Radiative Corrections to Nuclear Forces in the Pseudoscalar Meson Theory*

K. M. WATSON AND J. V. LEPORE
The Institute for Advanced Study, Princeton, New Jersey

(Received June 13, 1949)

The fourth-order corrections to nuclear forces in the charged and symmetrical pseudoscalar meson theories are obtained by the Feynman-Dyson method. All infinite parts are reconciled in terms of renormalization of mesonic and nucleonic mass and charge. The potentials obtained by a non-relativistic approximation yield ordinary spin independent forces which behave as $1/r^3$ in the non-relativistic region. This singularity is reduced to $1/r$ by relativistic effects. The potentials are in disagreement with experiment but the neutron-proton scattering problem is treated in order to obtain insight into the nature of relativistic corrections at 90 Mev, and it is shown that they are not small.

INTRODUCTION

THE success of the renormalization program[1] of quantum electrodynamics has given impetus to a re-examination of the difficulties of meson theories of nuclear forces. In particular, there are three questions to be asked in this connection: (1) Can the renormalization program of electrodynamics be successfully applied to any of the current meson theories? (2) In view of the largeness of the coupling constants describing the interaction of meson and nucleon fields, can one obtain valid solutions to problems concerned with these interactions? (3) Will conclusions drawn from these theories agree with experiment?

Case's[2] treatment of the anomalous magnetic moments of the neutron and proton indicates that there are examples for which an affirmative answer can be given to the first question; however, his results are not in agreement with experiment—in the pseudoscalar theory, for instance, the meson field seems to be too closely bound to the nucleon. It also appears that the renormalization of mass and mesonic charge will not always make all measurable quantities finite. Case[3] has found that one cannot obtain finite nucleon magnetic moments with a vector meson theory and tensor coupling to the nucleon field. Indeed, Dyson[4] has remarked that renormalization of mass and mesonic charge does not seem to be sufficient to remove all divergences in any theory for which a gradient coupling is used. Furthermore, Dyson[4] has shown that the scattering cross section of mesons by mesons is probably infinite for all present meson theories. It seems quite doubtful, then, that present treatments of field interactions will be successful when applied to meson fields.

The second question above poses formidable analytical difficulties, since one is accustomed to obtain solutions which are in the form of power series in the not-small coupling constants. In the hope of minimizing the uncertainty arising from the terms of higher order in the coupling constant it has appeared worth while to investigate the scattering of neutrons by protons to the fourth order in the coupling constant—previous treatments have considered only the second-order interaction between nucleons. That the deuteron problem can be expected to be quite sensitive to higher order radiative corrections follows from the fact that the least unsatisfactory nuclear potentials calculated in meson theory are too highly singular in regions of space outside the nucleon Compton wave-length to predict a reasonable model for the deuteron. Any possible tendency of higher order corrections to smooth out such singularities will thus be of critical importance. However, in the scattering problem the effective impact parameters tend to be greater than the de Broglie wave-length of the colliding particles, and so the scattering cross section can be expected to be somewhat less sensitive to corrections arising at very small distances of approach.

The pseudoscalar meson theory with pseudoscalar coupling was chosen because of its relative simplicity and because the fourth-order terms should be large compared to the second-order terms, since the Dirac matrix $\gamma_5(=0(v/c))$ occurs in the second order but not in the important fourth-order terms.

I. THE SCATTERING MATRIX

We assume the nucleon field to interact with a pseudoscalar meson field. The term in the Hamiltonian density representing the interaction of the fields is (we use units in which $\hbar=c=1$)

$$H(x)=if_\mu\bar{\psi}(x)\tau_\mu\gamma_5\psi(x)\phi_\mu(x). \quad (1)$$

τ_i $(i=1, 2, 3)$ are the isotopic spin matrices. $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$ where the γ_μ's are the Dirac matrices. ψ, $\bar{\psi}$ are the spinor operators of the nucleon field given by Schwinger,[5] and the ϕ_μ are the mesonic field variables. ϕ_1 and ϕ_2 are the field variables for the charged field; and ϕ_3 is that for the neutral field. The coupling constants f_μ are restricted by charge conservation to values for which $f_1=f_2$, but are otherwise arbitrary real quantities. The Schwinger-Tomonaga equation for the state function Ψ of the system is, in the interaction representation,

$$i[\delta\Psi/\delta\sigma(x)]=H(x)\Psi. \quad (2)$$

* This work was reported at the Washington Meeting of the American Physical Society (Phys. Rev. 76, 193 (1949)).
[1] See, for instance, F. J. Dyson, Phys. Rev. 75, 486 (1949).
[2] K. M. Case, Phys. Rev. 76, 1 (1949).
[3] K. M. Case, Phys. Rev. 75, 1440 (1949).
[4] F. J. Dyson, unpublished.

In this representation the commutation relations for the field variables are those for free fields, and have been given by Case,[2] Schwinger,[5] and others.

