On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay

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The method of subtraction fields in current meson perturbation theory is described, and it is shown that it leads to finite results in all processes. The method is, however, not without ambiguities, and these are stated. It is then applied to the following problems in meson decay: Decay of a neutral meson into two and three γ-rays, into a positron-electron pair, and into another neutral meson and photon; decay of a charged meson into another charged meson and a photon, and into an electron (or μ-meson) and neutrino. The lifetimes are tabulated in Tables I, II and III. The results are quite different from those of previous calculations, in all those cases in which divergent and conditionally convergent integrals occur before subtraction, but identical whenever divergences are absent. The results are discussed in the light of recent experimental evidence.

I. INTRODUCTION

RECENTLY Pauli and Villars[1] have shown that it is possible in electrodynamics to make the self energy of the light-quantum zero, by the use of some formal subtraction methods. One of these may most easily be understood as consisting in the introduction of several fictitious subtraction fields in addition to the electron-positron field. The idea, which is due to Rivier and Stücklberg,[2] is the following: The matrix element contains an infinite integral over the momenta of the intermediate virtual electron-positron pairs which are responsible for the self energy. To this matrix element are added and from it subtracted several others for the same process, in which however the virtual pairs have different masses. Since the infinities have the same structure, it is possible to choose the number and masses of the additional fields so as to make the expression converge. In the case of the photon self-energy, the conditions which are necessary to bring convergence are also sufficient to make it vanish. One may regard this procedure as a subtraction method; no real processes involving these additional fictitious fields, such as their self energy, or scattering are considered, and one requires the masses of the extra fields to be very large. It is also possible to treat the other infinite quantities[3] in electrodynamics, the electron self-energy, and the charge renormalizations in the same way. However, this is academic, since one may disregard them, finite or infinite. In meson theories this is not so. Divergencies of a sort that cannot be removed by name-calling occur,[4] especially the decay of mesons into other particles via an intermediate Fermi-Dirac (nucleon) field. We discuss these processes in this paper.

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[1] W. Pauli and F. Villars, Rev. Mod. Phys. 21, 433 (1949).

[2] D. Rivier and E. C. G. Stücklberg, Phys. Rev. 74, 218, 986 (1948).

[3] It has been shown by F. J. Dyson, Phys. Rev. 75, 486 (1949), that all the infinite quantities in the perturbation theory of quantum electrodynamics are either of the form of a correction to the mass of the electron or to its charge.

[4] Divergences of this sort have been exhibited by K. M. Case, Phys. Rev. 75, 1440 (1949), in the calculation of the magnetic moment of nucleons due to their tensor coupling to a vector meson field.

II. SUBTRACTION FIELDS

Since it is very convenient in these and other field theoretical problems to use the Feynman diagrams,[5] the reader is assumed to be familiar with this mode of computation. It is equivalent to the older methods. For purposes of illustration, consider the disintegration of a scalar meson into two lighter scalar mesons, via an intermediate neutron field, and the scalar interaction. The Feynman diagrams are as follows:

The initial meson, of four-momentum k makes a neutron-anti-neutron pair; then either the neutron or the anti-neutron can radiate the meson k_1, before the particles annihilate with the production of the other meson k_2. The two matrix elements are

$$\frac{gg'^2}{(8E_kE_{k1}E_{k2})^{\frac{1}{2}}}\int\frac{d^4p}{(2\pi)^4}Sp$$
$$\times\frac{[(p_\mu+k_{1\mu})\gamma_\mu+im][p_\nu\gamma_\nu+im][(p_\omega-k_{2\omega})\gamma_\omega+im]}{(p^2+m^2)[(p+k_1)^2+m^2][(p-k_2)^2+m^2]}$$
$$+\text{same term with } k_1,\, k_2 \text{ interchanged}$$
$$=\frac{gg'^2}{(2E_kE_{k1}E_{k2})^{\frac{1}{2}}}\int\frac{d^4p}{(2\pi)^4}(im)$$
$$\times\frac{[3p^2+2p(k_1-k_2)-m^2-k_1\cdot k_2]}{(p^2+m^2)[(p+k_1)^2+m^2][(p-k_2)^2+m^2]}.$$

The integral is logarithmically divergent. However, if it is now regarded as a function of m, the virtual nucleon mass, and we subtract and add other nucleon fields of much larger mass, m_i (m_0 is the mass of the neutron), the sum will be finite provided $\sum_i m_iC_i=0$. $C_i=\pm1$ and indicates whether the ith field is to be added or subtracted. However, there will be a term left of the form $\sum_i C_im_i \ln m_i$, which becomes infinite as the m_i ($i\neq0$) are made large, unless it is required that $\sum C_im_i \ln m_i=\text{const}$. This constant seems to be arbitrary, and as long as it is so, the subtraction is not

[5] F. J. Dyson, Phys. Rev. 75, 486 (1949).

