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On the Origin of Elements

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The abundances of elements and of isotopes indicate that heavy and light elements have been produced by difference processes. The origin of heavy elements is discussed in detail. It is assumed that the heavy elements were formed by a fission process from a neutron-rich nuclear fluid. Simple assumptions are made about this fission process and isotopic abundances are calculated for $62 \leqq Z \leqq 78$. The properties of the neutron-rich liquid and possible details of the fission process are discussed.

I. INTRODUCTION

REGULARITIES in the abundances of nuclear species form the experimental evidence on which any theory of the origin of elements must be based. The data[1] on the abundance of chemical elements are shown in Fig. 1. The most striking feature of this figure is the different behavior of light and heavy elements. All the abundant elements are light. Furthermore, the abundances of light elements vary erratically. The last high abundance peak lies at Fe and within the next six or eight elements the abundance dips by a factor 10^4. Somewhat arbitrarily, the boundary between light and heavy elements shall be fixed at the end of this decrease, namely at $Z=34$. The abundance of heavy elements is low and roughly constant. The observed fluctuations seem mainly to be due to stability questions: odd Z-values have a small abundance, nuclei with closed shells[2] a high one.

Isotopic ratios also behave differently for light and heavy elements. This regularity shows up for even Z-values for which as a rule several isotopes are known. For high elements the lightest isotope is usually abundant, the heaviest is often rare. For the heavy elements there exists a dissymetry in the opposite sense. The lightest, and often also the next to lightest, isotope is rare, whereas the heaviest stable isotope is, as a rule, quite abundant. The first element to which this rule applies is Se, with $Z=34$. Its lightest isotope, Se^{74}, has an abundance of 0.9 percent, which is to be compared with 9.3 percent for the heaviest isotope. Among the other heavy elements the lightest isotopes are always less abundant than 1.4 percent with the exception of five cases. Four of these have closed neutron shells. The heaviest isotope is never rare.

The experimental evidence permits us to put forth a number of simple qualitative arguments.

A. Light and heavy elements differ markedly both in abundance and isotopic ratios.

B. The light elements may have been formed by thermonuclear reactions. The apparently greater abundance of lighter isotopes may be explained by assuming that the build-up process consisted in adding protons to already existing nuclei. The great differences between the abundances of light nuclei may be explained by the sensitive dependence of the effective cross sections on the temperature and by the great variability of cross sections for different kinds of processes.

C. There is conclusive evidence that at the time of production of the heavy nuclei, the proportion of neutrons considerably exceeded that which is now present in nuclei. Evidence for this neutron-excess comes from two sources: first, without such a neutron excess it is not possible to understand that the heavy isotopes of heavy elements are much more abundant than the lightest isotopes. Second, in absence of a neutron excess it is very hard to find any method by which the heavy nuclei could be built up at all. The only alternative would be to build up the heavy nuclei by reactions between charged particles. Such reactions would, however, require extremely high temperatures, and at these temperatures it is not possible to prevent disintegration of uranium and other fissionable nuclei. A neutron-excess within the heavy nuclei may stabilize them against fission.

Of the three statements proposed above, the last one seems inescapable. In fact, this last conclusion forms a part of every theory on the origin of heavy elements which so far has been proposed.[3] In the following we shall restrict our attention to the formation of heavy elements. In particular, we are going to explore the hypothesis that the heavy nuclei were formed by the disintegration of a "cold" nuclear fluid containing a great excess of neutrons.

II. THE BREAK-UP HYPOTHESIS

The breakup of a primordial nuclear fluid is in many respects similar to the fission process. We shall of course not limit ourselves to the neighborhood of uranium, but assume the presence, at that stage, of much heavier nuclei. These nuclei break up due to their

[1] The values used here have been given by Harrison S. Brown, Rev. Mod. Phys. (to be published), whose work was in many details based on the publication of V. M. Goldschmidt (Det Novske Videnskaps, Akademi i Oslo, I Mat.-Naturv. Klasse 1937, No. 4).

[2] M. G. Mayer, Phys. Rev. **74**, 235 (1948).

