the measurements at 250 gauss, the hyperfine separation of the $^2P_{1/2}$ state was calculated to be 1500 ± 50 megacycles per second. Several attempts were then made to find transitions between $F=3$ and $F=2$. Both because of the lower power output of the Lighthouse Tube oscillator in the region around 1500 Mc and because of the lower transition probabilities of the lines relative to those of the $^2P_{3/2}$ state, as seen in Table I, no transition with $\Delta F = \pm 1$ was detected. The inaccurate value of $\Delta\nu$ obtained, however, serves as further indication that the element that has been studied is aluminum, since it overlaps the spectroscopic value found by Jackson and Kuhn.

Perturbations between Fine Structure Levels

We have so far ascribed the deviation from the interval rule of the hyperfine structure of the $^2P_{3/2}$ state to the effect of a nuclear quadrupole moment. The same effect, however, may be caused by perturbations due to neighboring fine structure levels. Casimir[9] has shown that levels with the same F and differing in J by 0 or 1 repel each other with an interaction energy of the order of (hyperfine splitting)^2/(fine structure separation). Thus the $F=3$ and $F=2$ levels of the $^2P_{3/2}$ state interact with the corresponding levels of the $^2P_{1/2}$ state. The total width of the hyperfine structure of the former state is about 900 Mc/sec. and the $^2P_{3/2}-{}^2P_{1/2}$ separation is about 3.36×10^{12} cycles. The magnitude of the repulsion is therefore 0.25 Mc or about one percent of the observed deviation from the interval rule.Thus we are justified in ascribing the latter to the effect of a nuclear electric quadrupole moment.

[9] H. B. G. Casimir, *On the Interaction between Atomic Nuclei and Electrons* (Teyler Tweede Genootschap, Haarlem, 1936).

ACKNOWLEDGMENT

The author wishes to thank Professor Jerrold R. Zacharias for his guidance and suggestions in the course of the problem. Thanks are due to Dr. Carrol W. Zabel for his generous help in the theoretical phases of the problem. To Drs. Darragh E. Nagle and Luther Davis, Jr., the author is indebted for general assistance in the use of the atomic beam apparatus for which they have been largely responsible.

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Extensive Air Showers at High Altitudes

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The latitude dependence, density properties, and lateral extension of extensive cosmic-ray air showers have been studied at 9200 m elevation, with coincidence counters in a B-29 airplane. On a flight between 0° and 63°N geomagnetic latitude, no significant variation of shower frequency with latitude has been detected.

Relations between the counting rates of coincidence counter sets of different orders from threefold to ninefold are in agreement with a power law density spectrum for the showers, with constant negative exponent. The measured value of this exponent is 1.73 ± 0.04 at 9200 m, 1.82 ± 0.07 at 11,000–12,000 m, and 1.53 ± 0.07 at 720 m. The counting rate of a threefold coincidence counter set was found to decrease 33 percent as the total spread of the counters was increased from 2.8 m to 13 m at 9200 m altitude. The corresponding decrease at 720 m is 21 percent. The altitude dependence of the extensive showers was measured up to 12,300 m, and a definite maximum shower rate has been found near an altitude of 8000 m.

I. INTRODUCTION

SINCE their discovery more than ten years ago,[1–3] large cosmic-ray air showers have been studied intensively in the lower atmosphere. However, the rapid increase in number of showers with altitude in the lower atmosphere indicates that most of them are being absorbed in this region, and that their source is high in the upper atmosphere. Investigations conducted nearer to the origin of the showers should provide a more sensitive test of hypotheses concerning their production and growth than similar studies at lower altitudes. Therefore, coincidence counter measurements of the properties of extensive air showers have been undertaken in a B-29 airplane flying between 6000 and 12,000 m above sea level. The altitude dependence has been measured up to 12,300 m. The density structure of the air showers was studied at 9200 m and higher altitudes, by determining the counting rate as a function of the number of counters in coincidence. Observations were also made of the variation of shower frequency with latitude and with total spread of the coincidence counters. Preliminary reports of the altitude dependence have been published previously.[4, 5]

* Now at Yale University, New Haven, Connecticut.

[1] K. Schmeiser and W. Bothe, Ann. d. Physik **32**, 161 (1938).

[2] Auger, Maze, and Grivet-Meyer, Comptes Rendus **206**, 1721 (1938).

[3] Kohlhörster, Matthes, and Weber, Naturwiss. **26**, 576 (1938).

[4] H. L. Kraybill and P. Ovrebo, Phys. Rev. **72**, 351 (1947).

[5] H. L. Kraybill, Phys. Rev. **73**, 632 (1948).

This paper contains a more complete account of the results of these experiments.