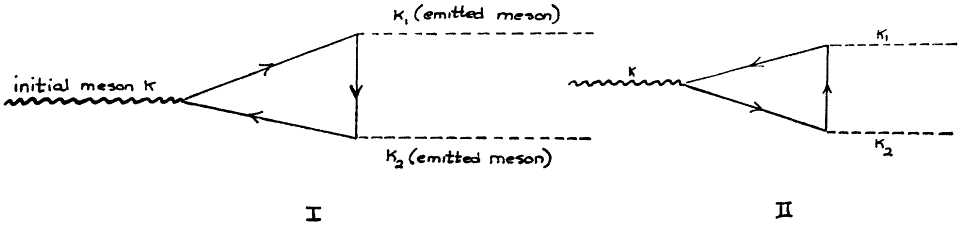

FIG. 1.

unique. In the following, it has been taken to be zero. This has the intuitively correct result that the final convergent matrix element will be small, if the intermediate mass m is large.

All infinities in field theory are similar to that of this example. Somewhere in the Feynman diagram there is a closed loop which gives rise to the infinite integral. Furthermore this loop contains a world line which begins and ends in the loop. In the above example this was the neutron line, in the case of the neutron self energy this is the meson line. The infinite integrals are either logarithmically, linearly, or quadradically divergent. They are always made finite by requiring a sufficient number of the following conditions for the masses of the additional fields of the type whose line begins and ends in the loop.

1. $\sum C_i=0$, 4. $\sum C_i \ln m_i=0$,
2. $\sum C_i m_i=0$, 5. $\sum C_i m_i \ln m_i=0$,
3. $\sum C_i m_i^2=0$, 6. $\sum C_i m_i^2 \ln m_i=0$.

Condition 1 has been used by Feynman. Pauli has required conditions 1 and 3 in the treatment of the photon self energy, and conditions 1 and 4 in the calculation of the electron self energy. Different problems require different conditions, but conditions 1–6 may be simultaneously fulfilled and always suffice.

As already pointed out in the example, the constant (zero in 4, 5, and 6) in the logarithmic condition is arbitrary although the choice which is made here is the simplest and leads to intuitively correct results.

If one applies this procedure to all virtual fields, then all Feynman diagrams give independently finite results. This is illustrated on a more complicated diagram:

It represents the scattering of a nucleon by another, through the intervention of two types of mesons, of mass κ and μ. There are two loops I_1 and I_2, giving rise to a divergent double integral. The only lines which begin and end in the same loop are those of the meson μ. The subtraction method described here requires that this matrix element be supplemented by $(n+1)^2-1$ others, in which the masses of the two intermediate μ-mesons range independently from 0 to n. One really should increase the number of diagrams still more, by also allowing the mass of the κ-meson to range over its values. However in the limit of large mass κ all such diagrams give the result zero, and one is left just with those diagrams in which the mass of the κ-meson is κ_0. Now all those diagrams are considered separately in which one of the μ-mesons, say the bottom one, has fixed mass μ_i and the top μ-meson mass runs from 0 to n. The integral I_1 is then finite, and if now μ_i is allowed to assume all its values, the double integral becomes finite. One then permits the extra masses μ_i, $i\neq 0$ to become very large and the matrix element becomes independent of them.

It does not seem possible, however, to put the method into Hamiltonian form, and this is so because some of the extra fields must be subtracted rather than added. It is rather an algorithm, defined only in terms of perturbation theory. There are no equations of motion, and therefore no rigorous solution with which the perturbation approximation can be compared. This has as its consequence that some results which follow immediately in the Hamiltonian formalism have to be reexamined. One of these, the unitarity of the "S" matrix, still holds true, and so do gauge and Lorentz invariance. Furthermore, because the formalism is convergent throughout, these properties cannot be lost during the calculation. How they may otherwise be lost in processes in which infinities occur is shown by Wentzel[6] in a calculation of the photon self-energy.

Although gauge invariance, Lorentz invariance and probability conservation are maintained in this subtraction procedure, at least one theorem, that of the equivalence of pseudoscalar and pseudovector coupling of the pseudoscalar meson to the nucleon, is not. It is lost for those processes which are convergent with pseudoscalar coupling, but divergent with pseudovector coupling before subtraction. For these cases the ordinary proof of equivalence is not rigorous, since divergent expressions occur. It is perhaps a serious difficulty, and does not seem to be connected with the lack of uniqueness of the logarithmic conditions. Of course, the equivalence is maintained for all those processes in which both couplings give convergent results before subtraction.

In this paper the word divergent is applied indiscriminately to all integrals which do not have a unique value regardless of whether or not it is possible to obtain finite values by choosing special integration

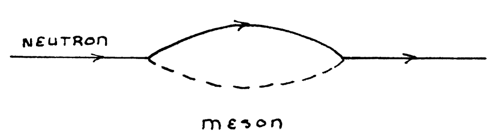

FIG. 2.

