Interaction between Elementary Particles. Part I

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An elementary particle is considered as a mathematical point which under an invariant law of communication involving a universal length a establishes a primary potential in space-time. Due to the peculiar communication law, $r^2-c^2t^2+a^2=0$ or $S^2=0$, in contrast to the light signal law $R^2=0$, the primary potential is without singularity at $r=0$. It becomes the background of a regular electrodynamic or mesonic field and automatically yields a finite radius a to the charge distribution, with a density falling off as $(a/r)^6$ at larger distances. The potentials of different particles are additive so that all quantities obey the superposition law. The total force on the Nth particle is produced under the same invariant communication law $S=0$ by integration of the force density k_N over the volume elements of the instantaneous rest system. K_N is the resultant of mutual and self-forces, $K_N=S_MK_{MN}$. Only retarded field interaction is admitted between different particles. The original production of the self-charge by the center and its force reaction on the center is derived from a heuristic scheme involving half-retarded, half-advanced potentials, as a substitute for the instantaneous reaction between the "parts" of a charge. In the dynamics of the particle a variable "acceleration mass" of field origin is added to the mechanical mass. An experimental determination of mass differences under various very high accelerations could reveal the unknown ratio between the field mass and the mechanical mass of a charged particle.

1. POINT PARTICLE *VERSUS* FINITE RADIUS

THERE are two ways of approaching the problem of elementary particles and their fields. When it became obvious that the primitive model of a charged ball was incompatible with relativistic dynamics, one turned to point particles as primary sources in spite of the field singularity at $r=0$. The last twenty years have seen heroic efforts of removing the divergencies inherent in the fields of point charges, first by arbitrarily cutting off undesirable terms, and lately by systematic elimination methods. Nevertheless, to many it seemed appropriate to construct a theory which would never introduce a cause for divergencies at least in the classical domain rather than to rely on laborious elimination methods at a later stage.

Dirac[1] tried to remove the point singularity from the very outset by assuming an effective potential equal to half the difference between retarded and advanced potential as responsible for the self-force of the electron. Dirac's theory led to an ever increasing self-acceleration of an isolated electron, and to a premonitory acceleration under an incoming light impulse due to signal communication *faster* than the velocity of light. Wheeler and Feynman in their absorber theory of radiation[2] assume half the sum of the retarded and advanced potential to be effective between *different* particles, maintaining that an accelerated isolated electron would not suffer a self-force at all.

The second approach, through unitary field theories, strives to replace the arbitrary construction of finite balls of charge by modified field equations which automatically produce stable charge concentrations of finite radius. The most famous of these attempts, the Born-Infeld theory[3] (1934) replaces the linear Maxwell scheme by non-linear field equations involving a basic length a. The same a then becomes the width of the resulting field maximum. However, non-linear theories in which resonance depends not only on frequency but also on amplitude, are unwieldy to quantization. Furthermore, the lack of superposition makes it impossible to define the individual fields of various particles at close distance from one another.

The following attempt of establishing an invariant theory of interaction between particles including self-reaction is related in various respects to the aforementioned theories, but tries to avoid some of the difficulties of the latter. Similar to Born-Infeld, we obtain a particle of finite electric radius a in an invariant fashion without introducing a special structural hypothesis; yet the linearity and superposition of the ordinary Maxwell field equations is not abandoned. Second, similar to Dirac, we admit advanced as well as retarded potentials for the communication "inside" of the particle, and retarded potentials for the interaction between different particles; but instead of half the difference we use half the sum of the two self-potentials, and yet we obtain a finite self-energy as well as a finite mutual energy between particles even at distances $r \ll a$ and $r=0$. It was Dirac's half-difference which seemed objectionable to Wheeler and Feynman until they were able to show that Dirac's results are derivable from their assumption of half the sum of the two potentials. In contrast to W. and F., however, we use only *retarded* potentials for the interaction between different particles and admit a self-force also for an isolated particle.

2. THE SIGNAL EQUATION OF A POINT PARTICLE

Let us start with an analogy. When a particle is tested as to the simultaneous values of its energy and

[1] P. A. M. Dirac, Proc. Roy. Soc. **167**, 148 (1938).
[2] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945).
[3] M. Born and L. Infeld, Proc. Roy. Soc. **142**, 410 (1934); **144**, 425 (1934); **147**, 522 (1934); **150**, 141 (1935).

momentum, e.g., by its Compton reaction to photons, then various E and p values are found to satisfy the relation

$$p^2-(E/c)^2+b^2=0, \quad (b=m_0c), \tag{1}$$

and the particle is said to have rest mass $m_0=b/c$. We know that particles exist which satisfy this relation, although with different values of the constant b. Figure 1 shows the two hyperbolic surfaces in E/c, p space with the gap $2b$ between positive and negative E/c-values. The 3-dimensional p-space is continuous.

