) Thus we may consider that the experimental result was explained by our formula (6).

By (6) we can estimate the contribution of the pressure absorption in the Herzberg's experiment.¹

If we assume that the change of P during the vibration is nearly equal to \bar{P} in the static state, the proportion of quadrupole absorption to pressure absorption is about

$$1: \frac{n}{R_0^6} \left(\beta^2 + \frac{11}{30} \gamma^2 \right) \doteq 1:10^{10}$$

at 10 atmos. (his experimental condition); thus the pressure absorption is predominant.

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Experimental Study of the Compton Effect at 1.2 Mev

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SINCE the Compton effect represents one of the most fundamental interactions between radiation and matter, it is desirable that the theory be compared carefully with experiment. The angular distribution of the scattered gamma-rays has been predicted by Klein and Nishina,¹ and while it is true that the total cross sections have been well checked by many investigators,² search of the literature reveals that angular distribution studies have been relatively neglected. Angular investigations in the x-ray region show reasonably good agreement with theory,³ and Chao,

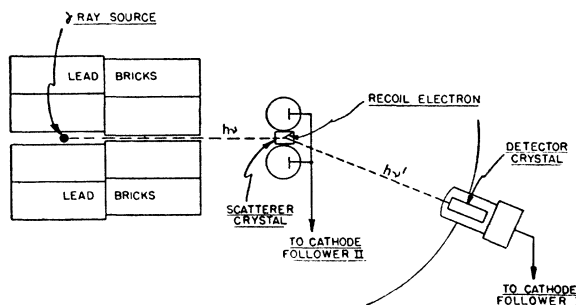


FIG. 1. Compton scattering apparatus for 1.24-Mev gamma-rays.

Bay and Szepesi, and Jordan⁴ have been experimental studies in the gamma-ray region. However, these latter results are not sufficiently accurate to provide a conclusive test of the Klein-Nishina formula. Recent developments in counting gamma-rays with scintillations make possible a more accurate experimental investigation of the angular scattering in the Compton effect. The preliminary results of such a study are presented below.

The apparatus used in this experiment has been briefly described previously.⁵ A sketch of the main features appears in Fig. 1. The quantity measured in the experiment is the number of coincidences between the recoil electron (counted by the "scatterer") and the scattered photon (counted by the "detector"). The number of coincidences has been studied as a function of the angular position of the detector. This number is corrected for the detector's efficiency and may be directly compared with the theoretical functions.

The source in this experiment has been a 20-millicurie Co⁶⁰ rod of dimensions $\frac{1}{8}$ inch diameter, $\frac{3}{8}$ inch length. Co⁶⁰ gamma-ray lines are present in equal strength and are of energies 1.169 and 1.331 Mev. The collimating channel is 10 inches long and of $\frac{1}{8}$ -inch \times $\frac{1}{8}$ -inch cross section in lead. A $\frac{1}{2}$ -inch stilbene cube serves as scatterer and the detector is a block of stilbene $\frac{1}{2}$ inch \times $\frac{1}{2}$ inch \times 1 inch. In this experiment the distance between scatterer and detector is 11.6 cm. The crystals as well as the 1P21 photo-multipliers are carefully selected, and in operation the detector photo-multipliers are cooled to dry ice temperature. A $\frac{1}{16}$ -inch aluminum sheet is placed between the scatterer and detector to isolate the detector from any scatterer recoil electrons. The coincidence method uses an oscilloscope viewed by an auxiliary counting photo-multiplier (not shown) and will be described elsewhere.