Since we are interested in a scattering problem, we shall use the scattering matrix of Dyson,[1] which is the operator transforming the state vector Ψ of the system from its initial value in the infinite past to its final value in the infinite future, according to Eq. (2). The

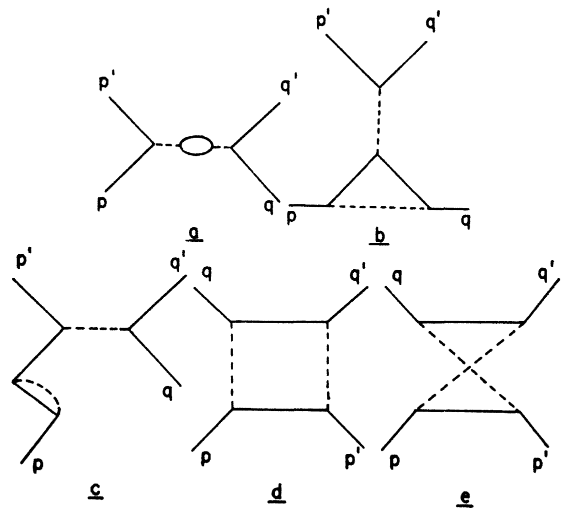

FIG. 1. Graph (a) gives the analog of the vacuum polarization of electrodynamics. A meson self-energy term is included in the contribution from this graph. (b) is the analog of the graph that accounts for most of the Lamb shift in electrodynamics. (c) contains the effects of the nucleon self-energy. (d) and (e) correspond to simple scattering processes in which two mesons are exchanged by the nucleons. (e) can be obtained from (d) by reversing the direction of the arrow on one of the nucleon lines, or analytically by taking the charge conjugate of one of the nucleon current operators in (d).

formal expression for the scattering matrix is

$$S=\sum_{n=0}^{\infty}(-i)^n\frac{1}{n!}\int_{-\infty}^{+\infty}dx_1\cdots\times\int_{-\infty}^{+\infty}dx_n P[H(x_1)\cdots H(x_n)],\quad (3)$$

where the P-bracket orders the product of the H's in such a way that those whose arguments lie on earlier time-like surfaces precede those on later surfaces. We restrict ourselves to the terms S_2 and S_4, which contain the coupling constant to the second and the fourth power, respectively.

We assume that the incoming nucleons have four-momenta p_μ, q_μ, while the outgoing nucleons have four-momenta p_μ', q_μ'. Then the second-order scattering matrix can be written immediately:

$$S_2=-i\tfrac{1}{2}(2\pi)^4(\bar{\psi}_{p'}\tau_\lambda\gamma_5\psi_p)(\bar{\psi}_{q'}\tau_\lambda\gamma_5\psi_q)(f_\lambda^2/\Delta p_\mu^2+\mu^2),\quad (4)$$

where μ is the meson rest-mass and $\Delta p_\mu=p_\mu'-p_\mu=q_\mu-q_\mu'$, since the four-momenta are restricted by energy-momentum conservation. ψ_p is the Fourier component of $\psi(x)$ referring to an electron with momentum p. It is implicitly defined by

$$\psi(x)=\int d^3p\psi_p\exp(ip_\mu x_\mu).$$

$\bar{\psi}_{p'}$ is defined similarly.

To find S_4 we must consider the Feynman-Dyson graphs shown in Fig. 1. (Heavy lines are nucleon lines, broken ones are meson lines.)

II. THE EVALUATION OF S_4

A: Graph (a)

One obtains[1]

$$S_a=-(\tfrac{1}{2})^5\int dx_1\cdots\int dx_4 f_\lambda f_\sigma f_\nu f_\rho\delta_{\lambda\sigma}\delta_{\nu\rho}\Delta_F(x_1-x_2)$$
$$\times\Delta_F(x_3-x_4)(\bar{\psi}(x_1)\tau_\lambda\gamma_5\psi(x_1))(\bar{\psi}(x_4)\tau_\rho\gamma_5\psi(x_4))$$
$$\times Sp\{S_F(x_3-x_2)\tau_\nu\gamma_5 S_F(x_2-x_3)\tau_\sigma\gamma_5\},\quad (5)$$

where $Sp\{\cdots\}$ means that one must take the spur of the isotopic spin and Dirac matrices occurring in the parenthesis. S_F and Δ_F are the functions given by Dyson,[1] and are

$$S_F(x)=2\left[\frac{i}{(2\pi)^4}\right]\int d^4k\exp(ik_\sigma x_\sigma)\frac{[i\gamma_\lambda k_\lambda+\kappa_0]}{k_\lambda^2+\kappa_0^2-i\epsilon},$$
$$\Delta_F(x)=-2\left[\frac{i}{(2\pi)^4}\right]\int d^4k\exp(ik_\sigma x_\sigma)\frac{1}{k_\lambda^2+\mu^2-i\epsilon},\quad (6)$$

where, as before, μ is the mesonic rest-mass, κ_0 is the nucleonic rest-mass, and ϵ is a positive, real parameter which tends to zero after the integrations are performed. The function of ϵ is to determine the manner by which one passes poles in the integrand. It can be shown that this choice of the sign of ϵ is equivalent to choosing outgoing waves for the scattered system and is a consequence of having fixed the initial state of the system in the solution (3) of Eq. (2).