[6] Gregor Wentzel, Phys. Rev. 74, 1070 (1948).

procedures. In this connection it should be pointed out that *all* infinities or ambiguities of this sort which are present in the old perturbation theory find their counterparts in the new perturbation theory of Tomonaga,[7] Feynman[8] and Schwinger.[9]

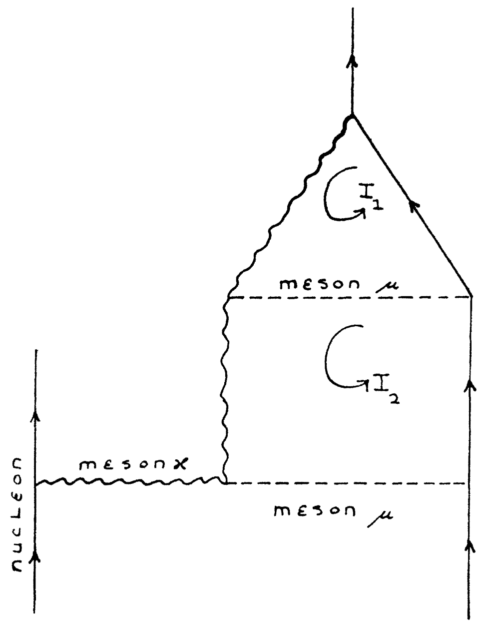

FIG. 3.

III. SOME MESON DECAYS NOT INVOLVING THE FERMI COUPLING OF THE NUCLEONS

The notation and the equations of motion are given in an appendix. Here it may suffice to point out that natural units are used, and that heavyside units are used throughout except for the electromagnetic field. There $e^2=1/(137)^{\frac{1}{2}}$. The nucleon-meson coupling constants have been left explicitly in the results; their values are not known because of the well-known failure of meson theory in the quantitative analysis of the nuclear force problem. However, they are believed to be of order of magnitude 1.

(A) Decay of a Neutral Scalar Meson into 2 Photons[10]

(1) Scalar meson with scalar coupling.

$$M=\frac{ge^2}{(2\kappa)^{\frac{1}{2}}\pi^4}A_\mu(k_1)A_\nu(k_2)[I_{\mu\nu}+J_{\mu\nu}],$$

where

$$I_{\mu\nu}=m\int_0^1 dx\int_0^x dy\int d^4p \times\frac{\{[m^2+p^2-\kappa^2y(1-x)]\delta_{\mu\nu}-4p_\mu p_\nu\}}{[p^2+m^2-\kappa^2y(1-x)]^3},$$

$$J_{\mu\nu}=\frac{-\kappa^2m}{2}\left(\delta_{\mu\nu}+\frac{2k_{1\nu}k_{2\mu}}{\kappa^2}\right)\int_0^1 dx\int_0^x dy\int d^4p \times\frac{1-4y(1-x)}{[p^2+m^2-\kappa^2y(1-x)]^3}.$$

The quantities k_1 and k_2 are the photons which appear as decay products; $k=k_1+k_2$ is the momentum of the decaying meson. $J_{\mu\nu}$ is convergent but $I_{\mu\nu}$ converges only conditionally. We therefore use the subtraction field technique. Regard M as a function of m and add and subtract auxiliary matrix elements such that condition (2) $\sum C_im_i=0$ is fulfilled. This suffices to determine M uniquely. $J_{\mu\nu}$ is unchanged in the limit of large masses. The formerly divergent part of $I_{\mu\nu}$, which is now finite, is

$$\int_0^1 dx\int d^4p\sum m_iC_i\frac{(p^2\delta_{\mu\nu}-4p_\mu p_\nu)}{[(p^2+m_i^2)-\kappa^2y(1-x)]^3}=0$$

the remaining part of I

$$\sum C_im_i\int_0^1 dx\int_0^x dy\int d^4p\frac{[m_i^2-\kappa^2y(1-x)]}{[p^2+m_i^2-\kappa^2y(1-x)]^3}=\frac{\pi^2}{2}\sum m_iC_i\int_0^1 dx\int_0^x dy=\frac{\pi^2}{4}\sum m_iC_i=0.$$

So that $I_{\mu\nu}=0$.