II. METHOD AND APPARATUS

Nine Geiger counters were mounted just beneath the roof of a B-29 airplane, as shown in Fig. 1. The counters were connected to five threefold coincidence circuits, as indicated in Fig. 2. Details of the counter geometry are given in Fig. 2a (arrangement A). Circuits numbered 2 and 4 measured the counting rate of three uniformly spaced counters with a total spread of 2.8 m. Circuits numbered 1 and 3 measured the counting rate of three counters with a total spread of 4.2 m. The spacings were chosen so as to afford the widest possible separation of the counters in the rear pressurized cabin of the airplane. Since circuits 1 and 3, and circuits 2 and 4, were connected to counter arrangements with identical geometry, a check of the functioning of each of these circuits was available. The circuit numbered 5 was connected to two counters in the rear cabin and to one counter in the front cabin, having a total spread of 13 m.

The circuits were of the usual Rossi type. All counters were shielded with metal boxes. A cathode follower mounted at each counter fed the pulses through shielded coaxial cable to the coincidence circuits.

In mounting the counters in the airplane, it was necessary to displace some of them by slight horizontal distances (up to 5 cm) from the positions shown in Fig. 2a. From the rather slight dependence of counting rate upon counter spread which was observed at all altitudes (see Fig. 7), it is considered that these small displacements had negligible influence upon the counting rates. Counters numbered 5 and 6 were displaced vertically a few centimeters in order that no set of three counters connected to one circuit should lie on a straight line. Thus there was a minimum possibility for a single particle to trip any of the circuits.

The counters were of all-metal construction with brass walls. The wall thickness was 0.8 mm. The active length was 32 cm and the inside diameter was 2.34 cm, so that the active area was 75 cm^2. They were filled with the usual nine to one mixture of argon and ethyl alcohol.

Coincidences from each threefold circuit were recorded as lines flashed by neon lamps on a moving photographic film. The channels for each coincidence ran side by side on the film, so that the simultaneous occurrence of any combination of coincidences could be detected. In this manner various orders of coincidence from threefolds to ninefolds were observed. For example, the simultaneous discharge of circuits 1 and 2 indicated a fourfold coincidence, and the simultaneous discharge of circuits 1, 2, 3, and 4 indicated an eightfold coincidence. A typical record is shown in Fig. 3. The error of resolution of the recorder was less than one percent for the threefold coincidences and was two to three percent for the higher order coincidences.

The errors due to counter deadtime and accidental coincidence counts were determined as follows: The counters were tested and found to have an efficiency of at least 99 percent for ionizing cosmic rays at Chicago, with a background of about 200 counts/min. The resolving times of the coincidence circuits were determined to be between 1.5 and 2.0 μsec., by measuring accidental counting rates when radium was placed near the counters. The effective total deadtime of counters and circuits was determined by measuring the inefficiency with high single counting rates in each counter. A value of 240 ± 20 μsec. was obtained for the deadtime.

Counting rates of individual counters were measured during the airplane flights with a portable pulse rate meter. The average rate of a single counter ranged from 50 to 35 counts/sec. at 9200 m elevation, between 63 degrees north geomagnetic latitude and the equator. At 12,300 m (41°N), the highest altitude which was reached, the average rate per counter was 85 counts/sec. Under these conditions, the threefold accidental counting rate, given by $3N^3T^2$, where N is the single counting rate and T is the resolving time, is 0.03 count/hr. Since the slowest threefold counting rate was 11/hr., accidental counts were always negligible. The deadtime inefficiency at the fastest single counting rate was 2.0 percent per counter. For most of the data, which were taken at an altitude of 9200 m, the deadtime correction was approximately 1.2 percent per counter.

III. LATITUDE DEPENDENCE OF AIR SHOWERS

The counting rate of the arrangement in Fig. 2a was measured as a function of latitude on a flight between

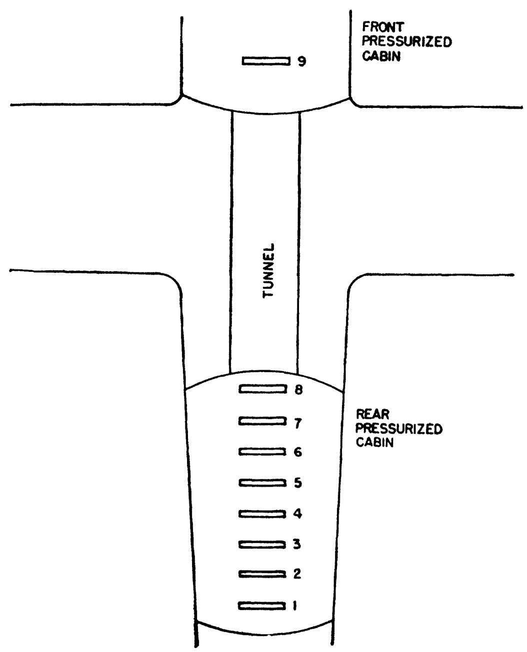

FIG. 1. Top view of B-29 airplane showing positions of shower counters (arrangement A).

northern Canada (63°N geomagnetic latitude) and Lima, Peru, on the magnetic equator. The airplane flew at a constant pressure altitude of 30,000 feet (9200±50 m), corresponding to a pressure of 22.6 cm of mercury. Since a barometric altimeter was used on the airplane, the flight was made at constant pressure rather than constant elevation above sea level. All altitudes stated in this article will refer to the reading of the altimeter, and will therefore denote a pressure which can be obtained from the standard pressure-altitude curve in terms of which the altimeter was calibrated.[6]