Using Eq. (6) and performing the spatial integrations in Eq. (5), we have:

$$S_a=-4f_\lambda^2(\bar{\psi}_{p'}\tau_\lambda\gamma_5\psi_p)(\bar{\psi}_{q'}\tau_\rho\gamma_5\psi_q)\frac{\delta_{\lambda\rho}}{[\Delta p_\mu^2+\mu^2]^2}f_\rho^2 I,\quad (7)$$

where

$$I=\int d^4l\frac{l_\alpha^2+\kappa_0^2-l_\alpha\Delta p_\alpha}{(l_\mu^2+\kappa_0^2)[(l_\mu-\Delta p_\mu)^2+\kappa_0^2]}\quad (8)$$

where the $-i\epsilon$ term of Eq. (6) is considered as being included in μ^2 and κ_0^2 when needed. I diverges quadratically, but Eq. (7) includes a correction to S_2 in Eq. (4) arising from the meson self-energy. Interpreting

[5] J. S. Schwinger, Phys. Rev. 74, 1439 (1948).

this mass correction as being included in the "experimental" value of the mass, one can remove the quadratic divergence from Eq. (8) as follows. We consider the change in S_2 resulting from varying μ:[6]

$$\delta S_2=(dS_2/d\mu_\rho)\delta\mu_\rho. \quad (9)$$

We insert for $\delta\mu_\rho$ in Eq. (9) the meson self-energy[7] to order f^2,

$$\delta\mu_\rho=(i/\mu)f_\rho^2[4/(2\pi)^4]I_0, \quad (10)$$

(the index ρ not summed) where I_0 is obtained from I (Eq. (8)) by replacing Δp_α by k_α, the energy-momentum vector for a free meson, obeying the relation $k_\mu^2=-\mu^2$. Considering the $\delta\mu_\rho$ as being included in μ, we subtract δS_2 from S_a. On doing this I is replaced by I', where

$$I'=I-I_0=I-I(\Delta p_\mu^2=-\mu^2). \quad (11)$$

I' diverges logarithmically, but it still contains a term which is to be interpreted as a coupling constant renormalization. Replacing f_λ in Eq. (4) by $f_\lambda+\delta f_\lambda$, and keeping only the linear term in δf_λ, we again obtain an expression $\delta' S_2$ of the form of S_a if

$$\delta f_\lambda=-i\frac{4}{(2\pi)^4}f_\lambda^3\left[\lim_{\Delta p^2\to-\mu^2}\left(\frac{I'}{\Delta p_\mu^2+\mu^2}\right)\right].$$

Again subtracting $\delta' S_2$ from $S_a-\delta S_2$ we have an expression with I' replaced by I'', where

$$I''=I'-(\Delta p_\mu^2+\mu^2)\lim_{\Delta p_\mu^2\to-\mu^2}\left(\frac{I'}{\Delta p_\mu^2+\mu^2}\right). \quad (12)$$

Using the Feynman representation for product denominators,

$$\frac{1}{ab}=\int_0^1\frac{dx}{[ax+b(1-x)]^2}, \quad (13)$$

$$I''=\tfrac{1}{2}i\pi^2[\Delta p_\mu^2+\mu^2]\int_0^1\ln\left[1+\frac{\Delta p_\mu^2}{\kappa_0^2}x(1-x)\right]dx + \text{terms of } 0\left(\left(\frac{\mu}{\kappa_0}\right)^2\right). \quad (14)$$

The final expression for S_a is obtained by replacing I by I'' in Eq. (7).[8]

B: Graphs (b) and (c)

Writing the term in the S-matrix arising from graph (b) and performing the integration over coordinates, we obtain

$$S_b=-\frac{f_\lambda^2 f_\sigma^2}{\Delta p_\mu^2+\mu^2}[\bar\psi_{p'}\tau_\lambda\tau_\sigma\tau_\lambda\gamma_5\psi_p][\bar\psi_{q'}\tau_\sigma\gamma_5\psi_q]U, \quad (15)$$

where

$$U=\int d^4k\frac{k_\alpha^2}{[(k_\mu+p_\mu')^2+\kappa_0^2][(k_\mu+p_\mu)^2+\kappa_0^2][k_\mu^2+\mu^2]} \quad (16)$$

is a logarithmically divergent integral.

