Transient Nutations in Nuclear Magnetic Resonance*

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Transient nutations of the resultant nuclear magnetic moment vector are set up by applying radiofrequency power in the form of pulses in the neighborhood of resonance ($\omega=\gamma H_0$). The nutations have an initial amplitude depending on the state of magnetization at the start of a pulse and on the proximity to resonance, and are damped by spin-spin and spin-lattice interaction. The thermal relaxation time can be directly found by observing the dependence of initial amplitude on the time between pulses. The spin-spin time constant T_2 can be found from the rate of decay even in the presence of normally disturbing inhomogeneity in magnetic field. Sensitivity is in many cases comparable to that obtained in the modulation method with narrow band amplifiers. The fast response due to the relatively wide band widths used can be applied to a rapid search for unknown resonances. The effects observed are in qualitative accord with predictions based on the Bloch theory.

I. INTRODUCTION

SINCE the original work on the detection of nuclear magnetic resonance in bulk matter[1,2] many improvements in technique and method have been introduced by various workers.[3–6] In most of the methods currently in use a steady state is set up which may be described as an equilibrium resulting from the competition between the applied radiofrequency magnetic field on the one hand which tends to equalize the populations of the Zeeman levels and thus to demagnetize the system and on the other hand the effect of the interactions between the spins and the thermal motions of the nuclei which tends always to restore the system to thermal equilibrium as governed by the Boltzmann distribution.

In case absorption is being detected an optimum radiofrequency power exists which gives maximum signal strength; in the case of dispersion a maximum signal results for large radiofrequency power. As Bloch[7] has shown the two maxima are the same being given in his notation by [8,9]

$$-v_{\max}=u_{\max}=\tfrac{1}{2}(T_2/T_1)^{\frac{1}{2}}M_0. \quad (1)$$

In many cases $T_2/T_1\ll 1$ and the maximum values of u and v are much less than M_0.

The steady state may be attained either by a very slow approach to resonance in which case the steady state conditions may be assumed to hold at each instant of time, or by a relatively rapid approach to the resonant conditions in which case transient motions of the magnetic moment vector will be set up eventually decaying to a steady state motion in a time determined by the relaxation constants T_1 and T_2. During these transients u and v may attain values much larger than the steady state maximum (1), approaching in fact the value M_0. It appears therefore that considerably larger signal strengths are in many cases available through such transients. This does not of course imply that more sensitivity may be expected to result from utilization of the transients, since the fact that they last for only finite times limits the minimum band widths that may be used in the amplifying and detecting circuits and thus limits the minimum noise power which determines the ultimate sensitivity. Nevertheless, as we shall show (Section VII), the sensitivity to be expected from detection of the transients is often comparable and in some cases superior to that obtainable from the steady state even with the use of band widths in the latter case as small as 10^{-1} cycle/sec.

Quite aside from the question of sensitivity the study of the transients appears capable of yielding, in a straightforward manner, data on the relaxation constants T_1 and T_2 which allows their separate evaluation. Furthermore, it appears to be possible by study of the transients in certain circumstances to measure T_2 even if this quantity cannot be determined from the steady state because its effect on the resonance width is obscured by inhomogeneities in the magnetic field. Finally the high sensitivity obtainable from the transients even with band widths of 10^3 cycles/sec. or more seem to offer some promise in their utilization in the search for unknown resonances, since the search can be conducted in far shorter times than is possible in the steady state case. We have made theoretical and experimental investigations of these transients and although our results so far are preliminary they seem to have sufficient interest to merit publication in advance of a more exhaustive study.

Before analyzing the effects that have been expected and observed from the transients under review it would

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[1] Purcell, Torrey, and Pound, Phys. Rev. 59, 37 (1946).
[2] Bloch, Hansen, and Packard, Phys. Rev. 69, 127 (1946).
[3] Bloembergen, Purcell, and Pound, Phys. Rev. 73, 679 (1947).
[4] R. V. Pound, Phys. Rev. 72, 527 (1947).
[5] A. Roberts, Rev. Sci. Inst. 18, 845 (1947).
[6] J. R. Zimmerman, and D. Williams, Phys. Rev. 73, 94 (1948).
[7] F. Bloch, Phys. Rev. 70, 460 (1946).
[8] In Bloch's paper the factor $\frac{1}{2}$ is missing evidently due to a computational error.
[9] This notation is explained in Section II of this paper.

be well to distinguish them from certain transient effects which have already been discussed and in some cases observed by other writers.

(a) In the introduction to his paper Bloch[7] suggests the possibility of observing nuclear induction in the absence of a r-f field by first disorienting the resultant nuclear moment with a strong field pulse. When this driving pulse has disappeared a signal resulting from the free precession of the nuclear moment should be observable. This effect to the best of our knowledge has not yet been observed. It differs from the transient effects treated here in that the latter occur before rather than after the driving pulse is extinguished; thus the motion of the nuclear moment vector is in the present case forced rather than free.

(b) In the same paper Bloch discusses in some detail the effects to be expected from an approach to resonance which is fast in comparison to the relaxation times T_1 and T_2 but slow in comparison to the nutation time $(\gamma H_1)^{-1}$ of the magnetic moment vector. This case gives rise to a nuclear induction which attains a maximum value near M_0 at resonance. The signal strength is thus large, but its duration is necessarily short compared with T_2. The theory of this effect is based on an approximate solution of Bloch's differential equations which is of such complexity as to make a detailed interpretation of experimental results in terms of the relaxation constants somewhat difficult. Bloch, Hansen, and Packard[10] have, however, obtained experimental results which show at least a qualitative similarity to the effects expected. In contrast, the effects considered here result from a sudden approach to the resonant condition taking place in a time short compared both to $(\gamma H_1)^{-1}$ and to T_2 and T_1. Once the resonant condition is thus established, the effective external parameters of the theory H_1, ω and H_0 are kept constant. The Bloch equations take in this case the form of linear equations with constant coefficients and are thus susceptible to exact solution in terms of exponential and sinusoidal functions of time, making the interpretation of the data relatively simple.