Table I compares the counting rates between 0° and 30°N with the rates between 30° and 63°N geomagnetic latitude, for different coincidence combinations. In all cases the counting rates are slightly higher in southern latitudes than in northern latitudes. The differences, however, are within the limits of statistical fluctuations. It is concluded that if a latitude effect of the usual type is present, with higher shower frequency in northern latitudes, it can hardly be as great as 10 percent. If a 10 percent latitude variation should actually exist, the probability of obtaining a deviation equal to or greater than that of the observed results would be one in 370. Further evidence of the lack of a geomagnetic latitude effect is provided by a single flight at 11,000 m altitude at Lima, Peru. The counting rate on this flight was slightly higher than the value given by the altitude curve of Fig. 8, which was based upon observations at 41°N.

The absence of a large latitude effect at these high altitudes confirms less accurate observations of Kraybill and Ovrebo,[4] and it indicates that if the primary particles producing the showers are charged, nearly all must have energies greater than 60 billion electron volts. This result is predicted by the hypothesis[7] that the showers arise principally by cascade multiplication from single high energy electrons near the top of the atmosphere, having an energy spectrum of the form $E^{-1.8}$. It has been shown[8] that showers detected by

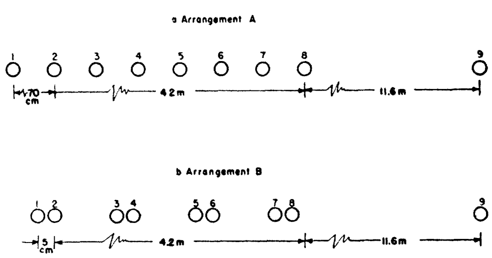

FIG. 2. Side view of coincidence counter arrangement. The following circuit connections were used: Circuit 1—counters 1, 5, 7; circuit 2—counters 3, 5, 7; circuit 3—counters 2, 6, 8; circuit 4—counters 4, 6, 8; circuit 5—counters 6, 8, 9.

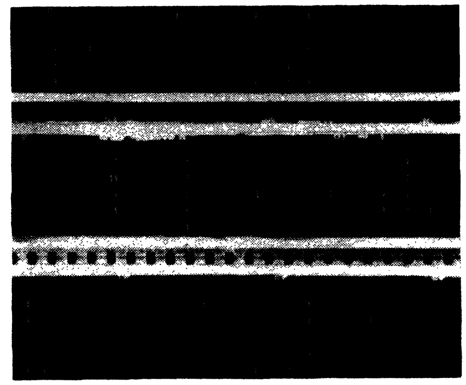

FIG. 3. Typical record of coincidences.

counters of the area used in these experiments would be produced by single electrons of energy greater than 10^{12} ev.

Although the earth's magnetic field should not produce an observable latitude effect upon the large showers, a small variation in counting rate might be expected on the basis of cascade theory, because of differences in air temperature at the same pressure at different latitudes. An increase of temperature at constant pressure reduces the air density and increases the lateral spread of the shower electrons. It can be shown from the single electron hypothesis that the counting rate of a coincidence counter arrangement measuring air showers should be roughly proportional to $\rho^{2(\delta-1)-\alpha}$, where δ is the exponent in the integral density spectrum of the showers falling at a given position, ρ is the air density, and $\alpha=(-d \ln C/d \ln L)$, where C is the counting rate and L is the separation of the counters. From Fig. 7, it is estimated that α has a value of the order of 0.2 for counters of about 2.8 m spread at 9200 m altitude. At the same elevation, δ was found to be 1.73 (see Table III), so that the counting rate should vary roughly as $\rho^{1.3}$ or as $T^{-1.3}$, where T is the absolute temperature of the air. Thus, for a temperature variation of 15°C at 9200 m, the counting rate might vary by approximately 8 percent.

The above latitude flight was made in early June, when the sun was overhead at a geomagnetic latitude near 30° north. The outside air temperature, which was indicated by the airplane thermometer, is plotted as a function of latitude in Fig. 4. Also plotted are the average counting rates for each latitude zone of twenty degrees. A curve is drawn showing the expected variation of counting rate due to temperature changes. It is clear that considerably more data are needed to establish the presence or absence of a correlation with temperature.

IV. DENSITY OF AIR SHOWERS

Table II shows the number of counts of the various combinations of circuits, obtained at 9200 m with the arrangement of Fig. 2a. The listed values have been

[6] These data are contained in Report No. 538 of the National Advisory Committee for Aeronautics.

[7] This assumption will be referred to hereafter as the "single electron hypothesis."