We now propose an analogous definition of a point particle in space-time:

When effects or "potentials" are received in a variety of world points r, t related by the equation

$$r^2-(ct)^2+a^2=0, \quad \text{or} \quad S^2=0 \quad (a=\text{signal radius}), \tag{2}$$

when these effects have their common source in a point particle located in the world point 0 ($r=t=0$). Figure 2 shows the two hyperbolic signal surfaces of the past and future characterizing a particle of signal radius a. Of course we do not know whether elementary particles actually possess signal surfaces $S=0$ rather than light cones $R=0$; yet Eq. (2) would be the simplest invariant way of introducing a finite radius a and eliminating divergencies without resorting to a structural hypothesis or to non-linear field equations. Equation (2) was first proposed by the author[4] in 1939 under the impression of Born's *Principle of Reciprocity*.

As seen from Fig. 2 there is an initial time lag, $t=a/c$, for signals or "potentials" emerging from the point particle even to reach the immediately surrounding points $r=0$. The communication time to distances r in general is $t=1/c[r^2+a^2]^{\frac{1}{2}}>r/c$ with asymptotic value $t=r/c$ for $r\gg a$. The "phase velocity" with which points along a radius are passed is $dr/dt=c[1+(a/r)^2]^{\frac{1}{2}}$, faster than c. Yet since this sudden spurt of communication over a range of order a (which is responsible for the lack of infinities at $r=0$) starts only after the initial time lag, the total time for a signal to reach a point r is always longer than r/c so that *the signal velocity is less than c.*

3. THE MAXWELL FIELD

Before applying the signal Eq. (2) to the theory of electric particles we comment on the Maxwell-Lorentz theory in general. An electromagnetic field F containing a continuous charge-current density J can be derived from a 4-potential ϕ, satisfying the Lorentz condition $\text{Div}\phi=0$, in the following fashion. The 6-vector F is obtained as

$$F=\text{curl}\phi, \tag{3}$$

from which the first Maxwell couple follows as an *identity*. The 4-vector J is *defined* as

$$4\pi J=\Delta iv F, \tag{4}$$

[4] A. Landé, Phys. Rev. **56**, 482 (1939). J. Frank. Inst. **231**, 63 (1941).

which is the same as the second Maxwell couple; the latter thus is but a *definition* of J in terms of F, in accordance with the experimental procedure. From (4) follows the continuity equation of J as an identity. Equation (4) may also be written as

$$4\pi J=-\Box^2\phi. \tag{5}$$

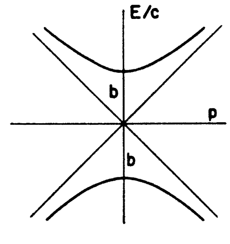

FIG. 1.

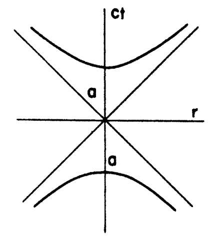

FIG. 2.

At last, the 4-vector Lorentz force density and work rate density of the field is defined as

$$k=[J,F], \tag{6}$$

from which follows the identity $k=-\Delta iv T$ where T is the symmetric stress-energy-momentum tensor of the field. These equations, or rather *definitions* in terms of the original 4-vector ϕ, hold for any divergenceless function $\phi(x_1\cdots x_4)$. One also may reverse the potential Eq. (5) to

$$\phi=\frac{1}{\pi}\int(Jd\Sigma/R^2), \tag{7}$$

where $d\Sigma=dx_1\cdots dx_4$ and R is the world distance between $d\Sigma$ and the field point. Whenever a J-distribution is given which satisfies the Maxwell theory, i.e., which is derived from any function ϕ with $\text{Div}\phi=0$, then this J may conversely be considered as a source "producing" a potential ϕ according to (7) with light velocity retardation ($R=0$).

We now introduce a special choice for the field potential ϕ surrounding an elementary *point particle* moving with any variable velocity $U_k=dx_k/d\tau$ ($\tau=$proper time) on any world line. If the point particle were a point charge $+\epsilon$ one would choose the Liénard-Wiechert potential

$$\phi=\mp\epsilon\left[\frac{U}{(R\cdot U)}\right]_{R=0}, \tag{8}$$

whose Div vanishes, but which is singular at the point charge. We replace (8) by

$$\phi=\mp\epsilon\left[\frac{U}{(R\cdot U)}\right]_{S=0}, \tag{9}$$

whose Div also vanishes. The subscript $S=0$ indicates that the primary potential ϕ in various world points is established by the point particle with a time retardation (or advancement in case of the lower sign) according to Eq. (2).