[3] G. Gamow, Nature **162**, 680 (1948); C. B. van Albada, Bull. Astr. Inst. Netherlands **X**, 374 (1946); and Astrophys. J. **105**, 393 (1947); Klein, Beskow, and Treffenberg, Arkiv. f. Mat. Astr. o. Fys., Bd. 33B, No. 1 (1946). For further references see "On the abundance and origin of elements," by E. Teller and M. G. Mayer, Report to the Solvay Congress, Brussels (1948).

high charge. The fragments have an excess of neutrons and are highly excited, and thus, neutron evaporation follows. The number of neutrons evaporated is assumed much greater than in fission. The residual nuclei and their β-decay products are assumed to be the present heavy nuclei in the region beyond $Z=34$.

One might expect that elements with $Z>137$ are excluded since, in a Coulomb field with $Z>137$ the electron orbits with $j=\frac{1}{2}$ can no longer exist outside of the nucleus. This actually need not be considered as a difficulty if the finite extension of the nucleus is taken into account. We shall return to this question in the following section.

Let us consider a nuclear fragment, which, at the moment of the breakup has a charge Z. Even Z values are produced as fission fragments more frequently than odd Z values. This is so because at great neutron excesses pairs of protons are likely to form configurations similar to α-particles, while a single proton can be found in a configuration similar to that of a triton nucleus. These configurations are likely to be better approximations to the actual wave functions in a nuclear fluid rich in neutrons than they are in atomic nuclei. The great difference in binding energy between the α-particle and the triton then helps to explain why more fragments with even Z-values are formed. We shall assume that such fragments are produced exclusively.

The neutron number associated with the fragment of charge Z is not fixed, but its average is determined by the properties of the large nucleus before the break-up. It seems plausible to assume that the ratio of neutron- to proton-number does not vary over a wide range. In the break-up process, energy may be distributed over many degrees of freedom, with the result that the probability of finding the fragment with various internal energies can be represented by a smooth curve, probably a Gaussian. In the neutron evaporations varying amounts of energy will be lost as kinetic energy. After a number of such evaporation processes a resultant fixed (not yet stable) isotope will be left with energy whose probability distribution is most likely to be Gaussian. Finally, not enough energy will be left to evaporate another neutron, and the process will stop. The probability $P(Z, N)$ of the process terminating at a definite isotope with neutron number N is

$$P(Z, N)=K_Z(E_{N,Z}-E_{N-1,Z}) \times\exp[-(1/\alpha^2)(E_{N,Z}-E_0)^2]. \quad (1)$$

$E_{N,Z}$ is the binding energy of an isotope containing N neutrons; E_0 is the binding energy of the nucleus for which $P(Z, N)$ is a maximum. Both E_0 and α, the spread of isotopes, are functions of Z. K_Z is a normalization factor. The next factor, $E_{N,Z}-E_{N-1,Z}$ is the binding energy of the last retained neutron. Formula (1) is valid only if the spread, α, corresponds at least to several units, in which case the probability of evaporating down to N, but not to $N-1$ neutrons, is proportional to the binding energy of the last neutron.

Some of the isotopes produced will be unstable with respect to β-decay, and a chain of these events produces the final stable nuclei.

The nuclear energies were obtained from the semi-empirical formula for the mass of an atom.[4]

$$M=A-0.00081Z-0.00611A+0.014A^{\frac{2}{3}} +0.083\left(\frac{A}{2}-Z\right)^2 A^{-1}+0.000627Z^2A^{-\frac{4}{3}}+\delta, \quad (2)$$

where $\delta=0$ for A odd, $\delta=-0.036A^{-\frac{3}{4}}$ for A even Z even, $\delta=+0.036A^{-\frac{3}{4}}$ for A even Z odd. From this formula, the value of Z for which the energy is a minimum at constant A can be calculated. These points of Z and A will be referred to as the stability line. The observed asymmetric distribution of elements can be explained only if the maximum of the Gauss distribution lies at higher neutron numbers than correspond to the sta-