As was pointed out by Fukuda and Miyamoto,[11] the same result may be obtained by an application of the principle of gauge invariance as a help in evaluating the integral. If $A_\mu(k_1)$ and $A_\nu(k_2)$ are permitted to undergo gauge transformations:

$$A_\mu(k_1)\rightarrow A_\mu'(k_1)+k_{1\mu}\Lambda(k_1),$$
$$A_\nu(k_2)\rightarrow A_\nu'(k_2)+k_{2\nu}\Lambda(k_2),$$

the principle of invariance requires

$$k_{1\mu}(I_{\mu\nu}+J_{\mu\nu})=k_{2\nu}[I_{\mu\nu}+J_{\mu\nu}]=0,$$

now $k_{1\mu}J_{\mu\nu}=k_{2\nu}J_{\mu\nu}=0$ already, since

$$k_{1\mu}\left(\delta_{\mu\nu}+\frac{2k_{1\nu}k_{2\mu}}{\kappa^2}\right)=0=k_{2\nu}\left(\delta_{\mu\nu}+\frac{2k_{1\nu}k_{2\mu}}{\kappa^2}\right)$$

[7] S. Tomonaga, Prog. Theor. Phys. **1**, 27 (1946). Koba, Tati, and Tomonaga, Prog. Theor. Phys. **2**, 101 (1947); **2**, 198 (1947). S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3**, 1 (1948).
[8] R. P. Feynman, Phys. Rev. **76**, 748 (1949).
[9] J. Schwinger, Phys. Rev. **74**, 1439 (1948); **75**, 651 (1949).
[10] J. R. Oppenheimer was the first to point out that present theory requires the γ-instability of neutral mesons coupled to nucleons. The calculations were first made by R. Finkelstein, Phys. Rev. **72**, 415 (1949).

[11] Fukuda and Miyamoto, Prog. Theor. Phys. (in press), were the first to notice that the old results were not gauge invariant. Their work formed the starting point of this research. I wish to thank H. Yukawa for making their results available to me before publication.

if one just remembers that $k^2=-\kappa^2=(k_1+k_2)^2=2k_1\cdot k_2$. So we must demand $k_{1\mu}I_{\mu\nu}=k_{2\nu}I_{\mu\nu}=0$. Now $I_{\mu\nu}$ is a function of m and κ only, and therefore must be $I_{\mu\nu}=f(\kappa,m)\delta_{\mu\nu}$ and $k_{1\nu}f(\kappa,m)=0$ requires $f(\kappa,m)=0$; $I_{\mu\nu}=0$. With both methods we have the result:

$$M=\frac{ge^2}{4\pi^2}\left(\frac{\kappa}{2}\right)^{\frac{1}{2}}mA_\mu(k_1)A_\nu(k_2)\left(\delta_{\mu\nu}+\frac{2k_{1\nu}k_{2\mu}}{\kappa^2}\right)\times\int_0^1dx\int_0^x dy\frac{1-4y(1-x)}{m^2-\kappa^2y(1-x)}$$

and from this the lifetime

$$\tau^{-1}\approx g^2e^4/\pi_3(\kappa/12m)^2\kappa=8\times10^{13}g^2\text{ sec.}^{-1}.$$

Here, as in all the following calculations, this is the first non zero term in an expansion in κ/m. κ is taken to be $300m_e$.

(2) Decay of pseudoscalar meson into two photons via pseudoscalar coupling.

$$M=\frac{mge^2}{\pi^4}\left(\frac{\kappa}{2}\right)^{\frac{1}{2}}\mathbf{k}_1\cdot\mathbf{A}_1\times\mathbf{A}_2\int_0^1dx\int_0^x dy\int d^4p\times\frac{1}{[p^2+m^2-\kappa^2y(1-x)]^3}.$$

This is uniformly convergent, gauge invariant and unchanged by the subtraction fields.

$$\tau^{-1}\cong\frac{g^2e^4}{\pi^3}\left(\frac{\kappa}{8m}\right)^2\kappa=1.8\times10^{14}g^2\text{ sec.}^{-2}$$

(3) Decay of pseudoscalar meson into two photons via pseuodovector coupling.

$$M=\frac{fe^2}{2\pi^4(2\kappa)^{\frac{1}{2}}}A_\mu(k_1)A_\nu(k_2)\int d^4p\times\left\{\frac{\frac{1}{4}\,\mathrm{Tr}\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu p_\beta k_{2\alpha}}{[(p+k_2/2)^2+m^2][(p-k_2/2)^2+m^2]}+\frac{\frac{1}{4}\,\mathrm{Tr}\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu k_{1\alpha}p_\beta}{[(p+k_1/2)^2+m^2][(p-k_1/2)^2+m^2]}-\frac{m^2\,\mathrm{Tr}\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu k_{1\alpha}k_{2\beta}}{(p^2+m^2)[(p+k_1)^2+m^2][(p-k_2)^2+m^2]}\right\}.$$