(c) Another transient effect was observed by Bloembergen, Purcell, and Pound[3] and called by them "the wiggles." It has been analyzed theoretically by Jacobsohn and Wangness.[11] It bears no relation to the effects considered here.

II. THEORY

The theory of the transients will be based on the general macroscopic theory of Bloch.[7] This theory (as pointed out by Bloch) is only semi-quantitative, but it has virtue of simplicity. It does not seem feasible at this time to undertake a more rigorous treatment. Not the least interest, perhaps, in an experimental investigation of the transients will be found in an elucidation of the limitations of the Bloch theory.

It is supposed that a sample of material with non-vanishing nuclear moments is placed in an applied magnetic field consisting of two components: a steady component of magnitude H_0 oriented in the z-direction and an oscillating component $2H_1\cos\omega t$ in the x-direction. As is well known the oscillating component can be further decomposed into two circularly polarized components in the x–y plane rotating in opposite directions about the z-axis. Only one of these, namely that one rotating in the same sense as the free Larmor precession of the nuclear moment, is effective in disorienting the nuclear moment; the other can and in fact will be completely ignored. The resultant nuclear moment M of the sample may be resolved into three components: M_z in the z-direction, u parallel to the effective rotating component of the magnetic field and v orthogonal to M_z and leading the effective rotating field component by 90°.

The time dependence of these components of magnetic moment is governed in the Bloch theory by the differential equations:

$$du/d\tau+\beta u+\delta v=0, \tag{2a}$$

$$dv/d\tau+\beta v-\delta u+M_z=0, \tag{2b}$$

$$dM_z/d\tau+\alpha M_z-v=M_0, \tag{2c}$$

where

$$\begin{aligned} &\tau=\gamma H_1 t, && \delta=(\omega_0-\omega)/\gamma H, \\ &\beta=1/\gamma H_1 T_2, && \omega_0=\gamma H_0, \\ &\alpha=1/\gamma H_1 T_1, && M_0=\chi_0 H_0, \end{aligned} \tag{3}$$

t is the time, γ the absolute value of the nuclear gyromagnetic ratio, and χ_0 the static susceptibility. The above notation is the same as that of Bloch, except that we use γ in place of his $|\gamma|$.

It is shown in the appendix that if w stands for any one of u/M_0, v/M_0, or M_z/M_0 the general solution of these equations may be written in the form

$$w=Ae^{-a\tau}+Be^{-b\tau}\cos s\tau+\frac{C}{s}e^{-b\tau}\sin s\tau+D. \tag{4}$$

The first three terms in (4) give the transient effects and the last term the steady state. The values obtained for D must of course agree as in fact they do with the Bloch steady state solution.[12] The constants a, b, s, A, B, C, and D of Eq. (2) are functions of α, β, and δ. Of these constants a, b, and s are independent of the initial conditions and are the same for all components of M; A, B, C, and D depend on which component w stands for, and all except D depend on the initial values u_0, v_0, and $(M_z)_0$ of u, v, and M_z. The factors a, b, and s and the coefficients A, B, C, and D are evaluated in the appendix, implicitly in the general case and explicitly in the special cases (a) $\delta=0$, i.e., the case of exact

[10] Bloch, Hansen, and Packard, Phys. Rev. **70**, 474 (1946).

[11] B. A. Jacobsohn and R. K. Wangness, Phys. Rev. **73**, 1509 (1948).

[12] See Eq. (56) in reference 7.

resonance; (b) $\alpha=\beta$ or $T_1=T_2$; and (c) in case the radiofrequency magnetic field H_1 is sufficiently large so that $\alpha\ll 1$ and $\beta\ll 1$.

In the following section the method now being used for experimental observation of the transients is outlined. As explained there the quantity directly observed is v as a function of time. The Bloch theory developed in the appendix gives for v in the three cases (m_0 is the initial value of M_z/M_0; it is assumed that $u_0=v_0=0$):

(a) ($\delta=0$)

$$-v/M_0=\frac{\alpha}{1+\alpha\beta}\left[1-e^{-(\alpha+\beta)\tau/2}\cos\left(1-\left(\frac{\alpha-\beta}{2}\right)^2\right)^{\frac{1}{2}}\tau\right]+\left[m_0-\frac{\alpha(\alpha+\beta)}{2(1+\alpha\beta)}\Big/\left(1-\left(\frac{\alpha-\beta}{2}\right)^2\right)^{\frac{1}{2}}\right]\times e^{-(\alpha+\beta)\tau/2}\sin\left(1-\left(\frac{\alpha-\beta}{2}\right)^2\right)^{\frac{1}{2}}\tau, \quad (5)$$

(b) ($\alpha=\beta$)

$$-v/M_0=\frac{\beta}{1+\beta^2+\delta^2}[1-e^{-\beta\tau}\cos(1+\delta^2)^{\frac{1}{2}}\tau]+\frac{1}{(1+\delta^2)^{\frac{1}{2}}}\left(m_0-\frac{\beta^2}{1+\beta^2+\delta^2}\right)e^{-\beta\tau}\sin(1+\delta^2)^{\frac{1}{2}}\tau, \quad (6)$$