From graph (c), we have

$$S_c=\frac{f_\rho^2 f_\sigma^2}{\Delta p_\nu^2+\mu^2}[\bar\psi_{q'}\tau_\sigma\gamma_5\psi_q]\{\bar\psi_{p'}\tau_\sigma\gamma_5\frac{[-i\gamma_\lambda p_\lambda+\kappa_0]}{p_\mu^2+\kappa_0^2}\times[I_1(i\gamma_\alpha p_\alpha+\kappa_0)+I_2\kappa_0]\psi_p\}, \quad (17)$$

where

$$I_1=\int_0 dx(1-x)\int\frac{d^4k}{[k_\mu^2+A]^2},\qquad I_2=\int_0^1 dxx\int\frac{d^4k}{[k_\mu^2+A]^2}, \quad (18)$$

and $A=\kappa_0^2x^2+\mu^2(1-x)$. The Feynman relation (13) has been used in obtaining Eq. (18). It appears most reasonable to evaluate the indeterminate form in Eq. (17) as

$$\frac{[-i\gamma_\lambda p_\lambda+\kappa_0][i\gamma_\sigma p_\sigma+\kappa_0]}{p_\mu^2+\kappa_0^2}\to 1. \quad (19)$$

The term in Eq. (17) that contains I_2 is due to the nucleon self-energy. This may be seen if we replace "the self-energy part"[1] of graph (c) by a term arising from an additional term in the interaction Eq. (2),

$$\delta H=\delta\kappa_0\bar\psi\psi,$$

where[9]

$$\delta\kappa_0=if_\sigma^2[I_2\kappa_0/(2\pi)^4]. \quad (20)$$

Substracting the contribution to Eq. (17) arising from $\delta\kappa_0$ removes the I_2-term from Eq. (17). The remainder, containing I_1, diverges logarithmically.

In electrodynamics the I_1 term in S_c cancels the diverging term in U, leaving a finite result.[10] Here, were we dealing only with neutral mesons ($f_1=f_2=0$), this would happen also. However, when a nucleon emits a charged meson, it changes its isotopic spin state. Reference to graph (b) indicates that this will modify its ability to interact with another nucleon. In physical terms, this represents a change in the probability that either of the original nucleons will remain in its initial isotopic spin state to interact with the other nucleon. This manifests itself in a change in the effective value of the coupling constants. Indeed, we

[6] Compare S. T. Epstein, Phys. Rev. **73**, 177 (1948).

[7] This can be easily obtained by adding a term $\mu_\rho\delta\mu_\rho\phi_\rho^2$ to $H(x)$ in Eq. (1) and equating to zero the resulting expression in S for the scattering of a single meson in interaction with its virtual nucleon field. This equation can be solved for $\delta\mu_\rho$.

[8] Essentially this result, as a correction to the f^2 nuclear potential, has been given previously by F. J. Dyson in a lecture at the Institute for Advanced Study.

[9] This can be obtained in the same manner as $\delta\mu_\rho$. See reference 7.

[10] J. S. Schwinger, Phys. Rev. **75**, 651 (1949).

replace the coupling constants f_λ in S_2 (Eq. (4)) by $f_\lambda+\delta' f_\lambda$, where

$$\delta' f_k = i\frac{f_k}{(2\pi)^4}\left\{\sum_{\rho=1}^{3} f_\rho^2 2I_1 + \left[\sum_{j=1}^{3} f_j^2 - 2f_k^2\right] U(\Delta p_\mu^2 = -\mu^2)\right\}, \quad (k=1,\,2,\,3). \quad (21)$$

The $\delta' f_\lambda$ terms are subtracted from Eqs. (15) and (17) and considered as included in the measurable values of the coupling constants. The remaining part of the scattering matrix is equivalent to Eq. (15) with U replaced by U^1, where

$$U^1 = U - U(\Delta p_\mu^2 = -\mu^2) = -i\pi^2 \int_0^1 \ln\left[1+\frac{(\Delta p_\mu)^2}{\kappa_0^2}x(1-x)\right]dx + \text{terms of order } (\mu/\kappa_0)^2. \quad (22)$$

The validity of the interpretation of expressions (21) as coupling constant renormalization is less clear than for the case of graph (a). In particular, it does not seem at all certain that when one goes to higher orders in the coupling constant that the combinations (21) will always re-occur correctly.

C: Graphs (d) and (e)

Graphs (d) and (e) correspond to scattering processes in which two mesons are exchanged by the interacting nucleons. Included in graph (d) is the second Born approximation of the f^2 potential. It can be expected, however, that for non-relativistic energies that vacuum fluctuation phenomena will give the largest contribution, since the matrix γ_5 is of the order v/c (v is the nucleon velocity) when it couples positive energy states (simple scattering of nucleon) and of order unity when it couples a positive to a negative energy state (pair creation). For this reason graphs (d) and (e) will give the major contribution to the total scattering matrix for non-relativistic energies. These graphs do not contain renormalization effects and lead to a finite scattering matrix.

Performing the integrations over coordinate space, using (Eqs. (6)), we obtain for S_d:

$$S_d = [\bar\psi_{p'}\tau_\sigma\tau_\lambda\gamma_\alpha\psi_p][\bar\psi_{q'}\tau_\rho\tau_\nu\gamma_\beta\psi_q] f_\lambda^2 f_\sigma^2 \delta_{\lambda\nu}\delta_{\sigma\rho} V_{\alpha\beta}, \quad (23)$$

where

$$V_{\alpha\beta} = \int d^4k \frac{k_\alpha k_\beta}{[(k_\mu+p_\mu)^2+\kappa_0^2][(k_\sigma-q_\sigma)^2+\kappa_0^2] \times [(k_\lambda+\Delta p_\lambda)^2+\mu^2][k_\rho^2+\mu^2]} \quad (24)$$

Again the $-i\epsilon$ term (Eqs. (6)) is considered as included in κ_0^2 and μ^2. It will be necessary here to make explicit use of $-i\epsilon$, since graph (d) contains intermediate states for which energy is conserved (i.e., it is possible to conserve energy for each of two successive scatterings).