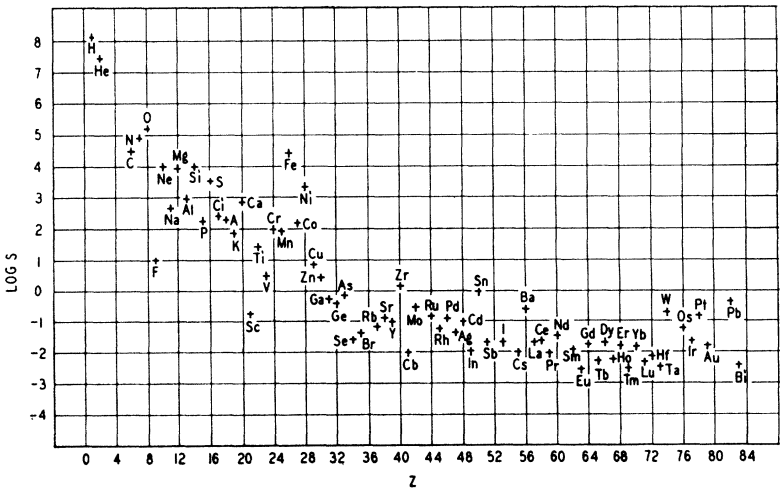

FIG. 1. Abundance of the chemical elements.

[4] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939); von Albada, Astrophys. J. **105**, 393 (1947).

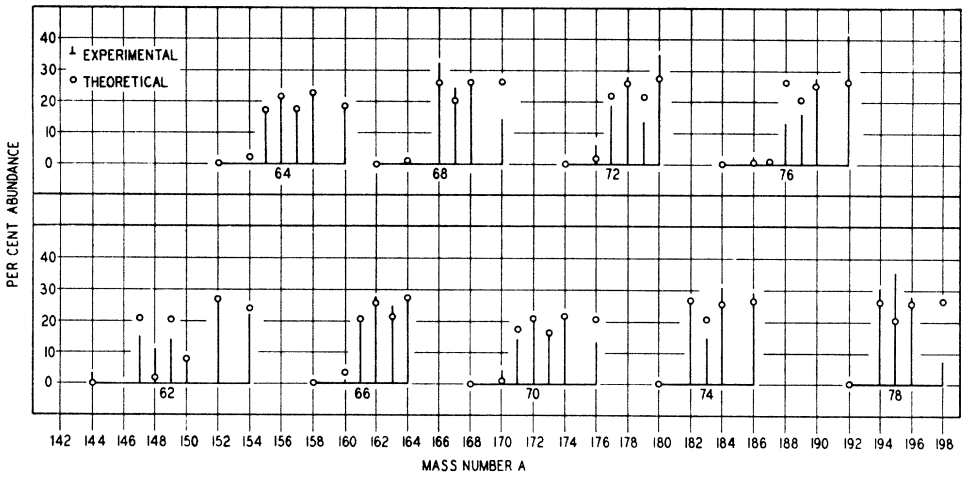

FIG. 2. Relative abundance of isotopes.

bility line. Formula (2) does not take into account fluctuations in binding energy. Such fluctuations occur near closed shells of 50, 82, or 126 like nucleons. In the range of nuclei $62 \leq Z \leq 78$ the influence of closed shells is minor. We have selected this region for a preliminary comparison. In order to fit the abundances with the minimum number of parameters, we assume that, if N, Z lies on the stability curve, $E_{N,Z} - E_0$ is a constant. In our calculations this constant was set equal to 0.03569 mass units.

The spread of the distribution, α, was chosen in such a way that the abundance of the lightest isotopes should agree as closely as possible with the actual abundances. In the calculations, $\alpha = 0.02415$ mass units was used.

As is seen from Fig. 2, there is a rough qualitative agreement between the calculated and observed isotopic distributions.

As a typical example we consider the case $Z = 62$. The agreement is satisfactory except for the isotopes of weight 144 and 148. The latter isotope is less abundant than the two neighboring odd isotopes. This remarkable behavior is reproduced and grossly exaggerated by theory.

The reason for this is that the isotope 148 is "shielded," whereas 147 and 149 are not. "Shielded" nuclei are those which cannot be formed by a chain of β-decays from elements containing 2, 4, 6, $\cdots$ less protons, since such chains would stop at stable isobars of lower Z value. In our theory, the isotope 148 is formed in the tail of the Gaussian for $Z = 62$, whereas the main contribution to the abundance of isotopes 147 and 149 comes from the β-decay of the corresponding isotopes of $Z = 60$, which are near the maximum of the Gaussian. The experimental abundance of the isotope 144 is 3 percent, whereas the calculation gives 0.0015 percent. This isotope has 82 neutrons and the relatively high abundance is connected with the stability of this shell.

TABLE I. Abundance of isotopes of odd mass number.