Of the three terms in the integral, the first two are only conditionally convergent, and the convergent last term is the term required by the theorem of equivalence of pseudoscalar and pseudovector coupling.[12] However, if subtraction fields are used, not only do the first two terms converge (to zero), but the last term is changed, so as to destroy the equivalence between the two types of coupling. The condition needed for convergence is (1) $\sum C_i=0$. Then

$$M=\sum C_i\frac{2fe^2}{(2\kappa)^{\frac{1}{2}}\pi^2}\int_0^1dx\int_0^x dy\frac{\kappa\mathbf{k}_1\cdot\mathbf{A}_1\times\mathbf{A}_2}{[1-\kappa^2/m_i^2y(1-x)]}\cong\frac{2fe^2}{(2\kappa)^{\frac{1}{2}}\pi^2}\frac{\kappa^2\kappa}{m^2}\frac{\mathbf{k}_1\cdot\mathbf{A}_1\times\mathbf{A}_2}{24},$$

$$\tau^{-1}=\frac{f^2e^4}{\pi^3}\left(\frac{\kappa}{m}\right)^4\frac{\kappa^3}{(24)^2}=(\kappa f)^2\times5.5\times10^{10}\text{ sec.}^{-1}$$

from the third term alone, without subtraction fields, we would get

$$\tau^{-1}\cong(2mf)^2\frac{e^4}{\pi^3}\left(\frac{\kappa}{8m}\right)^2\kappa=(2mf)^2\times1.8\times10^{14}\text{ sec.}^{-1}$$

in agreement with the equivalence theorem.

In this case it is, therefore, possible to derive two plausible results. The equivalence theorem is of course not rigorously valid here, since, because of the divergence of the pseudovector case, the arguments can be carried through only formally. However, it is a strong argument against this subtraction procedure that it does not maintain such general theorems. On the other hand, the result obtained on the basis of equivalence is also questionable, since it is independent of the mass of the intermediate nucleons, whereas intuitively we require longer lifetimes with heavier nucleons.

(4) Scalar meson with vector coupling, vector meson with both types of coupling.

These transitions are forbidden by the charge conjugation theorem of Furry.[13]

(5) Pseudovector meson with pseudovector coupling.

The matrix element of this process is infinite, but becomes finite and gauge invariant by the subtraction of fields obeying (1) $\sum C_i=0$; the result is zero for both longitudinal and transverse modes of the photon. It is not difficult to prove that the transition is forbidden in all orders of the mass ratio and coupling constants.**

(B) Decay of the Vector Meson into Three Photons

The matrix element is conditionally convergent. The integral is made convergent on the introduction of the subtraction fields. It can then be seen from gauge invariance arguments that the first non-vanishing term in the matrix element occurs in 4th order of the mass ratio. The matrix element for this process resembles very

[12] The equivalence of pseudoscalar and pseudovector coupling has been discussed by E. Nelson, Phys. Rev. **60**, 830 (1941); F. J. Dyson, Phys. Rev. **73**, 929 (1948) and K. M. Case, Phys. Rev. **76**, (1949).

[13] W. H. Furry, Phys. Rev. **51**, 125 (1937).

** *Note added in proof.*—It has been proven by E. Wigner that the two photon decay of the vector meson of either parity is forbidden by angular momentum conservation arguments. I am indebted to Professor Wigner for a private communication.

much that for the scattering of light on light; there also, for small momenta of the light quanta, the first non-vanishing term is of the fourth-order in the ratio of energy of light quantum to electron mass. Because of the difficulty of the problem we content ourselves with an order of magnitude estimate.

$$M \approx \frac{ge^3}{(\kappa)^{\frac{1}{2}}\pi^2} \frac{\kappa}{m^4}[\mathbf{k}_1\cdot\mathbf{k}_2\mathbf{A}_2\cdot\mathbf{A}_1\mathbf{A}_3\cdot\mathbf{k}_1-\mathbf{k}_1\cdot\mathbf{A}_2\mathbf{k}_2\cdot\mathbf{A}_1\mathbf{k}_1\cdot\mathbf{A}_3],$$

where we have assumed one of the two possible gauge invariant forms for the matrix element, and neglected possible numerical factors except the factors of π, which are easy to determine.

$$\sum M^2 = \frac{8g^2e^6\kappa|\mathbf{k}_1|^3|\mathbf{k}_2|^3}{M^8|\mathbf{k}_3|^3}(1+2\cos^2\theta-5\cos^4\theta+2\cos^6\theta),$$

where $\cos\theta = \mathbf{k}_1\cdot\mathbf{k}_2/|\mathbf{k}_1||\mathbf{k}_2|$,

$$\tau^{-1} = \frac{8g^2e^6}{6\pi m^8}\int d\mathbf{k}_1\int d\mathbf{k}_2\int \frac{d\mathbf{k}_3}{(2\pi)^5}\delta_4(k_1+k_2+k_3-k)$$
$$\times\frac{k_1{}^3k_2{}^3}{k_3}(1+2\cos^2\theta-5\cos^4\theta+2\cos^6\theta)$$
$$\cong \frac{g^2e^6}{\pi}\left(\frac{\kappa}{m}\right)^8 \kappa\times 10^{-5} = 5\times 10^3 g^2 \text{ sec.}^{-1}.$$

This is a very long lifetime, about $(\kappa/m)^8$ longer than the previous result of Finkelstein.[10] In Finkelstein's calculation the questionable convergence of the integrals was put into the background. It is then not surprising that the result is not gauge invariant and certainly wrong.