To evaluate $V_{\alpha\beta}$ we use the following generalization of Eq. (13).

$$\frac{1}{abcd} = 6\int_0^1 dx \int_0^x dy \int_0^y dz [az+b(y-z)+c(x-y)+d(1-x)]^{-4}. \quad (25)$$

Using Eq. (25) and performing the k-integration, we obtain

$$S_d = \pi^2 \frac{i}{2}\frac{f_\lambda^2 f_\rho^2}{\kappa_0^2}\{L_1(\bar\psi_{p'}\tau_\rho\tau_\lambda\gamma_\alpha\psi_p)(\bar\psi_{q'}\tau_\rho\tau_\lambda\gamma_\alpha\psi_q) + L_2\kappa_0(\bar\psi_{p'}\tau_\rho\tau_\lambda\psi_p)(\bar\psi_{q'}\tau_\rho\tau_\lambda i\gamma_\beta p_\beta\psi_q) + L_3(\bar\psi_{p'}\tau_\rho\tau_\lambda\gamma_\alpha(p_\alpha-q_\alpha)\psi_q) \times(\bar\psi_{q'}\tau_\rho\tau_\lambda\gamma_\beta(p_\beta-q_\beta)\psi_q)\}, \quad (26)$$

where

$$L_1 = \frac{1}{2}\int_0^1 dx\int_0^x dy\int_0^y dz\frac{1}{\Lambda}, \quad L_2 = -2\frac{dL_1}{d\kappa_0^2}, \quad L_3 = \frac{2}{\kappa_0^2}\frac{dL_1}{d\theta'}, \quad (27)$$

and

$$\Lambda = \{(x-y-z)^2+(1-x)(y-z)\theta+z(y-x)\theta' + \rho(1-x+y-z)-i\epsilon\}$$
$$\theta = \frac{(p_\mu'-p_\mu)^2}{\kappa_0^2}, \quad \theta' = \frac{(p_\mu-q_\mu)^2}{\kappa_0^2}, \quad \rho = \left(\frac{\mu}{\kappa_0}\right)^2. \quad (28)$$

Similarly, from graph (e) we obtain

$$S_e = -\pi^2\frac{i}{2}\frac{f_\lambda^2 f_\rho^2}{\kappa_0^2}\{N_1(\bar\psi_{p'}\tau_\rho\tau_\lambda\gamma_\alpha\psi_p)(\bar\psi_{q'}\tau_\lambda\tau_\rho\gamma_\alpha\psi_q) + N_2\kappa_0(\bar\psi_{p'}\tau_\rho\tau_\lambda\psi_p)(\bar\psi_{q'}\tau_\lambda\tau_\rho i\gamma_\beta p_\beta\psi_q) + N_3(\bar\psi_{p'}\tau_\rho\tau_\lambda\gamma_\alpha(p_\alpha-q_\alpha)\psi_q) \times(\bar\psi_{q'}\tau_\lambda\tau_\rho\gamma_\beta(p_\beta-q_\beta)\psi_q)\}, \quad (29)$$

where

$$N_1 = \frac{1}{2}\int_0^1 dx\int_0^x dy\int_0^y dz(1/\Gamma), \quad N_2 = -2(dN_1/d\kappa_0^2), \quad N_3 = +(2/\kappa_0^2)(dN_1/d\theta'), \quad (30)$$

and

$$\Gamma=\{(x-y+z)^2+(1-x)(y-z)\theta+(\theta'-\theta)z(x-y)+\mu^2(1-x+y-z)\}. \quad (31)$$

It should be noted that $i\epsilon$ has been set equal to zero in Eq. (31) since Γ is positive definite ($\theta'\geqslant\theta$). This corresponds to the fact that graph (e) has no intermediate states for which energy can be conserved, since it corresponds to processes in which two mesons are successively emitted.

In a non-relativistic approximation we can drop L_3 and N_3 terms and the γ_i's ($i=1$, 2, 3) in S_d and S_e. This gives

$$S_d=\pi^2\frac{i}{2}\frac{f_\lambda{}^2f_\rho{}^2}{\kappa_0{}^2}[L_1-\kappa_0{}^2L_2](\bar{\psi}_{p'}\tau_\rho\tau_\lambda\psi_p)(\bar{\psi}_{q'}\tau_\rho\tau_\lambda\psi_q),$$
$$S_e=-\pi^2\frac{i}{2}\frac{f_\lambda{}^2f_\rho{}^2}{\kappa_0{}^2}[N_1-\kappa_0{}^2N_2](\bar{\psi}_{p'}\tau_\rho\tau_\lambda\psi_p)(\bar{\psi}_{q'}\tau_\lambda\tau_\rho\psi_q). \quad (32)$$

Since these expressions do not contain spin operators, we conclude that there will be no tensor force between nucleons in a non-relativistic approximation for this theory.