Range of nuclear charge (even Z-values only)	$Z=32$ to $Z=40$	$Z=42$ to $Z=50$	$Z=52$ to $Z=60$	$Z=62$ to $Z=70$	$Z=72$ to $Z=80$
Average percentage of odd mass-number isotopes	9%	25%	19%	33%	27%

One striking regularity in the isotopic ratios is the relatively small abundance of odd mass-number isotopes among the elements of even Z. This is partly due to the fact that practically all even mass-number nuclei have even Z-values while one may assume that odd mass-number elements are about equally divided between elements of even and odd charge. If this were the full explanation and if even and odd mass-numbers were equally abundant one should expect the average percentage of odd isotopes for even Z to be 33 percent.

The abundances of odd isotopes among the heavy elements of even Z vary considerably. Averages of the percentage of odd isotopes are given in Table I.

The value of 33 percent is reached only in rare earth region. This may be due to a fluctuation. For the heavy elements, the average percentage of odd mass-number isotopes is closer to 25 percent than to 33 percent. Assuming that there are as many odd mass-number nuclei for odd as for even Z, the ratio of the number of all heavy nuclei with odd mass-number to those of even mass-number is 0.671. The same ratio for the region $62 \leqq Z \leqq 78$ is 0.86. The calculated value is 0.80. This calculated ratio does not depend on an adjusted parameter but on the difference in binding energy of even and odd neutrons as given by (2) and on the assumption of original creation of fragments with even Z only. The latter assumption is, of course, essential.

A more complete calculation of abundances should include the following effects.

a. Fluctuation in Binding Energies

In the absence of accurate mass determinations one should correct formula (2) for the presence of closed shells. According to the experimental stability line a defect in binding energy exists for several neutrons fol-

lowing a closed shell; a corresponding excess of binding energy is indicated for the last neutrons of a closed shell. The same statement holds for protons. Such changes in binding energy would appear explicitly in (1) and they also influence the value of E_0.

b. Delayed-Neutron Emission

Experience with nuclear fission shows that in some cases β-processes are followed by further neutron evaporation. In the two identified cases the neutrons evaporated were the 51st and the 83rd. The reduced binding-energy of neutrons beyond the 50th and 82nd is a factor favoring delayed neutron emission. Nuclei further from the stability line have a higher β-energy, and for these nuclei delayed neutron emission may be even more probable.

c. Neutron Capture

Some of the neutrons evaporated from the nuclear fragments could be recaptured by nuclei. This process would deplete nuclei with high capture cross sections and enrich the next heavier isotope, provided this isotope has a lower neutron cross section. It happens to be a fact that all the known neutron absorption cross sections of nuclei with 50 or 82 neutrons are low.

III. THE POLYNEUTRON MODEL

In order to obtain a model of a nucleus which can serve as the starting point for the break-up process we shall assume that an assembly of neutrons forms a nuclear fluid which will not spontaneously disintegrate into neutrons. The only limitation imposed on the size of this polyneutron is that its total mass should not exceed the mass of a star. For bigger masses, effects of gravitation and general relativity would become decisive. These we do not propose to discuss.

The polyneutron is unstable with respect to β-decay. Such a decay transforms a neutron into a proton and liberates energy equal to the sum of the mass difference of neutron and proton and the binding energy of a triton. Added terms, due to interaction of the neutrons with the triton are assumed to be smaller. This means that for each β-decay an energy of about 9 Mev is obtained. A second β-decay can form an α-particle liberating an energy of about 20 Mev.

If the disintegration electrons left the polyneutron, Coulomb repulsion would soon make further β-decays energetically unfavorable. For a large polyneutron there is, however, no reason why the electrons should not remain in the nuclear liquid and neutralize the charge of the α-particles. If no other process intervenes, the disintegrations will proceed until the electron density has risen to the point where the zero point energy of the electrons is 9 Mev, making further triton formation impossible.