For the vector meson with tensor coupling the lifetime should be of the same order of magnitude. For the pseudovector meson this process is forbidden by an analog of Furry's theorem. Since the two photon decay was already shown to be forbidden the first allowed transition is the 4 photon decay, with a somewhat longer lifetime than $10^{-3}\,g^{-2}$ sec.$^{-1}$.

(C) Decay of a Neutral Meson into Positron and Electron

A neutral meson may decay into a positron and electron by disintegrating into a virtual proton-antiproton pair. The pair annihilates with the emission of a virtual photon. The photon then disappears while creating the electron-positron pair. The processes are badly divergent, but made convergent by a liberal application of conditions 1–6. The results are given in Table I. It should be noted that the lifetime of the vector meson for this process is much shorter than its γ-decay lifetime, but that the pseudovector meson decay is forbidden. The most probable decay for the neutral pseudovector meson would seem to be into one quantum and an electron-positron pair.

IV. DECAY OF A CHARGED MESON INTO A NEUTRINO, AND EITHER ELECTRON OR SPIN $\frac{1}{2}(\mu)$ MESON

Because of its coupling to the nucleon field, and the Fermi coupling of the nucleons to the electron neutrino fields, a π-meson should be unstable against β-decay. Furthermore, it is not known if the μ-meson is coupled to the π-meson directly, or through the mediation of a Fermi coupling of the μ-meson, neutrino fields to the nucleons. In the latter case the $\pi-\mu$, ν-decay lifetime should be calculable in terms of the nuclear force constant g and the rate of κ-capture of μ-mesons. Unfortunately, perturbation theory for these roundabout transitions yields infinite results. The method of subtraction fields has therefore been used to calculate the lifetimes of such decays of Bose particles into two Fermi particles via an intermediate Fermi-Dirac field. The results for several types of π-mesons and Fermi couplings are given in Table II.

V. DISCUSSION OF THE RESULTS

Since the validity of the method has already been analyzed, we confine ourselves here to a discussion of the results on the assumption that the method is correct, in order of magnitude.

The calculations on the γ-instability of neutral mesons show that both scalar and pseudoscalar mesons can decay very quickly ($10^{-10}-10^{-14}$ sec.) into two photons, but that vector mesons decay into three photons with a long lifetime ($\sim 10^{-3}$ sec.) and that pseudovector meson decay into two or three photons is forbidden, and the 4 photon decay has an even longer lifetime. This should be compared with the observations on γ-rays at Berkeley.[14] Photons of energy in the

TABLE I. Two photon decay and electron-positron decay of neutral mesons.

Type of meson and coupling	Two-photon decay lifetime	Electron-positron decay lifetime
Scalar meson Scalar coupling	$\tau^{-1}=\frac{g^2e^4}{\pi^3}\left(\frac{\kappa}{12}\right)^2$ $=8\times10^{13}\,g^2$ sec.$^{-1}$	forbidden
Scalar meson Vector coupling	forbidden	$\tau^{-1}=e^4(\kappa f)^2\left(\frac{\kappa}{m}\right)^4\frac{\kappa}{450\times(2\pi)^3}$ $=8.7\times10^{10}(\kappa f)^2$ sec.$^{-1}$
Pseudoscalar meson Pseudoscalar coupling	$\tau^{-1}=\frac{g^2e^4}{\pi^3}\left(\frac{\kappa}{8m}\right)^2\kappa$ $=1.8\times10^{14}\,g^2$ sec.$^{-1}$	forbidden
Pseudoscalar meson Pseudovector coupling	$\tau^{-1}=\frac{g(\kappa f)^2}{\pi^3}e^4\left(\frac{\kappa}{m}\right)^4\frac{\kappa}{(24)^2}$ $=5.5\times10^{11}(\kappa f)^2$ sec.$^{-1}$	forbidden
Vector meson Vector coupling	forbidden	$\tau^{-1}=e^4g^2\left(\frac{\kappa}{m}\right)^4\frac{\kappa}{(2\pi)^3\times675}$ $=5.7\times10^{10}\,g^2$ sec.$^{-1}$
Pseudovector meson Pseudovector coupling	forbidden	forbidden

[14] H. York and B. Moyer, Phys. Rev. **76**, 187 (1949).

TABLE II. Lifetime for the decay of a meson of mass $300m_e$ into a meson of spin $\frac{1}{2}$ and mass 200, and a neutrino, or into an electron and a neutrino, via the fermi coupling of the nucleon. The value of g_f is taken in accord with the tritium-helium 3 β-decay. $\tau_{\mu\nu}$=mean lifetime of the $\pi\rightarrow\mu$, ν-decay. $\tau_{e\nu}$=mean lifetime of the $\pi\rightarrow e$, ν-decay $f=\frac{1}{2}-\frac{1}{2}\left(\frac{\kappa}{m_\mu \text{ or } m_e}\right)^2$.