(c) ($\beta\ll 1$)

$$-v/M_0=\frac{-\beta}{\delta^2+\beta/\alpha}-\frac{\delta(\beta-\alpha)^2}{1+\delta^2}\left(\frac{m_0}{1+\delta^2}-\frac{1}{\delta^2+\beta/\alpha}\right)e^{-a\tau}+\frac{1}{1+\delta^2}\left[\frac{(\beta-\alpha)\delta^2 m_0}{1+\delta^2}+\alpha\right]e^{-b\tau}\cos(1+\delta^2)^{\frac{1}{2}}\tau-\frac{m_0}{(1+\delta^2)^{\frac{1}{2}}}e^{-b\tau}\sin(1+\delta^2)^{\frac{1}{2}}\tau+O(\beta^2), \quad (7)$$

where

$$b=\beta-\frac{1}{2}\frac{\beta-\alpha}{1+\delta^2}+O[(\beta-\alpha)^3], \quad (8)$$

$$a=\frac{\beta+\alpha\delta^2}{1+\delta^2}+O[(\beta-\alpha)^3]. \quad (9)$$

If $m_0\sim 1$ the last term in the above expression for v is dominant and

$$v/M_0\cong-\frac{m_0}{(1+\delta^2)^{\frac{1}{2}}}e^{-b\tau}\sin(1+\delta^2)^{\frac{1}{2}}\tau, \quad (10)$$

or

$$v/M_0\cong-\frac{m_0\gamma H_1}{\Omega}\exp-\left[\frac{1}{T_2}-\frac{1}{2}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\left(\frac{\gamma H_1}{\Omega}\right)^2\right]t\times\sin\Omega t, \quad (11)$$

where

$$\Omega=(\gamma^2H_1^2+(\omega_0-\omega)^2)^{\frac{1}{2}}. \quad (12)$$

In cases (b) and (c) it will be seen that s is always real and thus the transients consist of damped oscillations superposed on an exponential decay; the latter (the term in A) vanishing at resonance. The oscillations can be interpreted as arising from a periodic nutational motion of the magnetic moment vector, damped by the relaxation processes.

It is obvious in fact that the transients must have this general form quite independent of any special predictions of the Bloch theory. In the absence of relaxation effects the motion of the magnetic moment vector is exactly determined by the second law of motion.[13]

$$d\mathbf{M}/dt=\gamma(\mathbf{M}\times\mathbf{H}). \quad (14)$$

Here $\mathbf{H}$ is the resultant magnetic field composed of H_0 (in the z-direction) and the effective rotating a.c. component H_1. Let us now transform to a system of axes rotating about the z-axis in the same sense as H_1 and with the same angular velocity ω. Let $D\mathbf{M}/dt$ be the velocity of $\mathbf{M}$ as viewed in the rotating system. Then by a familiar kinematical transformation we have

$$D\mathbf{M}/dt=d\mathbf{M}/dt+\mathbf{M}\times\boldsymbol{\omega},$$

or by (14)

$$D\mathbf{M}/dt=\gamma(\mathbf{M}\times\overline{\mathbf{H}}), \quad (15)$$

where

$$\overline{\mathbf{H}}=\mathbf{H}+\boldsymbol{\omega}/\gamma. \quad (16)$$

In the rotating system $\overline{\mathbf{H}}$ is constant in time and thus the solution of (15) can be stated at once: it consists in a uniform precession of $\mathbf{M}$ about $\overline{\mathbf{H}}$; the angle of the cone of precession being determined by the initial conditions. At the same time $\overline{\mathbf{H}}$ of course precesses about the z-axis with angular velocity ω. The motion of $\mathbf{M}$ about $\overline{\mathbf{H}}$ has the angular velocity

$$\Omega=\gamma|\overline{\mathbf{H}}|$$

or

$$\Omega=(\gamma^2H_1^2+(\omega_0-\omega)^2)^{\frac{1}{2}}. \quad (17)$$

The projection of this motion on the x-y plane will obviously result in oscillations in u and v with the angular frequency Ω. Near resonance $\Omega\ll\omega$ and the motion as viewed from the laboratory system appears as a slow nutation superposed on a rapid precession about the z-direction.

The effect of relaxation processes may be expected to take the form of a damping of these oscillations. The Bloch theory makes explicit predictions as to the form of this damping and also predicts (see Eqs. (5)–(12)) that the frequency of the oscillations is unaffected by relaxation in the case $T_1=T_2$ or in any case if $\beta\ll 1$. In

[13] Reference 7, Eq. (11).

some cases, however, e.g., if $\delta=0$ and $[(\beta-\alpha)/2]^2 \ll 1$ the damping may be sufficiently large so as to suppress the oscillations.

III. EXPERIMENTAL OBSERVATION OF THE TRANSIENT EFFECTS

It is clear that the transient situation may be set up by any approach to the resonant condition which is rapid compared to the nutation time $1/\gamma H_1$. To accomplish this any one of the three external parameters H_0, H_1, or ω may be varied with the time in the form of a pulse starting at say time $t=0$. In case H_0 or ω is varied: for time $t<0$ we must have $\delta=(\gamma H_0-\omega)/\gamma H_1 \gg 1$ and for $t>0$, δ should be of order unity or less. In case H_1 is pulsed, δ need not be large for $t<0$, but H_1 (for $t<0$) must be sufficiently small so that **M** is initially in the z-direction; at $t=0$, H_1 is increased to a relatively large value. There are experimental difficulties associated with the pulsing of any one of these parameters. In the case H_0 or ω is pulsed a very flat-topped rectangular pulse must be produced. It is especially difficult to get the required flatness with a sufficiently fast rise time in the case of H_0. On the other hand if H_1 or ω is pulsed there are difficulties associated with the response time of the radiofrequency circuits. In particular the inherent frequency sensitivity of radiofrequency bridge circuits is troublesome. However, in case H_1 is pulsed, the requirement of flatness of the pulse is considerably less severe than for H_0 or ω. For this reason we have chosen H_1 as the most suitable parameter for pulsing.

There is shown in Fig. 1 a block diagram of the apparatus now in use. In the interests of frequency stability we have chosen to pulse H_1 by gating a continuously operating oscillator. The oscillator is a R.C.A. 19427 which, although electron coupled, has excellent frequency stability. The gating modulator is shown in Fig. 2. It consists of a twin-T null network which is normally balanced to a null. Part of the effective conductance in the lower arm is provided by the output impedance of the 6J6 cathode follower. A negative gating pulse on the grid cuts the tube off and the impedance changes rapidly from 200 ohms to 500 ohms thus unbalancing the bridge.