[8] Mark M. Mills, thesis (California Institute of Technology, 1948—to be published).

corrected for deadtime errors. The combinations of coincidences which do not include circuit 5 all consist of counters having a spread between 2.8 and 4.9 m. Comparison of the counting rates of circuits 1 and 3 with those of circuits 2 and 4 shows that the counting rate of threefold coincidences is not highly sensitive to variations in counter spread within this range. Comparison of the counting rates for sixfold coincidences of somewhat different geometry (circuit combinations 13, 24, 14, and 23, in Table II) shows a similarly slight dependence upon total spread of the counters. It is therefore believed that at these high altitudes a large proportion of the recorded showers do not have strong variation of particle density over distances of the order of 3 to 5 m. If the integral density spectrum of the showers falling at a given point has the form $K(\Delta)^{-\delta}$, where K is a constant and Δ is the particle density, it is possible to compute the ratios of higher-fold coincidence counts to threefold coincidence counts, assuming that the separations between the counters may be neglected, so that the same density occurs at each counter. The counting rate of an n-fold coincidence set is given[9] by:

$$C_n=\delta\int_0^\infty(1-e^{-\Delta A})^n K\Delta^{-\delta-1}d\Delta, \quad (1)$$

where A is the counter area. An integration of this expression by parts,[10] and expansion of the binomial under the integral sign yields the formula:

$$C_n=nA^\delta[\Gamma(1-\delta)]\sum_{i=1}^{n}\frac{\lfloor n-1}{\lfloor i-1\,\lfloor n-i}(-1)^{i-1}(i)^{\delta-1}, \quad (2)$$

where Γ is the well-known gamma-function. The same method may be applied to calculate the counting rate of independent showers which discharged any one of the circuits 1, 2, 3, and 4, of Fig. 2a or any combination of these circuits. The formula is:

$$C_{1,2,3,4}=4A^\delta[\Gamma(1-\delta)][2\cdot 2^{\delta-1}-8\cdot 4^{\delta-1}+5\cdot 5^{\delta-1}+6\cdot 6^{\delta-1}-7\cdot 7^{\delta-1}+2\cdot 8^{\delta-1}]. \quad (3)$$

Curves of C_n/C_3 plotted against n, for different values of δ, are shown in Fig. 5. Plotted on the same figure are the experimental ratios taken from the data in Table II. A curve representing a constant δ-value can be drawn in agreement with the experimental points. Now, although Lewis[11] has shown that the separation of coincidence counters measuring showers cannot be neglected entirely at high altitudes, Mills[8] has computed an "effective" density spectrum in the form of a power law for counters with a separation of 2.8 m, up to an altitude of about 12,000 m. The experimental data plotted on Fig. 5 are considered to determine an effective δ-value for counters with a total spread of about four meters. At an elevation of 9200 m this value is 1.73.

Measurements were also taken using the arrangement shown in Fig. 2b, in which counters 1, 3, 5, and 7 were moved very close to counters 2, 4, 6, and 8, respectively. The distance between axes of the counters in a pair was then only 2 inches. This arrangement does not change the geometry of the threefold and fourfold coincidences, but it might increase the counting rate of sixfold and higher order coincidences, if marked spatial correlation, or clustering, were present in the showers. The points plotted on Fig. 5 show close agreement of the data from the two different arrangements, for all orders of coincidence. Hence there is no evidence of marked clustering in the spatial distribution of particles at these altitudes. Also, not many shower particles appear to enter the counters at angles greater than 60 degrees with the zenith, since such particles could pass through two counters at once in the closely spaced arrangement of Fig. 2b, and should have increased the number of higher order coincidences.

TABLE I. Latitude dependence of coincidence counting rates.

Latitude range	0–30 deg. N	30–63 deg. N
Total time in minutes	1063	1015
Threefold counts with average counter spread of 3.5 m (average of circuits 1, 2, 3, and 4)	24.6±0.8 per hr.	23.3±0.7 per hr.
Eightfold coincidences with counter spread of 4.9 m (simultaneous counts of circuits 1, 2, 3, and 4)	5.4±0.5	5.2±0.5
Threefold coincidences with counter spread of 13 m (circuit 5)	17.2±1.0	15.8±0.9
All independent counts or combinations of counts of circuits 1, 2, 3, and 4	59.0±1.8	55.4±1.8

Note: Errors listed are standard deviations due to statistical fluctuations.

Data taken at altitudes of 720 m and of 11,000–12,000 m are also shown in Fig. 5. Figure 6 shows coincidence ratios for counter combinations including counter nine, having a total spread of 13 to 16 m.

Table III summarizes the experimental values of δ obtained at different altitudes, compared with theoretical values deduced by Mills from the single electron hypothesis. The values of δ increase with altitude, as predicted by theory. However, the experimental values are lower than the theoretical values at high altitudes. Some of this difference may be due to the material above the counters, which averaged 1.8 g per cm^2 of duralumin and glass. Another factor is the approximation involved in assuming uniform density over all counters. Because of this approximation, it does not

[9] See, for example, Cocconi, Loverdo, and Tongiorgi, Phys. Rev. **70**, 841 (1946).
[10] Several authors have published calculations of this type. Compare A. Migdal, J. Phys. U.S.S.R. **9**, 183 (1945); J. Daudin, J. de phys. et rad. **8**, 301 (1947); Mark M. Mills, reference 8; J. E. Treat and K. I. Greisen, Phys. Rev. **74**, 414 (1948).
[11] H. W. Lewis, Phys. Rev. **73**, 1341 (1948).

appear that the differences between the theoretical and experimental δ-values at high altitudes are significant.