The maximum kinetic energy of the electrons is equal to the absolute value of their potential energy, $e\varphi$, and this quantity is connected to the number of electrons per unit volume n_e by the relativistic formula

$$n_e=(1/3\pi^2)(e\varphi/c\hbar)^3. \tag{3}$$

Near the surface of the polyneutron the electron distribution will protrude. The potential outside, φ, satisfies the relation

$$\Delta\varphi=-4\pi e n_e. \tag{4}$$

Using (3) we obtain the relativistic Thomas-Fermi equation

$$\Delta\varphi=(1/K^2e^2)\varphi^3. \tag{5}$$

Where K is a dimensionless number,

$$K=[(3\pi/4)(\hbar c/e^2)^3]^{\frac{1}{2}}=2462. \tag{6}$$

Assuming that φ depends only on the distance r from the surface of the polyneutron, neglecting effects of curvature, and assuming that the potential vanishes for $v\rightarrow\infty$, one obtains

$$\varphi=\sqrt{2}Ke/(r+r_0), \tag{7}$$

where the integration constant r_0 is determined by the condition of continuity at $r=0$. One obtains

$$r_0=(\sqrt{2}Ke/\varphi_i), \tag{8}$$

where φ_i is the constant value which the potential assumes in the interior of the polyneutron. If the β-decay has gone to completion, $\varphi_i=9$ million volts and $r_0=5.6\times10^{-11}$ cm. This thin layer of electrons is the rudiment of the extra-nuclear electrons in an atom.

The density of α-particles, n_α, inside the nucleus, is one-half that of the electrons.

$$n_\alpha=(1/6\pi^2)(e\varphi_i/c\hbar)^3=(2^{\frac{1}{2}}/4\pi)(K/r_0^3). \tag{9}$$

If φ_i is 9 million volts, $n_\alpha=1.60\times10^{+33}$ cm^{-3}.

The formation of the thin electron cloud near the surface lowers the surface energy σ of the polyneutron by the amount σ_e.

$$\sigma_e=-(1/6\pi)(K^2e^2/r_0^3), \tag{10a}$$

or

$$\sigma_e=-(\sqrt{2}K/12\pi)(\varphi_i e/r_0^2). \tag{10b}$$

Substituting the value $\varphi_i e=9$ Mev and using formulas (6) and (7) one obtains

$$\sigma_e=-4.24\times10^{17}\text{ erg cm}^{-2}=-0.265\text{ Mev barn}^{-1}. \tag{11}$$

This energy favors an increase in surface.

There will also be effects which tend to minimize the surface. For stable nuclei, these effects alone will be relevant. The corresponding surface tension σ_s can be obtained from the term proportional to $A^{\frac{2}{3}}$ in the semi-empirical formula (2). The assumptions that led to (2) give

$$\sigma_s=7.57\times10^{19}\text{ ergs cm}^{-2}=47.3\text{ Mev barn}^{-1}. \tag{12}$$

This surface tension is caused by the tendency of neutrons and protons to surround themselves with

other nucleons. A similar positive contribution to the surface tension must be expected in the case of the polyneutron. Such a contribution will decrease with decreasing neutron binding energy for two reasons. First, the energy needed to bring a neutron to the surface may be expected to be proportional to the binding energy. Second, the surface increase obtained in this way increases as the square of the average de Broglie wave-length of the neutron. If the kinetic energy of the neutron is assumed to be proportional to its binding energy, the surface increase is inversely proportional to the binding energy. Thus one is led to expect that σ_s of a polyneutron is proportional to the square of the binding energy of the neutrons within that structure. The positive surface tension σ_s will just balance the negative term σ_e if σ_s is about 180 times smaller than was calculated for stable nuclei. According to the above crude argument this will happen if the binding energy of a neutron in a polyneutron is 13.3 times smaller than that of a neutron in a stable nucleus. Using 8 Mev for the latter quantity one may estimate that 600,000 ev is this critical neutron binding energy for the polyneutron.

In order to investigate the surface stability, we shall briefly discuss the properties of surface waves. The frequency ω as a function of the wave-length λ, the density ρ, and the surface tension σ of the liquid is

$$\omega=(\sigma/\rho\lambda^3)^{\frac{1}{2}}. \tag{13}$$

The surface tension of the polyneutron, after the β-decay has proceeded for some time, is $\sigma=\sigma_s+\sigma_e$. We shall assume that from a certain point on the negative term σ_e is the bigger one of the two terms, that is, the total surface tension σ becomes negative. Surface waves will then no longer show a sinusoidal time dependence, but will grow exponentially as $e^{\omega t}$ with $\omega=(|\sigma|/\rho\lambda^3)^{\frac{1}{2}}$.