Forty-eight measurements of the number of coincidences per 200 seconds were made at each angular position examined. Plateaus in counting rate indicated that all scattering events were counted at each angular position except 20° and 15° where plateaus were not obtained. The number of events missed at 20° was probably not greater than 10 percent, however. In Fig. 2, which shows results obtained, the counting rates were corrected for the efficiency of the stilbene detector crystal which varies with the energy of the scattered photon. The well-established experimental total cross sections were used to make this correction. Other corrections applied to the experimental points in Fig. 2 have been made for: (a) the varying amount of absorption in the scatterer crystal which depends on the path traversed and photon energy and, hence, on the angular position of the detector; (b) the absorption of the scatterer photo-multiplier (glass envelope and contents) also in the path traversed by the scattered gamma-ray in some angular positions; and (c) the absorption of the thin-walled aluminum housings surrounding scatterer and detector

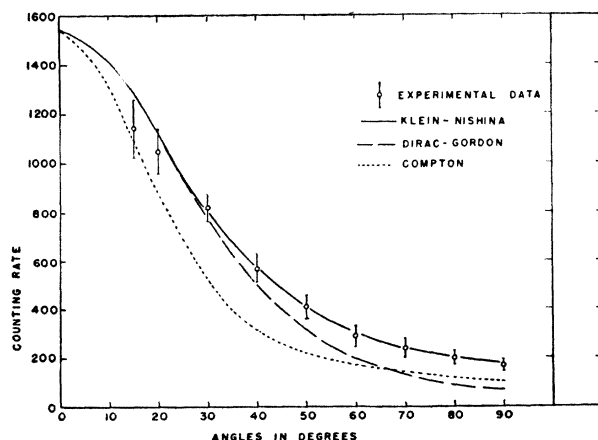


FIG. 2. Comparison of experimental and theoretical curves.

photo-multipliers as well as the absorption of the $\frac{1}{16}$ -inch aluminum sheet isolating the detector from scatterer electrons. The sum of corrections (a), (b), and (c) amounts to 9 percent at 20°, 21 percent at 50°, and 30 percent at 90°.

The theoretical curves plotted in Fig. 2 are the sums of the angular distributions of scattered photons for each of the two Co⁶⁰ gamma-rays. The strength of the source is known to only about 15 percent, and the normalization of the experimental curve has, therefore, been made with the closest theoretical Klein-Nishina curve within this 15 percent range. The Klein-Nishina curve is seen to fit the data much better than the older Compton and Dirac-Gordon curves. Further improvements in the experiment which will eliminate corrections (b) and (c) and determine absolute source strength are contemplated. The former improvements will reduce the total corrections to 9 percent at 50° and 11 percent at 90°.

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The Electromagnetic Separation of the Isotopes of Mercury

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THE natural isotopes of mercury have been enriched successfully in the mass spectrographs (calutrons) at the electromagnetic separation plant, Oak Ridge, Tennessee. Effort has been directed toward obtaining an enriched even-mass isotope with low odd-mass contamination, to minimize the isotope wave-length shift in the mercury spectrum, particularly the 5461A line. For this purpose the enriched sample of Hg 202, shown in Table I, has

TABLE I. Enriched and natural abundances of Hg isotopes in the sample of Hg 202.

Isotope	Enriched abundance	Natural abundance
Hg 196	0.014 percent	0.16 percent
Hg 198	0.092	10.02
Hg 199	0.113	16.92
Hg 200	0.781	23.10
Hg 201	0.365	13.22
Hg 202	98.06	29.72
Hg 204	0.574	6.84

been obtained. In this product the sum of the odd mass isotopes is 0.478 percent and the sum of the even mass isotopes is 99.522 percent.

In addition to the Hg 202 collection, Hg 198 has been enriched to >60 percent, Hg 199 to >70 percent, Hg 200 to >75 percent, and Hg 204 to >92 percent. Hg 196 and Hg 201 are still in process at this writing. Isotopic mixtures of high single isotope concentration are available on allocation from the Isotopes Division, Atomic Energy Commission, Oak Ridge, Tennessee.

The high purity Hg 202 reported in this letter has been examined in our spectroscopy laboratory by J. R. McNally, P. M. Griffin, and L. E. Burkhart and its characteristics are being discussed in a companion Letter to the Editor of the Journal of the Optical Society of America.

The successful enrichment of these mercury isotopes is the result of the cooperative efforts of the personnel of the Isotope Development Department. The contribution of the calutron crews, engineers, and shops are appreciated. These groups most directly