PHYSICAL REVIEW VOLUME 76, NUMBER 8 OCTOBER 15, 1949

On Magneto Optics of Neutrons and Some Related Phenomena

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(Received February 17, 1949)

Formulas are derived describing the phenomenon of total reflection of neutrons by a ferromagnet in dependence on the state of magnetization. It is shown how experiments of this kind can give information concerning the magnetic interaction between neutrons and atomic magnetic moments. A method to obtain well-polarized neutron beams with an intensity loss of only 50 percent of the primary beam is outlined. The effect of pure magnetic scattering and its influence on the transmission effect is discussed; an experimental arrangement for its observation is described. The significance of the approach to saturation in the single transmission effect is discussed in connection with the saturation theory of ferromagnets. Remarks are made concerning the possibility of detecting wall effects of magnetic domains through depolarization of neutron beams.

1. INTRODUCTION

THE present paper contains the discussion of a number of effects which are produced by the passage of neutrons through a ferromagnetic material. They are otherwise only indirectly connected with each other; some of them are more concerned with neutron properties, while others refer to information concerning the solid state which can be gained using neutrons as tools of exploration. Frequently in this paper we shall have occasion to refer to formulae derived and results obtained in three earlier papers[1] which shall be referred to in the following as I, II, and III.

2. REFRACTION, TOTAL REFLECTION AND POLARIZATION OF NEUTRONS BY FERROMAGNETS[2]

In paragraph 5 of III, we have derived expressions for the indices of refraction of neutrons passing through a (ferromagnetic) medium. This refraction is due to the scattering of the neutrons by the nuclei (and by the magnetic moments of the ferromagnetic atoms). Only the coherent scattering by the nucleus contributes to the index of refraction; the scattering due to the interaction of the neutrons of the atoms' magnetic moments is coherent in the case of ferromagnets but shows the added complication that it depends upon the relative orientations of the atomic magnetic moment and the neutron spin. If we resolve the spin of the incident neutron along the axis of magnetization, then a neutron of spin i has in general $2i+1$ indices of refraction which for $i=\frac{1}{2}$ are given by

$$n_\pm - 1 = \frac{2\pi N}{k^2}\left(C \pm \frac{e^2\gamma S}{mc^2}\right). \tag{1}$$

In (1) the following abbreviations have been used: $k=2\pi/\lambda$, N = number of atoms/cm^3, C = coherent scattering amplitude of nucleus, γ = neutron moment in nuclear Bohr magnetons, and S = spin of iron atom in units.

In the experiments of Fermi and Marshall,[3] use has been made of the index of refraction of neutrons to obtain for a number of elements total reflection of neutrons incident under a small glancing angle. It is clear that such experiments can be extended to cover the more involved case of total reflection by magnetized ferromagnets.

To sketch the arrangement in its simplest form, we allow neutrons to impinge upon a plate of iron which we assume to be magnetized. If, e.g., the direction of magnetization should lie in the plane of the plate, then neutrons with spins parallel or anti-parallel to the direction of magnetization will exhibit respectively indices of refraction given by

$$n_\pm - 1 = n_0 \pm \frac{2\pi N}{k^2}\frac{e^2\gamma S}{mc^2} - 1. \tag{2}$$

Here, n_0 stands for the index of refraction determined in Fermi's experiments. The critical glancing angles φ for neutrons with the two different spin orientations will, therefore, be given by

$$\varphi_\pm = 2(1-n_\pm)^{\frac{1}{2}}. \tag{3}$$

They will lie on both sides of the critical angle found by Fermi and Marshall. It is also obvious that for the *larger* of the two critical glancing angles the reflected beam will be *almost completely polarized*, while for the smaller critical angle both polarization states will occur in the reflected beam.

A numerical evaluation leads to the following data for iron as a reflector: $\varphi_+\sim 13$ minutes, $\varphi_-\sim 6$ minutes, $\varphi_0\sim 10$ minutes (unmagnetized state).

It seems from the estimate given that there should be no essential difficulty in carrying out the experiments in the way sketched. It will, of course, be necessary to avoid magnetic stray fields which would lead to a depolarization of the reflected neutrons. Techniques

[1] O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939); O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941); Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[2] O. Halpern, Phys. Rev. **75**, 343 (1949).
[3] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).

similar to those employed by D. J. Hughes[4] and his collaborators in measuring the double transmission effect of neutrons in ferromagnets should prove successful in the present case.