The exact integration of Eqs. (27) and (30) would present severe analytical difficulties. However, it is possible to obtain terms of order $(v/c)^2$ and higher. The leading term in L_1 is of order (c/v). It arises from the region of z-integration for which Λ is small. This term can be integrated explicitly. The next order terms in L_1 have the form

$$\ln[(\Delta p_\mu{}^2/\kappa_0{}^2)+\text{const.}],$$

and a term that is essentially a constant. The leading term of order (c/v) comes from the second Born approximation of the f^2 potential and is complex even in the limit $\epsilon\to 0$, because of the pole in the integrand.

Since Γ (Eq. (31)) is non-vanishing, N_1 does not contain the (c/v) term, but is otherwise similar to L_1.

In a non-relativistic approximation the leading terms in L_1 and N_1 are canceled by corresponding terms in $\kappa_0{}^2L_2$ and $\kappa_0{}^2N_2$ respectively (see Eqs. (32)). In lowest order approximation one obtains

$$L_1-\kappa_0{}^2L_2=-(N_1-\kappa_0{}^2N_2)\equiv\Delta(\theta)=-\frac{1}{4}\left\{\ln\rho+\left(\frac{\theta+4\rho}{\theta}\right)^{\frac{1}{2}}\times\ln\left[\frac{1+[\theta/(\theta+4\rho)]^{\frac{1}{2}}}{1-[\theta/(\theta+4\rho)]^{\frac{1}{2}}}\right]\right\}, \quad (33)$$

where ρ and θ are defined in Eq. (28).

We can obtain a non-relativistic expression for the momentum space representation of the potential energy of two nucleons (using Eq. (33)) as that expression which leads to Eqs. (32) in a Born approximation.[11]

$$(p'q'|V^4|pq)=-\frac{1}{4}\frac{1}{(2\pi)^3}\left[\frac{f_\lambda{}^2f_\mu{}^2}{(2\pi)^2}\right]\frac{\Delta(\theta)}{\kappa_0{}^2}\times\tau_\lambda{}^{(1)}\tau_\mu{}^{(1)}(\tau_\lambda{}^{(2)}\tau_\mu{}^{(2)}+\tau_\mu{}^{(2)}\tau_\lambda{}^{(2)})$$
$$=-1/(2\pi)^3(f^2/2\pi)^2\Delta(\theta)/\kappa^2$$
$$(\text{charged theory};\ f_1=f_2=f,\ f_3=0)$$
$$=-\tfrac{3}{2}[1/(2\pi)^3](f^2/2\pi)^2\Delta(\theta)/\kappa_0{}^2$$
$$(\text{symmetric theory};\ f_1=f_2=f_3=f), \quad (34)$$

where the superscripts (1) and (2) on the τ-operators imply that they refer to particles (1) and (2), respectively. The f^2 potential is non-relativistically:

$$(p'q'|V^2|pq)=-\frac{1}{8\kappa_0{}^2}\left(\frac{f_\lambda{}^2}{2\pi}\right)\frac{1}{2\pi^2}\frac{\sigma^{(1)}\cdot\Delta p\,\sigma^{(2)}\cdot\Delta p}{(\Delta p)^2+\mu^2}\tau_\lambda{}^{(1)}\tau_\lambda{}^{(2)}. \quad (35)$$

From Eq. (35) it is seen that in a non-relativistic approximation V^4 does not contain tensor forces, nor does it contain exchange forces. It is further seen from Eq. (33) that V^4 in coordinate space has a $1/r^3$ singularity at the origin (for $r\sim 1/\kappa_0$, relativistic corrections reduce this to a $1/r$ singularity). Each of these conclu-

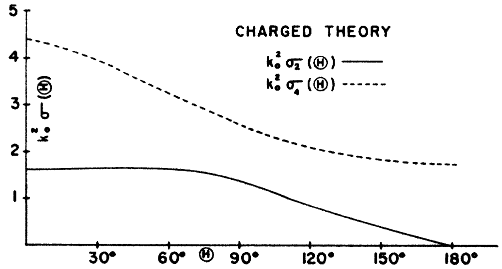

FIG. 2. Partial differential cross sections for the charged theory at 90 Mev contributed by the terms S_2 and S_4 in the scattering matrix taken individually.

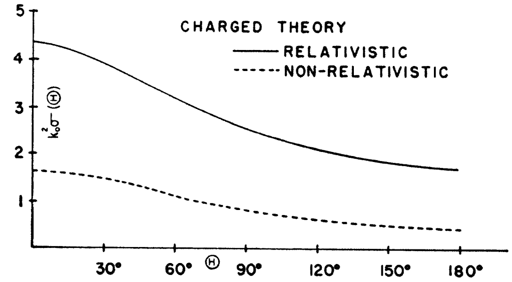

FIG. 3. Effect of relativistic corrections, as herein defined (page 17) on the partial cross section contributed by the S_4 term in the scattering matrix.