The resulting radiofrequency pulse is amplified and fed into a conventional bridge circuit shown in Fig. 3. The primary of the transformer is adjustable in position relative to the two secondaries, by means of a micrometer head. A second balancing adjustment is provided by a small trimming condenser across one of the tuned circuits. These adjustments are nearly orthogonal. By symmetrizing the two arms of this bridge especially with regard to the Q's of the two parallel tuned circuits, the bridge can be made sufficiently broad band to pass the pulse without appreciable distortion. The bridge is ordinarily balanced in the range between 40 db to 60 db. Because of absorption and dispersion by nuclear moments in the coil containing the sample under investigation, the radiofrequency pulse put out by the bridge is amplitude modulated; the modulation voltage being proportional to u or to v or to some mixture of the two depending on the character of the bridge balance.[14] In all work to date the bridge has been balanced in phase so that the output modulation is proportional to v.

The modulated radiofrequency pulse put out by the bridge is amplified and detected by a IN34 germanium crystal rectifier. This rectifier is biased negatively so that it removes the small modulation from the relatively large pedestal due to the unbalance of the bridge. The detected signal is amplified and displayed on the vertical plates of a cathode-ray tube. The horizontal linear sweep of this tube is triggered by the leading edge of the d.c. pulse which gates the modulator. The vertical deflection gives directly the quantity v as a function of time. Typical results are shown in Figs. 4 and 5. Figure 4 shows $-v$ as a function of time for $\delta=0$ due to proton resonance in 1.5 cc of glycerine at room temperature, and Fig. 5 shows a corresponding result for the same volume of distilled water at room temperature. In these examples the sweep duration is 0.01 second, the radiofrequency is 9 mc/sec., $H_0=2100$ gauss, and the radiofrequency magnetic field H_1 is 0.17 and 0.15 gauss, respectively. We will return to a discussion of these curves in Section VI.

The pulse durations required depend of course on the particular application being made. If measurements of the decay time of the transient is desired and if inhomogeneity effects are unimportant the pulse length must be of the order of T_2 or longer. If measurements are to be made of the relaxation time T_1 the pulse should last until the transients have ceased i.e., for a time long compared with T_2 (the effect of inhomogeneity on these considerations is discussed in Section VI).

IV. MEASUREMENT OF THERMAL RELAXATION TIME

The pulse technique in connection with the transients lends itself readily to a direct and straight forward measurement of the thermal relaxation time T_1, i.e., the time required to bring the spin system into thermal equilibrium with the lattice. For example suppose that the radiofrequency field H_1 is large enough to satisfy the inequality

$$\gamma H_1 \gg 1/T_2, \text{ i.e., } \beta \ll 1, \ \alpha \ll 1.$$

The solution of the Bloch equations which holds in this case is given in the appendix. At resonance ($\delta=0$) we have for M_z

$$M_z/M_0=m_0 \exp\left[-\frac{1}{2}\left(\frac{1}{T_1}+\frac{1}{T_2}\right)t\right] \times\left(\cos\tau+\frac{\beta-\alpha}{2}\sin\tau\right)+O(\alpha\beta). \quad (18)$$

[14] For a discussion of this effect of bridge balance see reference 3.

If the pulse duration is sufficiently long compared with the damping time $2T_1T_2/(T_1+T_2)$, the ultimate value of M_z will be very small compared with M_0. That is to say, the sample will have become nearly completely demagnetized. During the interval between the end of one pulse and the start of the next the sample will regain some magnetization because of the relaxation process. If, then, m_0M_0 is the value of M_z at the start of a pulse we shall have

$$m_0=1-e^{-T/T_1}, \tag{19}$$

where T is the time between pulses.

If m_0 has a substantial value, i.e., if $m_0\sim 1$, then from (10) it is evident that the initial amplitude of v will be closely proportional to m_0. Thus by observing the dependence of the initial amplitude of v on the time T between pulses and making use of (19) the quantity T_1 can be directly measured.

V. MEASUREMENT OF T_2

The quantity T_2 is a measure of the time required for a nuclear spin to become disoriented as a result of interactions with neighboring nuclei or with paramagnetic ions. This quantity can be measured by its effect on the damping of the oscillations in v. From Eq. (5) we see that this damping time T_0 is given by

$$1/T_0=\tfrac{1}{2}(1/T_1+1/T_2) \tag{20}$$

at resonance.

Inhomogeneities in H_1 or H_0 can also give rise to a damping of the oscillations in v. If these effects are unimportant, T_2 can be directly found from (20) once T_1 is known. In many cases $T_1\gg T_2$ and a knowledge of T_1 is unnecessary. The effect of inhomogenities is considered in the next section.

VI. EFFECT OF INHOMOGENEITIES IN H_1 AND H_0

The most important effect of inhomogeneities in H_1 or H_0 is the resultant distribution of nutational frequency (see Eq. (17)) over the sample. The signal from the whole sample can be regarded as due to a superposition of signals from the several parts of the sample, each part being in an essentially homogeneous field. Signals from the several parts will differ somewhat in frequency because of the variation of $\omega_0=\gamma H_0$ over the sample. Since they start out all in phase the initial integrated amplitude is large, but after a few oscillations phase differences develop which damp the resultant signal.

Consider first the effect of an inhomogeneity in H_1. The frequency of oscillation of v (nutational frequency) at resonance is $\gamma H_1/2\pi$. Consequently an inhomogeneity in H_1 of order ΔH_1 may be expected to result in appreciable damping in a time of order $1/\gamma\Delta H_1$.