The experiments sketched may also be useful for the purpose of testing the assumed interaction between the magnetic moments of the neutrons of the iron atom. Theory leads to the following expression for the amplitude of magnetic scattering C_m as function of the angle ϑ and the direction of magnetization $\boldsymbol{\kappa}$ relative to the directions $\mathbf{k}_0$, $\mathbf{k}$ of incident and scattered beam:

$$C_m=\frac{e^2\gamma S}{mc^2}2(\mathbf{s}\cdot\mathbf{q})$$

$$\mathbf{q}=\mathbf{e}(\mathbf{e}\cdot\boldsymbol{\kappa})-\boldsymbol{\kappa} \tag{4}$$

$$\mathbf{e}=\frac{\mathbf{k}_0-\mathbf{k}}{|\mathbf{k}_0+\mathbf{k}|}.$$

If, as assumed before, the direction of magnetization is lying in the plane of the mirror, the magnetic scattering amplitude becomes a maximum. It drops to its minimum value if $\boldsymbol{\kappa}\|\mathbf{e}_0$

$$C_m=0. \tag{5}$$

A study of the variation of the splitting of the two critical angles with changing orientation of the magnetic field would, therefore, allow a test of the theoretical expression (4) and of the assumptions made concerning the magnetic interaction function.

3. PURE MAGNETIC SCATTERING

In the arrangement originally proposed by Bloch,[5] the change in transmission of a neutron beam due to magnetization of the iron plate was ascribed to the interference of the scattering from the iron nucleus and from the magnetically active electrons of the ion shell. The fact that this phenomenon has two causes offered considerable difficulties to attempts aimed at obtaining quantitative theoretical expressions for this single transmission effect.

The pure magnetic scattering which is proportional to the square of the iron's magnetic moment did not enter into these calculations because it was assumed to be independent of the over-all state of magnetization of the material.

We shall show that due to the peculiar angle dependence of the magnetic scattering discussed in I §6 this simplification is not generally permissible; this will lead to a comparatively weak (negative) transmission effect linear in the thickness of the transversed material, in addition to the usually observed (positive) transmission effect which for complete saturation is quadratic in the thickness of the transmitted material.

[4] Unpublished results.

[5] F. Bloch, Phys. Rev. **50**, 259 (1936).

This linear transmission effect comes about as follows: The amplitude of the scattered wave containing nuclear as well as magnetic scattering is given by

$$C_s=C+C_m=C+(e^2\gamma S/mc^2)2\mathbf{s}\cdot\mathbf{q}.$$

Assuming as it is experimentally convenient that the direction of magnetization $\boldsymbol{\kappa}$ is perpendicular to the direction of the incident beam $\mathbf{k}_0$, we obtain for the scattered intensity

$$J\sim C^2+4C\frac{e^2\gamma S}{mc^2}\mathbf{s}\cdot\mathbf{g}+\left(\frac{e^2\gamma S}{mc^2}\right)^2 4(\mathbf{s}\cdot\mathbf{q})^2. \tag{6}$$

If the material is originally *unmagnetized*, then the directions of magnetization in the various domains make random angles with $\mathbf{k}_0$ and we obtain, therefore, for a *fixed* scattering angle an intensity given by

$$J\sim C^2+(e^2\gamma S/mc^2)^2\mathbf{q}^2. \tag{7}$$

The second part in (6) refers to the well-known positive transmission effect; we here want to note the actual value of the third term in the *unmagnetized* state which, after averaging, becomes

$$J\sim C^2+(e^2\gamma S/mc^2)^2\tfrac{2}{3}. \tag{7a}$$

If, on the other hand, the material is near magnetic saturation, then we are only allowed to average over the azimuth of the scattering angle, and obtain from (6) the following expression for the intensity

$$J\sim C^2+\left(\frac{e^2\gamma S}{mc^2}\right)^2\frac{1+\sin^2\vartheta/2}{2}. \tag{8}$$

The difference between the third terms in (7a) and (8) gives the previously discussed linear transmission effect. We find that even the sign of this difference varies with the scattering angle; it is positive for small and negative for large angles.