[11] That the potential can be obtained in this manner was first noted by Professor H. A. Bethe, Phys. Rev. 76, 191 (A) (1949).

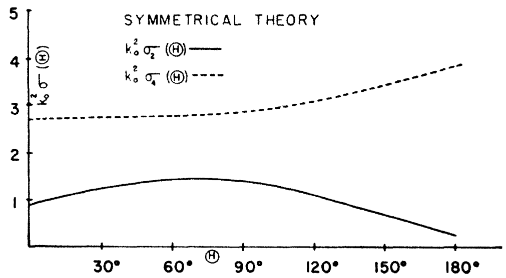

FIG. 4. Partial differential cross sections for the symmetrical theory at 90 Mev contributed by the terms S_2 and S_4 in the scattering matrix taken individually.

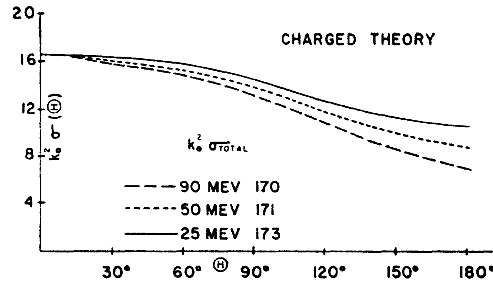

FIG. 5. Estimate of differential cross sections for the charged theory computed by using the eighth-order S-matrix (Eq. (A. 1)).

sions is in contradiction to experiment; however, it is not impossible that higher order radiative corrections might modify this result.

To see the importance of relativistic corrections, we investigate the differential scattering cross section.

III. SCATTERING CROSS SECTIONS

For the case of the charged theory the constant $f_3=0$ and $f_1=f_2=f$. As a consequence of the fact that only charged mesons are present the matrix element corresponding to diagram (b) vanishes identically since it is incompatible with electric charge conservation. For the problem of neutron-proton scattering (e) vanishes on the same grounds and (d) contributes. Similarly for the scattering of identical particles (d) vanishes and (e) contributes. The contributions of the two diagrams are, however, identical in a non-relativistic approximation (see Eq. (32)). After the infinite part of diagram (a) has been interpreted in terms of charge and mass renormalization it gives a very small effect on the cross section of the order of a percent.

The neutron-proton scattering cross section has been treated in the approximation of retaining consistently all relativistic effects of order $(v/c)^2$ in those terms of the scattering matrix up to the fourth power of the coupling constant. Figure 2 shows the partial cross sections at 90 Mev contributed by the second- and fourth-order terms taken separately. Thus in the center of gravity system

$$\sigma_2(\theta)=\pi^2E^2|S_2|^2,\quad \sigma_4(\theta)=\pi^2E^2|S_4|^2. \tag{36}$$

The flatness of the fourth-order curve and the forward maximum reflect the high singularity of the potential and the fact that only ordinary and not exchange forces are contributed by the fourth-order potential. The coupling constant was chosen as $(f^2/2\pi)=6.4$. We shall return to this point in a later section.

The importance of relativistic corrections is indicated in Fig. 3. Here the partial cross section contributed by the fourth-order term in the scattering matrix alone has been plotted with and without inclusion of relativistic terms. Since there is some ambiguity as to what one might mean by a relativistic correction, it seems worth while to discuss this point in some detail. As we have pointed out before (Eqs. (26) and (29)), the scattering matrix can be written in terms of the two invariant collision parameters θ, θ' which represent the square of the momentum transfer divided by κ_0^2 and the maximum value of this quantity respectively. In terms of the angle of scattering Θ there exists the following relation:

$$\theta=\theta'\sin^2(\Theta/2). \tag{37}$$

In terms of these parameters $\sigma_4(\Theta)$ may be written

$$\sigma_4(\theta,\theta')=\sigma_{40}(\theta)+(\theta')^{\frac{1}{2}}\sigma_{41}(\theta,\theta')+\theta'\sigma_{42}(\theta,\theta')+\cdots.$$

The terms in $(\theta')^{\frac{1}{2}}$ and θ', etc., can be said to represent relativistic corrections since in the center of gravity system

$$\theta'=4p^2/\kappa_0^2\sim(v/c)^2, \tag{38}$$

where $\mathbf{p}$ is the momentum of one of the colliding particles.

If we now let θ' tend toward zero the terms involving σ_{41} and σ_{42}, etc., tend to zero whereas σ_{40} remains finite and is identical with the partial cross section which would have been contributed by the non-relativistic potential (Eq. (34)). The meaning of the non-relativistic cross section σ_{40} seems rather unique when the limiting process is characterized in terms of the invariant quantity θ'. The condition for the validity of our non-relativistic cross section is simply

$$\theta'\ll 1\quad\text{or}\quad (2p)^2\ll\kappa_0^2. \tag{39}$$

Inspection of the curves in Fig. 3 shows that relativistic effects, when so defined, are rather large. The result is, at first sight, surprising, but it must be remembered that at 90 Mev $\theta'\sim\frac{1}{5}$ and therefore the terms σ_{41} and σ_{42} need not be small.