V. LATERAL SPREAD OF AIR SHOWERS

A brief measurement was made of the threefold coincidence counting rate at 9200 m, with counter spreads shorter than 2.8 m. The principal purpose of these measurements was to determine whether any local showers were affecting the threefold rate of the counters with a spread of 2.8 m. The arrangement is shown in Fig. 7. Two counters were placed at a fixed separation of 1.4 m, and a third counter was placed at varying distances (D) from a point midway between the fixed counters. The counting rate is plotted in Fig. 7 as a function of the distance D. The increase in counting rate, as D is changed from 2.1 m to zero, is 12 percent. The small size of this increase indicates that at 9200 m altitude no appreciable fraction of the counts recorded when D equals 2.1 m are caused by local showers generated in the airplane.

The variation of coincidence counting rate with D at an altitude of 720 m is also plotted on Fig. 7, after multiplication of the counting rates at the lower altitude by a constant factor to facilitate comparison with the data at 9200 m. The counting rate decreases as D is increased from 2.1 m to 12.3 m, by 33 percent at 9200 m, and by 21 percent at 720 m. It appears that the decrease of shower frequency with increasing counter separation is somewhat more rapid at high elevations than near sea level, although the observed difference is subject to considerable statistical error.

This result has been predicted qualitatively by Lewis from the single electron hypothesis. He showed that the axes of the recorded showers will fall nearer to the counter arrangement at high altitudes, and therefore the counters should be in a region of more rapidly varying particle density. As a result, the counting rate

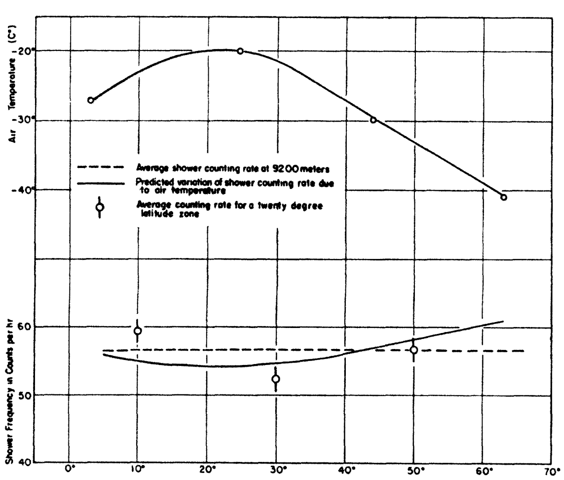

FIG. 4. Dependence of shower frequency and of air temperature on geomagnetic latitude at altitude of 9200 m.

TABLE II. Counting rates of the combinations of circuits in arrangement A at 9200 meters, for all latitudes between 0°N and 63°N geomagnetic latitude.

Circuit combination	Counts[a]	Order of coincidence	Average counting rate per hour[b]	Standard deviation
1	779			
2	855	3	24.0	±0.7
3	777			
4	806			
5	555	3[c]	16.5	±0.7
12	455			
34	433	4	13.4	±0.5
35	308			
45	291	4[c]	9.8	±0.5
345	213	5[c]	6.5	±0.4
13	249			
23	269	6	7.9	±0.4
14	252			
24	262			
15	169	6[c]	5.4	±0.4
25	179			
123	212			
124	209	7	6.5	±0.4
134	201			
234	213			
135	142			
235	152	7[c]	4.4	±0.35
145	134			
245	138			
1234	177	8	5.6	±0.4
1235	123			
1245	116	8[c]	3.8	±0.3
1345	120			
2345	126			
12345	107	9[c]	3.4	±0.3
Independent combinations of 1, 2, 3, and 4	1955	—	56.5	±0.3

[a] For a total time of 2078 minutes.
[b] Corrected for deadtime errors.
[c] Total counter spread is 13–16 m.

should decrease more rapidly with increased counter separation at high altitude.

VI. ALTITUDE DEPENDENCE OF AIR SHOWERS

The altitude variation of the counting rate of the counter arrangement of Fig. 2b was measured between 4600 m and 12,300 m (15,000 to 40,000 feet) at 41°N. The counting rate in the lower atmosphere was measured at 720 m pressure-altitude, at China Lake, California. Table IV gives the rates observed for a threefold arrangement with total spread of 2.8 m (circuits 2 and 4). The values are plotted on Fig. 8, together with theoretical altitude curves calculated by Mills and by Lewis. The theoretical curves were normalized to fit the experimental point at 720 m. Also plotted are measurements previously obtained by Kraybill and

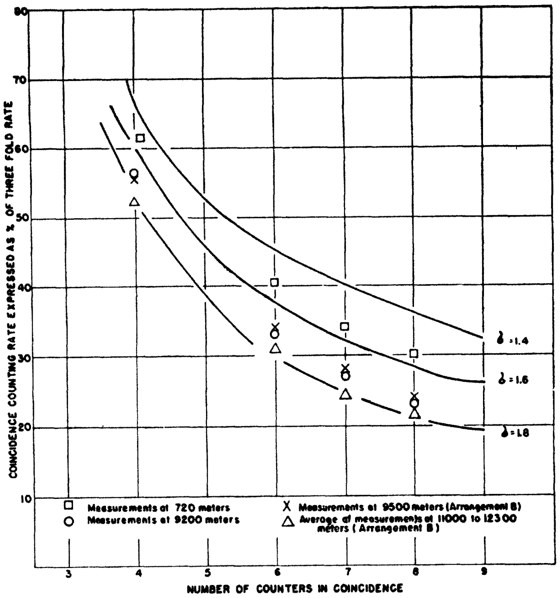

FIG. 5. Dependence of coincidence counting rate upon the number of counters in coincidence, for counter spreads of 2.8 to 4.9 m.