The effect of inhomogeneity in H_0 is somewhat different, however, since the frequency of oscillation does not depend on H_0 at resonance. In fact from (17) the angular frequency is

$$\Omega=(\gamma^2H_1^2+(\gamma H_0-\omega)^2)^{\frac{1}{2}}.$$

Thus if H_1 is large compared with the inhomogeneity of H_0 there will be relatively little variation in Ω over the sample. Investigation in fact shows that if a gaussian distribution in H_0 is assumed with full width at half maximum of ΔH_0 the signal resulting at resonance after integration over H_0 will be: (it is assumed that $\beta\ll 1$ and that $(H_1/\Delta H_0)^2\gg 1$)

$$v/M_0\cong\frac{-m_0}{[1+(t/T_H)^2]^{\frac{1}{2}}}\cdot e^{-t/T_0}\sin(\gamma H_1t+\varphi), \tag{21}$$

where T_0 is the natural damping time (Eq. (20)),

$$T_H=\frac{5.6H_1}{\gamma(\Delta H_0)^2}, \tag{22}$$

and φ is a slowly varying phase factor.

It is apparent from this expression that if (as assumed) $H_1\gg\Delta H_0$ we shall have $T_H\gg 1/\gamma\Delta H_0$. Consequently the damping due to H_0 can be made negligibly small for H_1 sufficiently large. Furthermore, it does not appear that this general qualitative result depends on any special assumptions of the Bloch theory.

Of course it does not pay to increase H_1 indefinitely, but only until the effect of inhomogeneity in H_1 is about that of H_0. By concentrating the sample near the center of the r-f coil the inhomogeneity in H_1 can be considerably reduced at the expense of signal strength. It

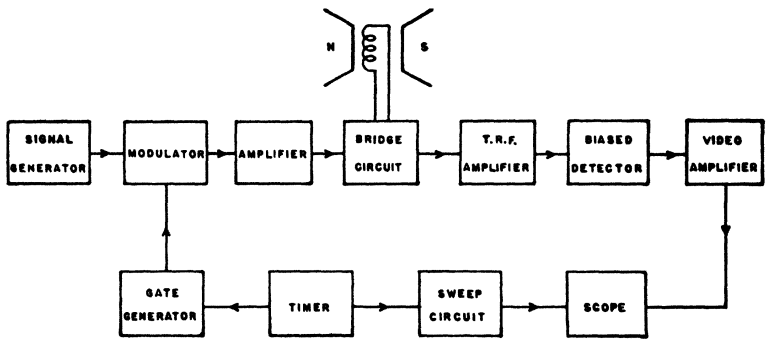

FIG. 1. Block diagram of circuitry used in observation of the transients.

would seem in fact that the effects of inhomogeneities can be reduced to the point where very much larger values of T_2 can be measured than can be found by line-width determinations.

This possibility has not been completely explored as yet. However, if we consider the observed damping in the case of distilled water (Fig. 5) it appears that the signal has been attenuated by about 25 percent in a time of 10^{-2} second. In this case since T_1 and T_2 are known[15] to be long compared with 10^{-2} sec. all the observed damping must be attributed to inhomogeneities. The actual inhomogeneity in H_0 in this case was found by a measurement of line width in the conventional manner to be 0.03 gauss. This gives for $1/\gamma\Delta H_0$ a value of 1.2×10^{-3} sec., a time very much shorter than the observed damping time. On the other hand the value of T_H (Eq. (22)) is 5×10^{-2} sec. A value of T_H as large as this would not produce appreciable damping in a time of 10^{-2} sec. according to Eq. (21). Thus, it would appear that most of the observed damping in this case must be attributed to inhomogeneity in H_1. This conclusion is strengthened by the observed fact that an increase in H_1 resulted in a shorter damping time contrary to the expected result if inhomogeneity in H_0 were responsible for the damping. Further evidence is provided by the marked increase in damping off resonance. This effect is discussed in Section VIII.

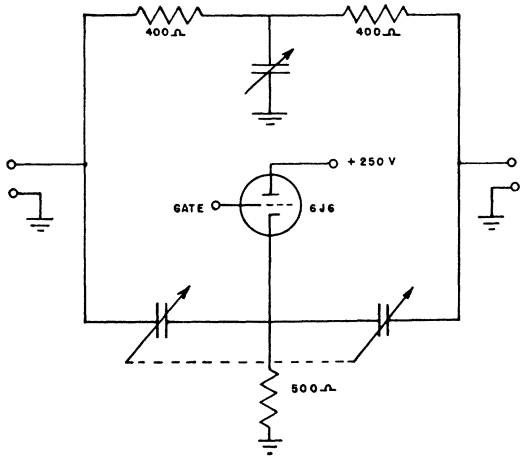

FIG. 2. Twin-T network used to pulse-modulate the radiofrequency power.

The damping in the case of glycerine (Fig. (4)) is seen to be considerably greater than for water, the extra attenuation being evidently due to natural damping. By comparing[16] the damping for glycerine with that for water for the same values of H_1 and H_0, we have obtained a value for the natural damping time for glycerine at room temperature of

$$T_0=2\left(\frac{1}{T_1}+\frac{1}{T_2}\right)^{-1}=1.1\times10^{-2}\text{ sec.}$$

The theory of Bloembergen *et al.*[3] leads to the conclusion that $T_1\sim T_2$ in this case. This assumption together with their experimental value of T_1 at room temperature is in reasonable accord with the observed value of T_0. We cite this datum not for any particular intrinsic interest but rather as an illustration of the potentialities of the pulse method in the measurement of abnormally large values of T_2.

The conventional method for the measurement of T_2 by line width observations, assuming our inhomogeneity of 0.03 gauss, would be limited to values of T_2 less than about 10^{-3} sec.