To demonstrate the influence of pure magnetic scattering most clearly, it is advisable to make this term as large as possible, while, at the same time, reducing the amount of the positive transmission effect. We remember for this purpose that the positive transmission effect is extremely sensitive to deviations from saturation; theory[1] and experiment[6,7] show in full agreement that for some materials a deviation from saturation of several percent is sufficient to reduce the transmission effect by one or two orders of magnitude.

The following experimental arrangement is, therefore, proposed to test the theory: A wave-length close to 4A is chosen; in that case the scattering occurs only under about 180°, i.e., only the first Debye ring is formed. We learn from III that the amplitude of scattering in this case amounts to 1.14C. The form factor of the

[6] Bloch, Nicodemus, and Staub, Phys. Rev. **74**, 1025 (1948). There also earlier literature.

[7] Hughes, Wallace, and Holtsmann, Phys. Rev. **73**, 1277 (1948).

magnetic scattering is in this case as shown in III or in the paper by M. Hamermesh[8] given by $\sim\frac{1}{4}$. The total purely magnetic scattering cross section, under these assumptions, takes on the value 8.10^{-25} unmagnetized and 12.10^{-25} cm^2 magnetized. This compares with a total nuclear scattering cross section of $\sim 12.10^{-24}$ cm^2. Deferring discussion of the interference term, we would, therefore, predict a negative transmission effect of $\sim$3.2 percent/cm.

The interference term is theoretically, as well as experimentally, of course, of higher order of magnitude if saturation is approached; but for incomplete saturation, e.g., of 90 percent, the depolarization of the neutrons inside of the iron is so strong as to almost annihilate the influence of the interference term. Theory and experiment show that the positive transmission Δ effect for incomplete saturation is given by (see II)

$$\Delta = \Delta_\infty(2/pd) = (w^2 d/p).$$

Here w is the cross section due to the middle term in (6) and p is the coefficient of depolarization which is proportional to the deviation from saturation. For $\lambda \sim 4$A and 90 percent saturation $w \sim 0.9$, $p \sim 200$ and Δ therefore $\sim$0.4 percent/cm which constitutes only a correction to the previously calculated negative transmission effect.

A determination of the transmission effect as function of magnetization should, therefore, lead to an isolation of the pure magnetic scattering and permit a direct experimental determination of the form factor of scattering for the first Debye ring. The experiment could, of course, be extended to smaller wave-lengths, but since very soon other Debye rings will appear, which, according to (8), would diminish the effect, it would soon become impracticable.

The effect here discussed is also of some significance in the evaluation of experiments near saturation. It is true that then the well-known positive transmission effect will greatly outweigh the negative transmission effect here discussed, but, since this negative transmission effect is *always* present, the true theoretical value of the positive transmission effect is larger than the observed experimental value by the amount of the previously neglected negative transmission effect. The structure of (8) indicates that this correction will only be of real numerical significance if sufficiently long wave-lengths are used.

4. REMARKS ON THE LAW OF MAGNETIZATION NEAR SATURATION

It is well known that the simple picture of rotation of the vector of the magnetic moment of the individual crystallites into the direction of the external field leads to a saturation law of the form

$$M = M_\infty[1-(a/H^2)], \tag{9}$$

for high field strengths this theoretical law is not confirmed by experiment, which instead gives a behavior of the type

$$M = M_\infty[1-(b/H)]. \tag{10}$$

Attempts to derive the $1/H$ term have been unsuccessful.

The experiments of Hughes[7] and his collaborators have thrown new light on the situation and, at least in a negative sense, have eliminated a number of theoretical possibilities. They indicate that any successful theory will have to derive the empirically observed saturation law on the basis of crystallites of the same order of magnitude as the microscopically measured grain sizes. Special assumptions ascribing slow saturation to smaller splinters which are exposed to additional crystal forces are ruled out by the observations. This conclusion can be established, as follows: The experiments indicate that the positive transmission effect approaches its asymptotic value according to the law $1-b/H$. The transmission effect, on the other hand, depends only on the depolarization due to deviation from ideal saturation. As predicted in II and verified in reference 7 the depolarization follows different laws, depending upon the size of the individual domain. If the domain, i.e., in our case the individual crystal grain, satisfies the relation

$$\delta/v \sim T, \tag{11}$$

δ = domain length, T = Larmor period, v = velocity, then the law of depolarization is given by e^{-px}; if, on the other hand, the domain size satisfies the inequality

$$\delta/v \ll T, \tag{12}$$

then the law of depolarization takes on the form $\exp[-px(\delta/vT)^2]$. In this second case, the depolarization is obviously much smaller than in the first one; the saturation value of the transmission effect would, therefore, be approached much earlier than in the first case.