Once this potential is established it produces, or defines, a perfectly normal Maxwellian field sustaining a certain charge distribution J. In this Maxwell field (7) is still the inversion of (5).

4. STATIC SOLUTION

When the point particle is at rest in the 0-point of space at all times t' one has $U_1=U_2=U_3=0$, and

$$\left.\begin{array}{l} U_4=ic,\quad S^2=0=r^2-c^2(t-t')^2+a^2, \\ \text{hence} \\ R_4=ic(t-t')=\pm i(r^2+a^2)^{\frac{1}{2}}, \\ (R\cdot U)=R_4U_4=\pm c(r^2+a^2)^{\frac{1}{2}}. \end{array}\right\} \tag{10}$$

The scalar electric potential $\varphi=\phi_4/i$ and the density ρ become

$$\varphi=\frac{\epsilon}{(r^2+a^2)^{\frac{1}{2}}},\quad \rho=\frac{\epsilon}{4\pi}\frac{3a^2}{(r^2+a^2)^{5/2}} \tag{11}$$

regular at $r=0$. At a large distance from the point particle (which is not a point charge) the condition $S=0$ is identical with $R=0$ so that our results coincide with those of point charges in the limit of $r\gg a$. The result (11) is the first test passed by the theory. It holds for retarded as well as advanced effects, and also for half their sum. (Half the difference would yield $\varphi=0$ and $\rho=0$ everywhere; we therefore reject this possibility.)

A nucleon of "charge" g may become the source of a *meson field* derivable from the primary potential

$$\phi=g\mp[U/(R\cdot U)\exp\{(R\cdot U)\kappa/c\}]_{S=0}. \tag{12}$$

In the static example (10) it yields

$$\varphi=g/(r^2+a^2)^{\frac{1}{2}}\exp\{-(r^2+a^2)^{\frac{1}{2}}\kappa/c\}. \tag{12'}$$

5. THE FORCE DENSITY ON THE SELF-CHARGE

If we assume that the charge-current density J_N which the Nth particle surrounds itself is obtained from half the sum of retarded and advanced potentials (9), as a substitute for the instantaneity within a point charge, we obtain in various space-time points:

$$\phi_N{}^{\text{self}}=\tfrac{1}{2}(\phi_{N,\,\text{ret}}{}^{\text{self}}+\phi_{N,\,\text{adv}}{}^{\text{self}})=\text{self-potential} \tag{13}$$

$$F_N{}^{\text{self}}=\tfrac{1}{2}(F_{N,\,\text{ret}}{}^{\text{self}}+F_{N,\,\text{adv}}{}^{\text{self}})=\text{self-field} \tag{13'}$$

$$J_N=\tfrac{1}{2}(J_{N,\,\text{ret}}+J_{N,\,\text{adv}})=\text{current density}. \tag{13''}$$

Only the resulting J_N is used further on, namely, as a source of an ordinary *retarded* potential ϕ_N and field $F_N=\text{Curl}\phi_N$. The same J_N is the source of ϕ_N but half a source, half a sink of $\phi_N{}^{\text{self}}$. ϕ_N, and F_N are "physical," i.e., derived from J_N, $\phi_N{}^{\text{self}}$ and $F_N{}^{\text{self}}$ "virtual," i.e., constructed as a heuristic scheme for deriving J_N.

Force and work rate density are defined in the Maxwell-Lorentz theory as $k=[J,F]$. Correspondingly when F is the total physical field$=S_MF_{MN}+F^{\text{ext}}$ we define the force density k_N which ultimately contributes to the total force K_N on the Nth particle

$$k_N=\tfrac{1}{2}k_{N,\,\text{ret}}+\tfrac{1}{2}k_{N,\,\text{adv}} \tag{14}$$

according to the division of J_N in two parts in (13''). Here we have

$$k_{N,\,\text{ret}}=[J_{N,\,\text{ret}},F] \tag{15}$$

and similarly $k_{N,\,\text{adv}}$. Another possibility would be to assume that the total force is the charge multiplied by the field at the point particle itself.

So far, this theory may be summarized as follows. In order to surround a point particle with a finite charge-current density J_N dependent on the world line of the center, we introduce a heuristic Maxwellian scheme involving retarded and advanced potentials and leading to $\phi_N{}^{\text{self}}$ and $F_N{}^{\text{self}}$ and then to J of (13''). The latter, irrespective of its mode of calculation,* becomes the source of an ordinary *retarded* potential ϕ_N and a corresponding field F_N, differing from the "virtual" $F_N{}^{\text{self}}$. The physical F_N alone produces, when acting on the J_M of another particle, a contribution K_{NM} to the force on the latter.