Type of Fermi coupling: Scalar	Vector	Tensor	Pseudoscalar	Pseudovector
Scalar meson, scalar coupling				
$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^4\left(\frac{\kappa}{2\pi}\right)^5\frac{f^2}{400}$ $\tau_{e,\nu}=0.16\ g^{-2}$ sec. $\tau_{\mu\nu}=0.54\ g^{-2}$ sec.	forbidden	$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^4\left(\frac{\kappa}{2\pi}\right)^5\frac{f^2}{25}$ $\tau_{e\nu}=1.0\times10^{-2}\ g^{-2}$ sec. $\tau_{\mu\nu}=3.3\times10^{-2}\ g^{-2}$ sec.	forbidden	forbidden
Pseudoscalar meson, pseudoscalar coupling				
forbidden	forbidden	forbidden	$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^4\left(\frac{\kappa}{2\pi}\right)^5\frac{f^2}{3600}$ $\tau_{e\nu}=1.5\ g^{-2}$ sec. $\tau_{\mu\nu}=4.9\ g^{-2}$ sec.	$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^2\frac{\kappa^5}{(2\pi)^5}\frac{(1-2f)}{36}f^2$ $\tau_{e\nu}=0.37\ g^{-2}$ sec. $\tau_{\mu\nu}=3.1\times10^{-3}\ g^{-2}$ sec.
Pseudoscalar meson, pseudovector coupling				
forbidden	forbidden	forbidden	$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^2\frac{\kappa^7}{(2\pi)^5}\frac{f^2}{900}$ $\tau_{e\nu}=.010/(\kappa g)^2$ sec. $\tau_{\mu\nu}=.033/(\kappa g)^2$ sec.	$\tau^{-1}=g^2g_f^2\left(\frac{\kappa}{m}\right)^4\frac{\kappa^7}{(2\pi)^5}f^2\frac{(1-2f)}{900}$ $\tau_{e\nu}=3.3\times10^{-4}/(\kappa g)^2$ sec. $\tau_{\mu\nu}=2.8/(\kappa g)^2$ sec.
Vector meson, vector coupling				
forbidden	$\tau^{-1}=\frac{g^2g_f^2}{225}\left(\frac{\kappa}{m}\right)^4\left(\frac{\kappa}{2\pi}\right)^5f^2\frac{(3-2f)}{3}$ $\tau_{e\nu}=0.32\ g^{-2}$ sec. $\tau_{\mu\nu}=0.73\ g^{-2}$ sec.	$\tau^{-1}=\frac{g^2g_f^2}{9}\left(\frac{\kappa}{m}\right)^2\frac{\kappa^5f^2}{(2\pi)^5}(3-4f)$ $\tau_{e\nu}=1.0\times10^{-4}$ sec. $\tau_{\mu\nu}=4.7\times10^{-4}$ sec.	forbidden	forbidden

neighborhood of 70 Mev are observed to come from the target of the cyclotron. Now if the decay of a meson of mass $300m_e$ were into three or more photons, then the energy of each photon would be smaller on the average. Furthermore, the lifetime would be so great that the meson would decay at a great distance (many meters) from the target, and consequently not be observed. If these interpretations of the experiments and the theory are correct, one is led to conclude that these neutral mesons are either of the scalar or pseudoscalar type.

The disintegration of the π-meson into electron and neutrino or μ-meson and neutrino is also interesting in the light of recent experiments. It is known that the μ-decay of the π-meson proceeds at least by a factor $\sim$100 more rapidly than its β-decay. Now the two constants which enter into the π-meson β-decay are known approximately: g from the strength of the nuclear forces and g_f from the β-decay of light nuclei. Previously the calculations were hampered by the divergences, but conventional momentum cut-off procedures,[15] give a lifetime for the process about the same as the experimental $\pi-\mu$-decay time, 10^{-8} sec. However, the more definite calculations made here and listed in Table III give much longer values, therefore disposing of this difficulty.

In the case of the coupling of the π-meson to the μ-meson one has 3 experimental data: (1) the rate of $\pi-\mu$-decay (2) the rate of μ-capture from the κ-orbit of a nucleus (3) the strength of nuclear forces. Previously it has been considered possible to explain the experiments in two ways.

1. The π-meson is coupled to the nucleons, and the nucleons to the μ-meson-neutrino field. The π-meson then decays via intermediate nucleon pairs, and μ-capture is direct.

2. The π-meson is coupled to both nucleons and to the μ-meson-neutrino field. $\pi-\mu$, ν-decay is then a direct process, but μ-capture is via an intermediate π-meson field. The $\pi\rightarrow\mu$, ν-decay in case 1 suffers from the infinity difficulties, but its order of magnitude had been estimated by cutting off at large momenta. Because the value of the π-nucleon coupling constant g is so near to unity, both pictures had been about equally successful in agreeing with experiment. However, if one accepts the subtraction methods here described, the $\pi\rightarrow\mu$, ν-decay lifetime in picture 1 becomes much too long compared to μ-capture, and only picture 2 which contains no infinities and is unaffected by these results, is consistent.