The fact now that, as shown in reference 7, the *depolarization effect* follows a law of the type (11) in which δ is of the order 10^{-3} cm indicates that the investigated materials have domain sizes satisfying (11). It is, therefore, not possible to assume a certain percentage of the material to be present in the forms of microscopic grains which only very slowly take on the saturation value of magnetization; these small grains could perhaps account for a slow saturation as determined by measurements of the magnetic flux, but would be quite unobservable in the transmission effect. The fact that the saturation of magnetization follows the same law as the saturation of the transmission effect constitutes a cogent argument for the identity of the domains that are responsible for both phenomena.

5. THE INFLUENCE OF WALLS SEPARATING DOMAINS

In II theoretical formulas and experimental methods have been described to determine the sizes and orienta-

[8] M. Hamermesh, Phys. Rev. **61**, 17 (1942).

tions of individually magnetized domains in various parts of the magnetization curve and their dependence upon external parameters. Experiments carried out along these lines have partially been published;[6,7] additional observations on similar effects, although presented at meetings of the American Physical Society, have not yet appeared in print.

In a recent note, Newton and Kittel[9] attempt to extend the method of II for the purpose of measuring the wall thickness between individual domains present for very small external fields. We should like to comment on some aspects of this question.

In II the influence of the domain walls was neglected for all parts of the magnetization curve; this is rigorously justified for *sizable* external magnetic fields when the domains are practically wiped out and only crystallite boundaries remain, and it seemed justified for most cases of practical interest because the time of passage through a domain wall (at very low magnetic fields) was assumed to be so small that the processes could be considered practically instantaneous. On the basis of estimates given by Becker and Doering[10] we assumed the domain walls to have a thickness of 30 to 100 lattice distances. For average neutron velocities of the order of 10^5 cm/sec. the time spent in traversing a domain wall is of the order of 2.10^{-11} sec. which is very small compared to a Larmor period which is about 2.10^{-8} sec. in iron and still larger in other ferromagnets. In addition to it, the depolarization by the domains themselves, which we assumed to be large compared to the domain walls, made it only consistent to neglect the wall effect.

Newton and Kittel do not specify the type of experiment which they have in mind; we shall, therefore, try to discuss their suggestions on a basis which seems to us most favorable for their attempt. It is obviously necessary that the domains themselves shall not contribute to depolarization if one wants to isolate the wall effects. This makes it necessary to have the magnetization of the domains lined up in directions parallel and anti-parallel to each other, a case which we discussed in II, paragraph 5. Unless such an orientation of domains is obtained, the wall effect will always be small compared to the domain effects, and the experiment of Newton and Kittel will, therefore, become impractical.

Treating, on the other hand, the just-mentioned case of parallel and antiparallel domains, we derived the result that, apart from wall effects, the component of the spin of the incident neutrons parallel to the direction of magnetization should pass through unchanged, while the components perpendicular are depolarized. If the experiment of Newton and Kittel should, therefore, be carried out, it would be necessary to transmit a beam of neutrons polarized, e.g., parallel to the hexagonal axis of a cobalt crystal; the beam then would show depolarization which could be ascribed to the wall effect, provided that the domains are really lined up in the way described. The difficulty encountered in these experiments has been partly already discussed by Newton and Kittel; namely, the smallness of the passage time through a domain wall compared to the Larmor period. Newton and Kittel estimate the passage time on the basis of values for the wall thickness which are considerably larger than those suggested by Becker and Doering; we have no personal opinion as to which values are preferable. The large values claimed for permalloy do not seem to us to be very helpful because we do not know whether it will be possible to achieve a perfect orientation of the domains parallel and antiparallel to each other in the case of the cubic crystal permalloy.

The extension of our method to the measurement of wall thicknesses would, of course, be highly welcome; the remarks here made are intended to serve the purpose of pointing out several requirements that are necessary prerequisites for such experiments.

[9] R. Newton and C. Kittel, Phys. Rev. **74**, 1604 (1948).
[10] Becker and Doering, *Ferromagnetismus* (Edward Brothers, Ann Arbor, Michigan, 1943), p. 189.