5. THE SELF-FORCE ON A PARTICLE

When asking for the self-force and work rate K_N on the Nth particle, it is logical that the k_N-contributions in the surrounding volume elements should react on the center according to the same communication law, $S=0$, which the center originally employed in establishing J_N. We tentatively define the four components of $K_N{}^0$ in the instantaneous rest system of the center by an integral of $k_N{}^0$ over the volume elements dV^0 of the same rest system:

$$K_N{}^0=\int[k_N{}^0]_{S=0}dV^0. \tag{16}$$

More specifically we assume that $k^0{}_{N,\,\text{ret}}$ is reported to the center in an advanced fashion, and $k^0{}_{N,\,\text{adv}}$ in a retarded way:

$$K_N{}^0=\frac{1}{2}\int\{[k^0{}_{N,\,\text{ret}}]_{\text{adv}}+[k^0{}_{N,\,\text{adv}}]_{\text{ret}}\}dV^0, \tag{17}$$

with the integrand given in (15) or (15') or (15''). This definition provides a maximum of simultaneity in spite of the finite communication velocity to and from the center. K_N is clearly divided into self-, mutual, and pure field contributions:

$$K_N=K_{1N}+K_{2N}+\cdots K_{NN}+\cdots+K_N{}^{\text{ext}}. \tag{18}$$

K_{MN} depends only on the world lines of the particles M and N.

The (apparent) trouble with the above definition (17) is that the fourth component of $K_N{}^0$ which repre-

* $\phi_N{}^{\text{self}}$ is defined by means of the rule $S=0$. Yet the relation between $\phi_N{}^{\text{self}}$ and J, as well as between J and ϕ is that of a communication law $R=0$.

sents i times the work rate of the field on the particle, may have a finite value even in the instantaneous rest system of the center, which is not the rest system of the surrounding J_N distribution. Of course, $K_4{}^0$ vanishes in static examples and in case of a constant acceleration. However, during times of a varying acceleration (in general, when there are non-vanishing odd order time derivatives of the acceleration), the field works on the particle even in the instantaneous rest system of the center. The particle then accumulates the well-known "acceleration energy" (G. E. Schott[5]) which is carried along by the particle and contributes to its inertia. During times of decreasing acceleration

The equations of motion of a particle with τ = proper time are

7. DYNAMICS

The equations of motion of a particle with τ = proper time are

$$dP/d\tau = K, \tag{19}$$

where K now is the 4-vector of force work rate resulting from both mechanical and field forces, and P is the momentum-energy vector of the particle satisfying the invariant relation $P^2+(m_0c)^2=0$. Since the work rate may possess a finite value even in the instantaneous rest system of the particle, the scalar m_0 is not a constant but rather a function of the proper time τ along the world line, representing mechanical plus variable "acceleration mass." When splitting P in two factors, namely, a scalar m_0 and a 4-vector acceleration $dU/d\tau$, (19) becomes

$$m_0(dU/d\tau)+U(dm_0/d\tau)=K. \tag{20}$$

In the instantaneous rest system where $(U_4)^0=ic$ and $(dU_4/d\tau)^0=0$ the four components of (20) read

$$[m_0(dU_1/dt)]^0=(K_1)^0, \quad \cdots, \quad [ic(dm_0/dt)]^0=(K_4)^0. \tag{21}$$

The four right-hand sides are sums of external mechanical plus self- and mutual field forces (the field forces depend only on the world lines. On the left of (21) one has the "unknowns" m_0, $U_1U_2U_3$ and their derivatives with respect to the time. Finding solutions in a system of several particles, each subjected to four equations of the form (21) is a matter of great complexity. Our aim was only that of deriving the field forces consisting of self- and mutual parts in a system of charged particles led along *prescribed* world lines.

8. CONCLUDING REMARKS

We have attempted to consider particles as mass points which react on each other at a distance through forces calculated by means of an intermediate kinematic scheme representing a superposition of self and mutual fields. Essentially this is a *unitary theory* of interaction between particles in which the fields are subordinate entities with no degrees of freedom of their own (unless one admits also pure fields in superposition).

Radiation theory rests on the dualistic view of particles and field representing separate entities, with a mutual perturbation energy being responsible for acts of emission and absorption. This view can be applied also to our theory as an approximation, when regarding the combined field of other particles as an "external pure field." In this approximation the potential energy-momentum of the particle, instead of being the product $(\epsilon\phi\cdot U)$ at the place of the center, will be an integral over the distribution of $(J\cdot\phi)$ in space. In case of an external field of wave-length $\lambda \ll a$ the resulting perturbation energy will be much smaller than in case of a point charge, and the integration over the whole radiation spectrum will behave as though it were cut off at wave-lengths $\lambda \sim a$.

[5] G. E. Schott (Cambridge University Press, London, 1912). Refer also to the remarks of Dirac, reference 1, p. 155.