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APPENDIX

1. Units, notation, equations of motion. Natural units are used.

$\hbar=c=1$; $e=1/(137)^{\frac{1}{2}}$.
M=matrix element.

[15] R. Christy, Seminar at the Institute for Advanced Study, April, 1949.

m = nucleon mass.
κ = mass of the decaying meson.
μ = mass of the product meson, if any.
$\psi(x)$ = nucleon wave function.
$\bar{\psi}(x) = \psi^*(x)\gamma_4$.
γ_μ, $\mu = 1, 2, 3, 4 = 4$ Dirac matrices, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$.
$\phi(x)$, $\phi^*(x)$ = meson wave functions.
$A_\mu(x)$ = electromagnetic 4 potential.
$k = 4$ momentum.
$\mathbf{k} = 3$ momentum.
$A_\mu(k) = 1/V \int_V A_\mu(x) e^{-ik_\nu x_\nu} d_3x$,
= fourier component of vector potential.
$\partial_\mu = \partial/\partial x_\mu - ieA_\mu$.

The Lagrangians are the following

(a) Proton, field, $L = \bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x)$.
(b) Free neutron field, $L = \bar{\psi}(x)(\gamma_\mu(\partial/\partial x_\mu) + m)\psi(x)$.
(c) Free electromagnetic field,

$$L = -\frac{1}{8\pi}\left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}\right)\left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}\right).$$

(d) Free neutral scalar or pseudoscalar meson field,

$$L = -\frac{1}{2}\left[\frac{\partial\phi}{\partial x_\mu}\frac{\partial\phi}{\partial x_\mu} + \kappa^2\phi^2(x)\right].$$

(e) Free neutral vector meson field,

$$L = -\frac{1}{2}\left[\frac{1}{2}\left(\frac{\partial\phi_\mu}{\partial x_\nu} - \frac{\partial\phi_\nu}{\partial x_\mu}\right)\left(\frac{\partial\phi_\mu}{\partial x_\nu} - \frac{\partial\phi_\nu}{\partial x_\mu}\right) + \kappa^2\phi_\mu\phi_\mu\right].$$

(f) Free charged scalar or pseudoscalar field,

$$L = -\left[\frac{\partial\phi^*}{\partial x_\mu}\frac{\partial\phi}{\partial x_\mu} + \kappa^2\phi^*\phi\right].$$

(g) Free charged vector field,

$$L = -\left[\frac{1}{2}\left(\frac{\partial\phi_\mu^*}{\partial x_\nu} - \frac{\partial\phi_\nu^*}{\partial x_\mu}\right)\left(\frac{\partial\phi_\mu}{\partial x_\nu} - \frac{\partial\phi_\nu}{\partial x_\mu}\right) + \kappa^2\phi_\mu^*\phi_\mu\right].$$

(h) Interaction between neutral scalar mesons and protons,

$$L = -g\phi(x)\bar{\psi}(x)\psi(x).$$

(i) Interaction between neutral pseudoscalar meson and proton,

$$L = -ig\phi(x)\bar{\psi}(x)\gamma_5\psi(x) - if(\partial\phi/\partial x_\mu)\bar{\psi}(x)\gamma_5\gamma_\mu\psi(x).$$
$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4.$$

(j) Interaction between neutral vector field and proton,

$$L = -g\phi_\mu(x)\bar{\psi}(x)\gamma_5\psi(x).$$

(k) Interaction between charged scalar meson and nucleon,

$$L = -g\phi(x)\bar{\psi}(x)\tau\psi(x) - g\phi^*(x)\bar{\psi}(x)\tau^*\psi(x).$$

(l) Interaction between charged pseudoscalar meson and nucleon,

$$L = -ig\phi(x)\bar{\psi}(x)\gamma_5\tau\psi(x) - ig\phi^*(x)\bar{\psi}(x)\gamma_5\tau^*\psi(x) - if(\partial\phi/\partial x_\mu)\bar{\psi}(x)\gamma_5\gamma_\mu\tau\psi(x) - if(\partial\phi^*/\partial x_\mu)(x)\bar{\psi}(x)\gamma_5\gamma_\mu\tau^*\psi(x).$$

(m) Interaction between charged vector meson and nucleon,

$$L = g\phi_\mu(x)\bar{\psi}\gamma_\mu\tau\psi - g\phi_\mu^*(x)\bar{\psi}(x)\gamma_\mu\tau^*\psi.$$

(n) Scalar Fermi interaction of nucleons,

$$L = g_f\bar{\psi}\tau\psi\bar{\phi}\tau^*\phi + g_f\bar{\psi}\tau^*\psi\bar{\phi}\tau\phi$$