For the symmetrical theory $f_1=f_2=f_3=f$. Here due to the presence of the neutral meson all the diagrams play a role. It turns out, however, that after identifications of charge and mass renormalization terms in diagrams (a) and (b) that they contribute less than one percent to the cross sections. Barring the inconsequential effect of the Lamb shift and vacuum polarization terms there are, in the non-relativistic limit, no exchange forces contributed by the fourth-order potential in the symmetrical theory (Eq. (34)).

The partial cross sections $\sigma_2(\Theta)$, $\sigma_4(\Theta)$ obtained in the case of the symmetrical theory are shown in Fig. 4. They have been computed in the same approximation as the corresponding terms in the charged theory. Again the flatness of the fourth-order curve reflects the high singularity of the potential. The fact that the forward-backward cross-section ratio for this curve is about one must be attributed to exchange forces introduced by relativistic corrections. The coupling constant $(f^2/2\pi)\sim 4.8$ was determined roughly by the requirement that the fourth-order partial cross section agrees with the corresponding quantity in the case of the charged theory.

We are indebted to Professor Bethe for sending his results to us in advance of publication and to Professor J. R. Oppenheimer for helpful criticism.

This work was carried out while one of us (K.W.) was supported by a grant-in-aid through the Institute for Advanced Study supported by the Atomic Energy Commission and the other (J.V.L.) was the holder of an Atomic Energy Commission Postdoctoral Fellowship.

APPENDIX

The S-matrix which has so far been obtained may be used to compute the cross section up to terms proportional to f^8. There are, of course, other terms involving f^8 due to higher order terms in the S-matrix. An attempt was made to improve the present approximation by taking the potential which we have obtained (Eqs. (34) and (35)) and solving the equation of motion again.

$$i(\partial\Psi/\partial t)=V\Psi.$$

This equation may be solved in the same manner as Eq. (2) and the S-matrix can be found. We may now identify those terms in S corresponding to the terms S_2 and S_4 which have been correctly dealt with. Thus an S-matrix of the form,

$$S=S_2+S_4+S_6'+S_8', \tag{A.1}$$

can be found. Of course, the terms in S_6' and S_8' depend on the singular potential (Eq. (34)) so that it is necessary to cut off the integrations. This cut-off was arbitrarily chosen as the nucleon Compton wave-length. The results are not sensitive to this choice since the relativistic effects would introduce a cut-off in this region anyway.

A coupling constant $f^2/2\pi=6.4$ was estimated in this way by fitting the total cross section at 90 Mev reported by Segrè and his collaborators.[12] The results are given in Fig. 5. The total cross sections are nearly energy independent in the range considered.

[12] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949).

PHYSICAL REVIEW VOLUME 76, NUMBER 8 OCTOBER 15, 1949

The Photo-Disintegration of the Deuteron

INGVAR F. E. HANSSON,
Institute of Mechanics and Mathematical Physics, Lund, Sweden

AND

LAMEK HULTHÉN,
Royal Institute of Technology, Stockholm, Sweden
(Received May 9, 1949)

The photo-magnetic and photoelectric cross sections have been calculated for γ-energies 2.62, 2.76 and 6.2 Mev, using different values of the deuteron binding energy (2.19 and 2.24 Mev). The calculations were carried out for a symmetrical theory with meson potential (Møller-Rosenfeld theory), the corresponding neutral theory (ordinary force) and a third version with no ^3P-interaction. Non-central forces were neglected. Results are given for two ranges, equivalent to meson masses 200 and 300. The influence of the γ-ray momentum on the angular distribution of the photo-nucleons should be noted.

FROM the point of view of nuclear force theory the photo-disintegration of the deuteron is one of the fundamental experiments. Measurements of the total cross section and the angular distribution of the photo-neutrons (or -protons) for moderate energies are now in progress in several laboratories,[1–5,*] and we hope that the theoretical results presented in this note will be of some use for the interpretation of the experiments.

We have calculated the photo-magnetic and photoelectric cross sections, σ_m and σ_e,** for the γ-energies mainly used in experiments so far: 2.62, 2.76 and 6.2 Mev. As a theoretical basis we have employed the Møller-Rosenfeld (MR) theory and, for comparison, the corresponding neutral theory (N) and a third version (O) where the interaction in the ^3P-state is assumed to be zero. The three cases can be characterized

[1] Wilson, Collie, and Halban, Nature **163**, 245 (1949).
[2] N. O. Lassen, Phys. Rev. **74**, 1533 (1948); Phys. Rev. **75**, 1099 (1949).
[3] B. Hamermesh and A. Wattenberg, Phys. Rev. **75**, 1290 (1949).
[4] Meiners, Smith, and Slack, Phys. Rev. **75**, 1632 (1949).
[5] Snell, Barker, and Sternberg, Phys. Rev. **75**, 1290 (1949).
* We are indebted to Dr. Hans Halban, Oxford, and Dr. N. O. Lassen, Copenhagen, for kindly informing us of experimental results before publication.

** For a survey of the theory of the photo-disintegration, see e.g., L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948), pp. 132–135, 175–179, 452–453.