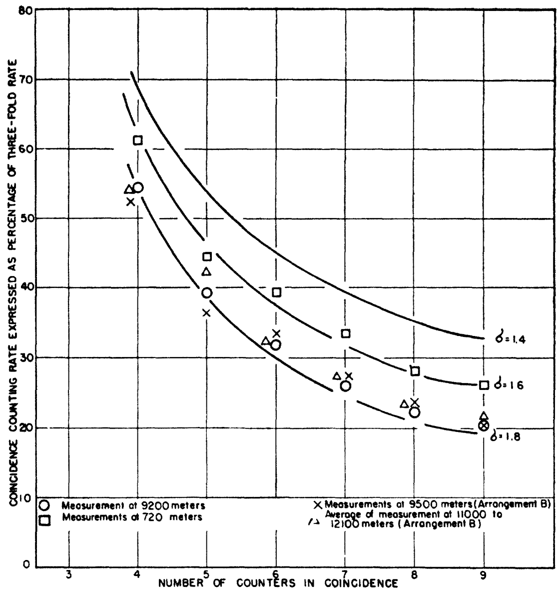

FIG 6. Dependence of coincidence counting rate upon the number of counters in coincidence, for counters with total spread of 13.0 to 16.5 m.

Ovrebo at Wright Field, Ohio (51°N), using a different threefold geometry, with total spread of 3.5 m. The data at Wright Field were multiplied by a constant factor of 0.82, in order to correct the ground counting rate for the different geometry and slightly larger counter area (80 cm^2). The rates at high altitudes are then in reasonable agreement with the more recent observations. The later measurements are considered more reliable, since they represent the average of two separate sets of counters with identical geometry, which gave consistent counting rates.

The altitude data for threefold coincidence counters with a total spread of 13 m (Fig. 2, circuit 5) are also plotted in Fig. 8. The counting rate at 9200 m for the 13 m counter spread is 24 times the rate at 720 m. The counting rate at 9200 m for counters with 2.8 m spread is 28 times the rate at 720 m. There is no indication that the shape of the curve for the counters with large extension differs at high altitudes from the curve for counters of shorter extension.

The theoretical curves in Fig. 8 represent independent calculations by Lewis and by Mills for threefold counter arrangements, assuming electrons incident at the top of the atmosphere with a power law energy spectrum of the form $E^{-1.8}$. Lewis used an analytical method to calculate the counting rate of three isotropically sensitive counters separated by a negligibly small distance. His assumptions appear to be applicable to the author's experimental arrangement in the lower half of the atmosphere. In regions less than 11 radiation units below the top of the atmosphere, he considers his values to be higher than actual counter measurements would indicate, because of his neglect of the finite separation of the counters.

Mills has made a careful numerical computation specifically designed for the threefold counter arrangement used by this author, at atmospheric depths between 5 radiation units and 30 radiation units. He used the following expression for the total number (N) of electrons in a shower of energy E_0, at a depth of t radiation units:[12]

$$N(E_0, t)=\left[\frac{1}{(2\pi)^{\frac{1}{2}}}\right]\left(\frac{1}{s}\right)\frac{H_1(s)K_1(s,-s)}{[\lambda_1''(s)t+1/s^2]^{\frac{1}{2}}} \times\left(\frac{E_0}{\epsilon}\right)^s \exp[\lambda_1(s)t]. \quad (4)$$

TABLE III. Theoretical and experimental values of the exponent in the air shower density spectrum.

Altitude	720 m	9200 m	9500 m	11,000–12,000 m
Theoretical value of δ computed by Mills for counters of 2.8 m spread	1.41	1.94	1.95	2.01
Experimental values of δ obtained from Fig. 5 for counters of about 3.5 m spread	1.53±0.07	1.73±0.04	1.72±0.04	1.82±0.07
Experimental value of δ computed from the ratio $\frac{C_{1234}}{C_3}$	1.51±0.05	1.73±0.03	1.70±0.03	1.76±0.06
Experimental values of δ obtained from Fig. 6 for counters of 13 to 16 m spread	1.58±0.08	1.75±0.05	1.75±0.05	1.74±0.09

Note: The indicated errors are standard deviations due to statistical fluctuations of a single point on the graph. The values were actually obtained from consideration of several partially interdependent points. Therefore, actual statistical errors should be somewhat less than those listed above.