It is interesting to note that the effects of inhomogeneities can in a sense be suppressed in certain cases even if H_1 is smaller than the inhomogeneity in H_0. If for example $T_1\sim T_2$ and both T_1 and T_2 are sufficiently long (e.g., as in distilled water) so that $\beta\ll1$ and $\alpha\ll1$ even for $H_1\ll\Delta H_0$, the resultant signal to be expected can be obtained by integrating (see Eq. (5))

$$-v/m_0=\frac{m_0}{(1+\delta^2)^{\frac{1}{2}}}e^{-\beta\tau}\sin(1+\delta^2)^{\frac{1}{2}}\tau$$

over δ. The resultant signal will be

$$v_r=\int_{-\infty}^{+\infty}v(\delta)f(\delta)d\delta,$$

where $f(\delta)d\delta$ gives the distribution of δ over the sample due to inhomogeneity. In this case $f(\delta)$ varies slowly compared with v and

$$-v_r/M_0\simeq\pi f(0)m_0e^{-\beta\tau}J_0(\tau),$$

where $J_0(\tau)$ is the Bessel function of order zero.

The fact that the natural decay ($e^{-\beta\tau}$) is exponential while the decay due to the inhomogeneity given by the Bessel function $J_0(\tau)$ is slow ($\tau^{-\frac{1}{2}}$) permits their separation and the evaluation of β and hence of T_2.

VII. SENSITIVITY

It was mentioned in the introduction that considerably greater signal strengths are available in many cases from the transient than from the steady state resonance. On the other hand, the band width of the amplifying and detecting circuits are necessarily larger for the transients because of their finite durations. The ultimate sensitivity is determined by the ratio of signal-to-noise power at the indicator. We first estimate the amount of signal power available.

The tuned circuit of which the coil containing the sample forms a part will be assumed to have an unloaded quality factor Q_0 and an unloaded shunt con-

[15] Bloembergen *et al.* (reference 3) find $T_1\sim2$ sec. and predict that $T_2\sim T_1$.

[16] This comparison is possible since the two samples were located in the same position relative both to the magnet and to the radiofrequency coil.

ductance g. A change ΔV in the voltage across this circuit gives rise to an available power of

$$(P_s)_p=(\Delta V)^2/16g. \tag{23}$$

On the other hand if ΔV is caused by nuclear spin resonance

$$\Delta V=4\pi\chi''\zeta Q_0 V=2\pi Q_0 v\zeta(V/H_1), \tag{24}$$

where V is the undisturbed pulse r-f voltage, $\chi''=v/2H_1$ is the imaginary part of the nuclear magnetic susceptibility, ζ is the "filling factor" of the r-f coil (see appendix to reference 3) and v and H_1 have the meanings assigned to them in Section II. In case H_1 is sufficiently large so that $\gamma H_1 \gg 1/T_2$ and if the time between pulses $\gg T_1$, v (see Eq. (10)) consists of a damped sinusoid with initial amplitude of

$$|v_m|=M_0=\chi_0 H_0, \tag{25}$$

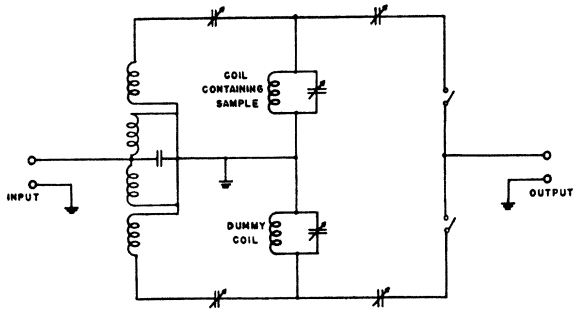

FIG. 3. Radiofrequency bridge circuit.

where χ_0 is the static susceptibility given by

$$\chi_0=N\gamma^2 I(I+1)\hbar^2/3kT, \tag{26}$$

where N is the number of spins per unit volume, I the nuclear spin, and T the absolute temperature.

The factor V/H_1 occurring in (24) may be found from the definition of Q_0:

$$Q_0=\omega W/P_0, \tag{27}$$

where

$$W=(1/2\pi)H_1^2 V_c \tag{28}$$

is the energy stored in the tuned circuit, V_c is the effective volume of the coil and

$$P_0=V^2/2g \tag{29}$$

is the power dissipated in the unloaded circuit. From (27) (28) and (29) we find

$$V^2/H_1^2=\omega V_c g/\pi Q_0. \tag{30}$$

From (23), (24), (25), (26), and (30) we obtain

$$(P_s)_p=\frac{1}{72}\zeta^2 Q_0 V_c N^2\gamma^2 I^2(I+1)^2\hbar^4\nu_0^3/k^2T^2. \tag{31}$$

Here $\nu_0=\omega/2\pi=\gamma H_0/2\pi$. Comparing (31) with the expression derived by Bloembergen *et al.*[17] for the

[17] Appendix to reference 3. We have put their factor $A=1$.

FIG. 4. Proton resonance in glycerine (exact resonance). Sweep time=0.01 sec., $H_1=0.17$ gauss, $\nu_0=9.0$ mc/sec.

available signal power $(P_s)_m$, in the modulation method we find

$$(P_s)_p=16(T_1/T_2^*)(P_s)_m. \tag{32}$$

Here T_2^* is the line width parameter which is equal to T_2 if the inhomogeneity in H_0 does not contribute to the line width and is of order $1/\gamma\Delta H_0$ if the inhomogeneity ΔH_0 is the major factor.

The factor $16T_1/T_2^*$ in (32) can be interpreted as arising in part from the fact that in the transient $v_m=M_0$ while in the steady state $v_m=\frac{1}{2}(T_2^*/T_1)^{\frac{1}{2}}M_0$ (Eq. (1)). This contributes a factor $4T_1/T_2^*$. The remaining factor of 4 is accounted for by the fact that in the modulation method the maximum signal occurs half-way down the resonance curve and thus has an amplitude of $v/2$.