This statement is correct only for sufficiently long wave-lengths. For short wave-lengths, σ must be considered as a function of λ. In fact, if λ is small compared to r_0, the thickness of the electron cloud, the electronic energy does not contribute to the surface tension, and σ therefore is positive. The effective σ, to be inserted in (13), will be zero at an intermediate wave-length, which one would expect to be of the order r_0, unless the positive and negative parts of the surface tension are very closely balanced. The instability will lead to the fastest break-up of the surface where $\omega=(|\sigma|/\rho\lambda^3)^{\frac{1}{2}}$ has its maximum value. This will happen at a wave-length somewhat longer than that at which σ vanishes.

Because of this surface instability, droplets will break off. According to the above argument the order of magnitude of the droplet diameter is r_0. The charge Z_p contained in the droplet volume is, according to (9),

$$Z_p=\frac{\pi}{6}r_0{}^3 2n_\alpha=\frac{2^{\frac{1}{2}}}{12}K=\frac{1}{12}\left[\frac{3\pi}{2}\left(\frac{\hbar c}{e^2}\right)^3\right]^{\frac{1}{2}}=290. \tag{14}$$

The value of Z_p depends only on universal constants and not on the question of how far the β-decay has progressed before the break-up.

Formula (14), in addition to representing only a crude estimate, disregards the fact that our nuclear fluid carries a surface charge. This positive surface charge is equal to the charge of the electrons which protrude from the polyneutron. Substituting (7) into (3) and integrating over r from 0 to ∞, one gets for ξ, the surface charge per unit area

$$\xi=\frac{\sqrt{2}\,Ke}{4\pi\, r_0{}^2}. \tag{15}$$

A droplet of radius R will carry a part of this surface charge. One will expect, however, that its amount will be a fraction β of $4\pi R^2\xi$ because during the formation of the droplet the surface stretches and the same number of positive charges are spread over a larger area. One may assume $\frac{1}{4}<\beta<\frac{3}{4}$. The charge number $Az(R)$ as a function of the radius is, therefore,

$$Z_p(R)=2^{\frac{1}{2}}K[\tfrac{2}{3}(R/r_0)^3+\beta(R/r_0)^2]. \tag{16}$$

The energy released upon formation of a droplet consists of three terms. These are caused by the additional extrusion of electrons, by the increase in surface energy, and by Coulomb repulsion between positive charges. (For the low Z_p values to be considered the electrostatic term involving electron-electron interaction in the product nucleus is negligible.) It is probable that the β-decay will proceed beyond the point where the two surface energies balance and at the time of breakup one expects $\sigma_s<|\sigma_e|$. For the purpose of an estimate, we set, however, $\sigma_s=|\sigma_e|$. The total energy released in the droplet formation, $E(R, Z_p)$ is then

$$E(R, Z_p)=\frac{K^2e^2}{r_0}\left[2^{\frac{1}{2}}\frac{Z_p}{K}-\frac{2}{3}\left(\frac{R}{r_0}\right)^2\frac{3}{5}\left(\frac{Z_p}{K}\right)^2\frac{r_0}{R}\right]. \tag{17}$$

The values of the charge, Z_p, the energy, the energy per unit charge and per unit volume are collected in Table II for different values of β and R. (The energy is given in units of $e\varphi_i$ and R in units of r_0.)

Assuming that a constant fraction of the energy appears as energy of translation, the droplet velocity

TABLE II. Values of the charge, Z_p, the energy, the energy per unit charge and per unit volume for different values of β and R. (The energy is given in units of $e\varphi_i$ and R in units of r_0.)

β	R/r_0	Z_p	$E/e\varphi_i$	$\frac{E}{Z_p(e\varphi_i)}$	$\frac{3r_0{}^3E}{4\pi R^3 e\varphi_i}$
0.25	0.1	11	−0.79	−0.072	−188
0.25	0.2	53.4	+4.50	+0.084	+134
0.25	0.5	508	128.7	0.254	246
0.50	0.1	19.7	7.45	0.377	1784
0.50	0.2	88.2	35.1	0.398	1046
0.50	0.5	725	254	0.350	485
0.75	0.1	28.4	15.4	0.543	3690
0.75	0.2	123	63.6	0.517	1899
0.75	0.5	943	346	0.367	663

is proportional to the square root of the energy per unit volume. High-velocity droplets are formed faster and, therefore, in greater numbers. Table II shows that for small values of β the energy of formation of small droplets is negative, which means that such droplets would not form. For larger values of β greater velocities are obtained for smaller droplets. Thus, most droplets may be formed with charges smaller than those given in (14). The energy per unit volume does not depend sensitively on Z_p and so the whole range of known nuclear charges may be obtained. Charges smaller than 34 may be obtained but the great abundance of these light nuclei cannot be explained by this process. Nuclei with $Z_p>92$ are also formed. These will eventually decay, probably by fission, into known elements. For sake of definiteness we assume that the bulk of heavy elements is formed by neutron-evaporation from primary droplets.