[12] The symbols of Eq. (4) are those used in reference 14. The letter s denotes the well-known parameter of cascade theory. It is expressed implicitly in terms of E_0 and t by the equation

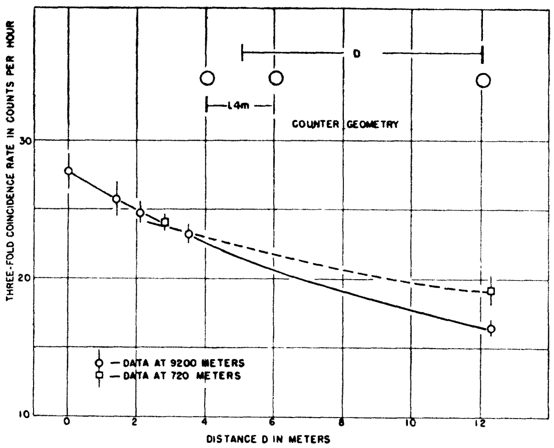

FIG. 7. Threefold coincidence counting rate at 9200 m as a function of separation of counters.

This formula is based upon the work of Snyder and Serber[13] and is given in a review article by Rossi and Greisen.[14] For the lateral distribution of the shower electrons at the maximum of the shower, he made an empirical approximation to the curve computed by Moliere.[15] This lateral distribution, for showers not at their maximum, was modified[16] by the factor $1/r^{1-s}$. The density spectrum produced by showers striking a single point at a given depth in the atmosphere was then calculated, for particles incident from the zenith direction. Mills modified this vertical density spectrum by an approximate method to take account of the counter separation. The result was an "effective" density spectrum for showers incident from one single direction, in which the same density was considered to exist at each of the three counters. This effective density spectrum for showers from a single direction was then integrated over all directions in the hemisphere about the zenith direction, taking into account the increase in effective thickness of the atmosphere with zenith angle, and the change in counter separation and effective area of the counters with incident direction of the shower. The result of this integration could be represented as a power law integral density spectrum for showers striking a point in the atmosphere from all directions. From this spectrum the counting rate of three counters was calculated, according to the formula (1).

The theoretical curve of Mills, calculated for counters with 83 cm² area and 2.8 m spread, is shown as the solid line in Fig. 8. The dotted line shows the curve computed by Lewis for 100 cm² counters and negligible separation. The theoretical curves agree rather closely in general shape. The difference in the height of the maximum in the two curves is due partly to the larger counter area assumed by Lewis. For counters of 80 cm² area, the height of the maximum of Lewis' curve should be reduced by about 8 percent. A more important factor in the difference between the two curves lies in Lewis' neglect of the finite separation of the counters.

The experimental increase in counting rate at high altitudes is about 2.6 times the increase predicted by Mills' calculation. Mills has estimated that neglect of fluctuations and his approximate treatment of the effect of counter separation may cause his results to be too low at the maximum of his curve by a factor of approximately 1.5. There is another factor, however, which causes the maximum in the curve of Mills, as well as the curve of Lewis, to be too low relative to the sea level value. Formula (4), which was used by Mills, expresses the number of electrons in a shower as a function of the thickness of matter in terms of radiation units, where one radiation unit (X_0) is defined by the expression:[17]

$$1/X_0 = 4\alpha(N/A)Z^2 r_0^2 \ln(183Z^{-\frac{1}{3}}). \qquad (5)$$

This formula is based upon the original calculations of Bethe and Heitler[18] for bremsstrahlung and pair pro-

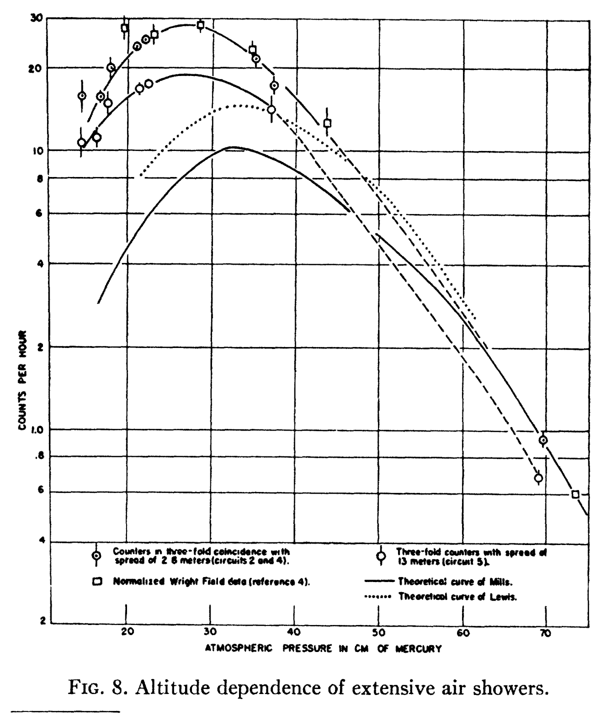

FIG. 8. Altitude dependence of extensive air showers.