The ratio of available noise power in the two methods, assuming equal noise figures, will be simply given by the ratios of the noise-band-widths, B_p for the pulse method and B_m for the modulation method:

$$N_p/N_m=B_p/B_m. \tag{33}$$

In the pulse method B_p has an optimum value which depends on the conditions of measurement. If for example we are dealing with the case $\beta\ll 1$ and if the band width is limited by a RC filter in the video amplifier the optimum band width is about $\gamma H_1/4$. With this band width the available signal power is reduced to half the value it would have for a wide band and thus if ρ is the ratio of signal-to-noise in the pulse method to that in the modulation method we obtain in this case

$$\rho=(32T_1B_m/\gamma H_1 T_2^*). \tag{34}$$

In this case it would be possible to adjust H_1 so that if $T_1\sim 1$ sec., $\rho\sim 1$ for $B_m=\frac{1}{10}$ sec.$^{-1}$. Thus for $T_1\sim 1$ sec. the sensitivity available is comparable to that of the modulation method with a narrow band width amplifier.

Of course the comparison of signal-to-noise ratios for

FIG. 5. Proton resonance in distilled water (exact resonance). Sweep time 0.01 sec., $H_1=0.15$ gauss, $\nu_0=9.0$ mc/sec.

the two methods is only qualitative at best because of the different methods of presentation. The actual comparison of sensitivities in the two cases is controlled to a considerable extent by psychological factors which are difficult to assess.

For example: according to (31) the signal strength in the pulse method does not improve if T_1 is decreased by the addition of a paramagnetic catalyst. Actually, however, the sensitivity does improve somewhat with short T_1 since the larger pulse repetition frequencies which become possible in this case permit visual integration of noise.

A further improvement in sensitivity at the expense of some distortion can be obtained by using after the detector a tuned amplifier tuned to the frequency $\gamma H_1/2\pi$ in place of the low pass video amplifier.

VIII. OTHER FEATURES AND LIMITATIONS

In Figs. 6 and 7 the appearance of the oscillations in v in glycerin is shown somewhat off resonance. Otherwise the conditions are the same as for Fig. 4. The steady magnetic field is displaced off resonance by 0.21 gauss in Fig. 6 and by 0.51 gauss in Fig. 7. As expected from the theory the frequency of the oscillations has increased following the theoretical formula (17).

The attenuation has markedly increased from resonance to 0.2 gauss off resonance. This effect which is also observed in the case of distilled water cannot be explained by the theoretical variation of the attenuation constant as given by Eq. (8). In the present case $T_1 \sim T_2$ so $\beta \sim \alpha$ and b should be nearly independent of δ. Even in the extreme case of $\alpha=0$, there would be at most a factor of two between the attenuation time on and off resonance. The observed increase can be accounted for on the basis of the inhomogeneity effect discussed above. Off resonance the dominant term in the expression for the nutational frequency is $\gamma H_0-\omega$ rather than γH_1 and the inhomogeneity in H_0, therefore, exerts a relatively strong effect on the damping. On this basis it would be expected that once $\gamma H_0-\omega > \gamma H_1$ a further increase in $(\gamma H_0-\omega)$ would not greatly affect the damping time. This expectation is confirmed by comparison of the damping in Figs. 6 and 7.

The proximity to resonance may be estimated by observation of any one of three quantities (*a*) initial amplitude, (*b*) nutational frequency, (*c*) attenuation time. In the case of glycerin or water the attenuation time is the most sensitive criterion. In the case of a substance for which $T_2 < 1/\gamma\Delta H$ this will probably not be true, however. According to Eq. (10) the initial amplitude should decrease in going off resonance in the same proportion that the nutational frequency increases. The nutational frequency can, however, be observed with much greater sensitivity than the amplitude and will probably be the most sensitive criterion for proximity to resonance in case $T_2 < 1/\gamma\Delta H$. In Figs. 6 and 7 the initial amplitude decreases faster than the nutational frequency increases because of the action of a RC filter in the video amplifier.

According to Eq. (5) the angular nutational frequency at resonance is

$$\Omega_{\text{res}}=[\gamma^2 H_1^2-\tfrac{1}{4}(1/T_2-1/T_1)^2]^{\frac{1}{2}}. \tag{35}$$

If $T_1=T_2$ or, in any case, if $\gamma H_1 \gg \frac{1}{2}(1/T_2-1/T_1)$ we should expect that $\Omega_{\text{res}}=\gamma H_1$. This prediction has been tested and verified in the case of the water resonance. There is a possibility that $1/T_2-1/T_1$ could be measured by means of Eq. (35). This could be done, however, only for small H_1 and in this case the nutational period becomes large compared with the damping time leading to a suppression of the oscillations.

It does not appear that the pulse method will be useful in ascertaining the structure of resonance lines, since the resolving power is in general somewhat less than that of conventional methods.

In the search for unknown resonance the pulse method should be useful because of its very fast response. Its utility in this application is not vitiated by the fact that ordinarily a time $>T_1$ must elapse between pulses, since (*a*) in this case relatively short pulses which do not saturate the sample are permissible and (*b*) in the act of searching no power is absorbed by the sample until one is close to resonance. We have not as yet attempted to exploit this application.

ACKNOWLEDGMENTS

We wish to acknowledge our gratitude to Professor F. G. Dunnington and the other members of the Rutgers physics research staff for much helpful assistance and encouragement, and especially to Professors H. S. Sommers, and P. R. Weiss for their stabilization of the electromagnet. We have benefited from the technical assistance of Messrs. Monroe Sheppard, Jay Thomas, Sidney Gray, and Ralph Campbell.