In discussing formation of droplets we neglected motion of charges by exchange. No proof can be given that this motion is faster than the motion of the nucleons.

During the formation of droplets many neutrons are present for each pair of protons. Under these conditions groups resembling α-particles will be formed and it will take considerable energy to separate pairs of protons. Thus Z_p will be even and the final abundance for even and odd mass numbers will be different. The observed fission processes of uranium leads to equal abundances of even and odd mass numbers indicating that in this case fission products have even and odd Z values with roughly equal probabilities. This may be due to the fact that in uranium α-particles are less good sub-units.

The primary droplets evaporate neutrons. It is of interest to estimate the energy available for this process. In the fission of uranium considerably less than half of the available energy appears as internal energy of the fragments. Because of the looser structure of the droplets considered by us, a greater fraction of the energy given in (17) may appear as internal energy. We shall arbitrarily assume that the energy will be divided equally between translational motion and internal degrees of freedom.

In Section II it was assumed that neutrons are evaporated from the droplets until the internal energy is exhausted. We shall now compare the available internal energy with the binding energy of neutrons from the semi-empirical formula (2). For a given Z there exists a maximum neutron number $N_{\max}$ beyond which neutrons are no longer bound. Setting $A_{\max}=Z+N_{\max}$ we can determine $A_{\max}$ from

TABLE III. Values of neutron numbers $N_{\max}(Z)$ and of the energy required to evaporate neutrons from a nucleus with $N_{\max}$ neutrons until stability is reached. (All energies in Mev.)

Z	$N_{\max}(Z)$	$E(N_{\max})-E_{\text{stability}}(Z)$	$\frac{1}{2}E(Z_p)$ $\beta=0.25$	$\beta=0.5$	$\beta=0.75$
40	97.5	153	10	70	97
60	151.8	209	26	106	144
80	208.1	245	45	143	190
100	266	269	66	179	235

$$(Z/A_{\max})^2(0.083+0.000209A_{\max}^{\frac{2}{3}}) -0.0093A_{\max}^{-\frac{1}{3}}-0.00569=0. \quad (18)$$

As a crude estimate we assume that $E(N_{\max}, Z)-E(N, Z)$ is the energy needed to reach the nucleus N, Z by evaporation.

Table III contains, in column 2, the neutron-numbers $N_{\max}$. The following column gives the energy, according to formula (2), which is required to evaporate neutrons from a nucleus with $N_{\max}$ neutrons until the stability curve is reached. It is seen that these energies are somewhat greater than $\frac{1}{2}E(Z_p)$ obtained from (17) and (16). Thus it is plausible that the evaporation stops before the stability-line is reached. In fact, if $\beta=0.5$ or 0.25 is used, the evaporation would stop at a much greater distance from the stability curve than was assumed in Section II. The above results depend on many crude assumptions:

(1) Formula (2) has been used beyond its range of proved applicability. It has been assumed that for $N>N_{\max}$ the binding energy is zero.

(2) The primary droplets might have come off with $N<N_{\max}$.

(3) It was assumed in the calculation that at the time of break-up the maximum electronic energy in the polyneutron is 9 Mev.

(4) We have made specific assumptions about the surface energy, the value of β and the distribution of droplet energy between internal and kinetic energy.

All that can be stated is that the model here discussed is compatible with the assumption of Section II concerning the center of the Gaussian. The half-breadth of the Gaussian used in II corresponds to about 37 Mev. It seems likely that a considerable or even preponderant part of this is due to a variation of energy given to the primary droplets.

It is evident that the above proposals cannot be considered as proved. It seems, however, of interest that the assumption of a great and indefinite number of excess neutrons leads to a break-up process and to a distribution of isotopes which is consistent with our knowledge of isotopic abundances of heavy elements.

We wish to thank Robert W. Christy who carried out the calculations on which Fig. 2 has been based.