$t = (-1/\lambda_1'(s))[\ln(E_0/\epsilon) - (1/s)]$. ϵ is the critical energy of electrons in air, and $\lambda_1(s)$, $H_1(s)$, and $K_1(s, -s)$ are functions tabulated in reference 14.

[13] H. Snyder, Phys. Rev. **53**, 960 (1938); R. Serber, Phys. Rev. **54**, 317 (1938).

[14] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

[15] G. Molière, Naturwiss. **30**, 87 (1942); W. Heisenberg, *et al.*, *Vorträge über Kosmische Strahlung* (Verlag, Julius Springer, Berlin, 1943; Dover Publications, New York, 1946).

[16] Here r denotes the lateral distance from the axis of the shower, in terms of "lateral units," as defined in *Cosmic Radiation* (Dover Publications, New York, 1946). Compare Pomeranchuk, J. Phys. U.S.S.R. **8**, 17 (1944); A. Migdal, reference 12.

[17] Here α denotes the fine structure constant, $r_0 = e^2/mc^2$, Z is the atomic number, A is the atomic weight, and N is Avogadro's number. X_0 is expressed in grams per cm².

[18] H. Bethe and W. Heitler, Proc. Roy. Soc. **146**, 83 (1934).

duction processes in the Coulomb fields of nuclei. It fails to take account of the contribution of the fields of the atomic electrons to these processes. It has been suggested[19] that the effect of the atomic electrons may be approximately accounted for by re-defining the radiation unit in terms of the following formula:

$$1/X_0=4\alpha(N/A)Z(Z+1)r_0^2\ln(183Z^{-\frac{1}{3}}). \quad (6)$$

The form of expression (4), giving the total number of electrons in a shower in terms of radiation units, is then left unchanged. But the number of radiation units in the atmosphere is increased from 24 to 27.5. The effect of this change upon the theoretical curves plotted in Fig. 8 will be to place the maxima of the curves at slightly higher altitudes, and to increase the value of the calculated counting rate at the maximum, compared with the value at sea level. Both of these effects tend to counteract the discrepancies between the theoretical and experimental curves in Fig. 8.

VII. DISCUSSION

The experimental results indicate that the extensive air showers at altitudes of 9200 m have lower average density than the showers near sea level. The density of showers detected by counter spreads of 13 to 16 m at 9200 m altitude does not differ greatly from that of showers detected by counter spread of 3 to 5 m. The decrease in the number of recorded showers with increasing counter separation is slightly more rapid at 9200 m than at sea level. The large showers do not exhibit an observable latitude effect at 9200 m altitude. These effects are predicted qualitatively by the hypothesis that the air showers arise from single electrons near the top of the atmosphere with an energy spectrum of the form $E^{-1.8}$.

The frequency of large air showers as recorded by coincidence counters has been shown to have a maximum value near an altitude of 8000 m, for threefold counters with a total spread of 2.8 m. The altitude curve for counters of 13 m extension is similar to the curve for counters with shorter spread, except for a slightly lower ratio of the counting rate at high altitudes to the rate at sea level. Theoretical calculations based upon the single electron hypothesis also lead to an altitude curve having a maximum in the upper atmosphere. The above experimental results do not indicate a mechanism for the large air showers essentially different from that of an electron cascade arising from one or a few particles starting near the top of the atmosphere.

TABLE IV. Altitude dependence of threefold coincidence counters with spreads of 2.8 m and of 13 m (circuits 2 and 4, and circuit 5, respectively).

Altitude	Pressure	Geomagnetic latitude	Time	Counting rates of spread of 2.8 m	Threefold coincidences spread of 13 m
720 m	69.9 cm	41 N	20820 min.	0.88±0.04 hr.$^{-1}$	0.70±0.05 hr.$^{-1}$
5650	37.3	41 N	158	17.5±1.8	13.5±2.3
6130	34.9	41 N	203	22.0±2.0	data missing
9200	22.6	9–63 N	2078	24.8±0.7	16.6±0.7
9500	21.5	35–50 N	1474	24.2±0.8	16.1±0.9
11000	17.1	0	266	20.0±1.8	14.7±1.9
11400	16.2	41 N	556	16.0±1.0	11.0±1.1
12300	14.5	41 N	97	16.1±2.3	10.6±2.6

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[19] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939); E. Clemental and G. Puppi, Nuovo Cimento **4**, 277 (1947). Compare also J. A. Richards and L. W. Nordheim, Phys. Rev. **74**, 1106 (1948); R. W. Williams, Phys. Rev. **74**, 1689 (1948).

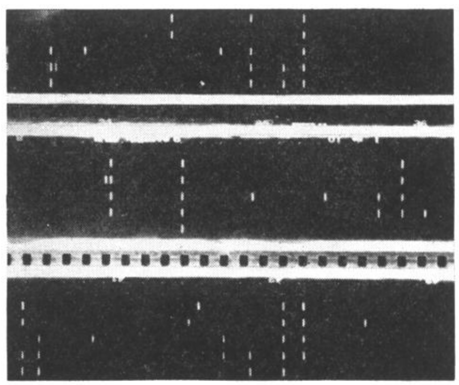

FIG. 3. Typical record of coincidences.