APPENDIX

The Bloch equations (Eqs. (2) of the text) with constant coefficients are most conveniently handled by operational methods making use of the Laplace transform:

$$\bar{f}(p)=\int_0^\infty f(\tau)e^{-p\tau}d\tau.$$

The Laplace transforms of the Bloch equations are

$$\left.\begin{aligned}(p+\beta)\bar{u}+\delta\bar{v}&=u_0M_0,\\ -\delta\bar{u}+(p+\beta)\bar{v}+\bar{M}_z&=v_0M_0,\\ -\bar{v}+(p+\alpha)\bar{M}_z&=\alpha M_0/p+m_0M_0,\end{aligned}\right\} \tag{36}$$

where $\bar{u}$, $\bar{v}$, and $\bar{M}_z$ are the transforms, and u_0M_0, v_0M_0, and m_0M_0, the initial values of u, v and M_z, respectively.

The solutions of (16) are

$$\left.\begin{aligned}p\Delta(p)\bar{u}/M_0&=u_0p[1+(p+\alpha)(p+\beta)]+\delta\alpha+pm_0+v_0,\\ p\Delta(p)\bar{v}/M_0&=u_0\delta p(p+\alpha)+v_0p(p+\alpha)(p+\beta)\\ &\qquad -(\alpha+m_0p)(p+\beta),\\ p\Delta(p)\bar{M}_z/M_0&=u_0\delta p+v_0p(p+\beta)+(\alpha+m_0p)[(p+\beta)^2+\delta^2],\end{aligned}\right\} \tag{37}$$

where

$$\Delta(p)=(p+\alpha)(p+\beta)^2+p+\beta+\delta^2(p+\alpha) \tag{38}$$

is the determinant of the coefficients in (36).

FIG. 6. Proton resonance in glycerine. Conditions same as in Fig. 4 except that H_0 is displaced off resonance by 0.21 gauss.

FIG. 7. Proton resonance in glycerine. Conditions same as in Fig. 4 except that H_0 is displaced off resonance by 0.51 gauss.

If now we let w stand for anyone of the components u/M_0, v/M_0 or M_z/M_0 it is clear that

$$\bar{w}(p)=g(p)/p\Delta(p), \tag{39}$$

where $g(p)$ is a cubic form in p, different for the three components $\bar{u}$, $\bar{v}$, and $\bar{M}_z$. Now the equation $\Delta(x)=0$ will have at least one real negative root. Let this root be $-a$, then $\Delta(p)$ can be factored into

$$\Delta(p)=(p+a)[(p+b)^2+s^2], \tag{40}$$

where it is not implied that s is necessarily real. The expression (39) for $\bar{w}(p)$ can now be expanded in partial fractions:

$$\bar{w}(p)=\frac{A}{p+a}+\frac{B(p+b)+C}{(p+b)^2+s^2}+\frac{D}{P}. \tag{41}$$

The inverse transform of this expression is

$$w=Ae^{-a\tau}+Be^{-b\tau}\cos s\tau+(C/s)e^{-b\tau}\sin s\tau+D. \tag{42}$$

The first three terms give the transient effects and the last term the steady state. The coefficients A and D are given by

$$D=\lim_{p\to 0}[p\bar{w}(p)],$$
$$A=\lim_{p\to -a}[(p+a)\bar{w}(p)].$$

The other coefficients are most conveniently found by applying the initial conditions directly to (42). We find

$$D=g(0)/a(b^2+s^2), \tag{43}$$
$$A=-g(-a)/a[(b-a)^2+s^2]. \tag{44}$$

(a) for $w=u/M_0$

$$B=-A+D+u_0,\quad C=aA+bB-\beta u_0-\delta v_0, \tag{45}$$

(b) for $w=v/M_0$

$$B=-(A+D)+v_0,\quad C=aA+bB-m_0+\delta u_0-\beta v_0. \tag{46}$$

(c) for $w=M_z/M_0$

$$B=-(A+D)+m_0,\quad C=aA+bB+\alpha(1-m_0)+v_0. \tag{47}$$

The time constants b and s are most conveniently calculated by expanding (38) and (40) in powers of P and equating coefficients. We find

$$\begin{aligned}2b+a&=2\beta+\alpha,\\ b^2+s^2+2ab&=2\alpha\beta+\beta^2+\delta^2+1,\\ a(b^2+s^2)&=\alpha(\beta^2+\delta^2)+\beta.\end{aligned} \tag{48}$$

With the aid of the last equation the expression for D becomes

$$D=g(0)/[\alpha(\beta^2+\delta^2)+\beta]. \tag{49}$$

Substituting $g(0)$ as obtained from (37) we find for the steady state solutions

(a) $w=u/M_0$ $\quad D=\alpha\delta/[\alpha(\beta^2+\delta^2)+\beta]$, (50a)

(b) $w=v/M_0$ $\quad D=-\alpha\beta/[\alpha(\beta^2+\delta^2)+\beta]$, (50b)

(c) $w=M_z/M_0$ $\quad D=\alpha(\beta^2+\delta^2)/[\alpha(\beta^2+\delta^2)+\beta]$. (50c)

These agree with steady state solutions given by Bloch. The other coefficients are more difficult to find in the general case since they depend on the solutions of the cubic $\Delta(x)=0$. However, there are certain special cases in which this cubic has simple roots and these cases are adequate to cover most conditions of physical interest.

(a) If $\delta=0$ (the condition for exact resonance) a real root of $\Delta(x)=0$ is $x=-\beta$. Then

$$a=\beta \tag{51a}$$

and from (48)

$$b=\tfrac{1}{2}(\alpha+\beta), \tag{51b}$$
$$s^2=1-\tfrac{1}{4}(\beta-\alpha)^2. \tag{51c}$$

The coefficients A, B, and C for the various components of magnetic moment are in this case (assuming $u_0=v_0=0$)

(1) $w=u/M_0$

$$A=B=C=0, \tag{52}$$

(2) $w=v/M_0$

$$A=0,\quad B=\alpha/(1+\alpha\beta),\quad C=\alpha(\alpha+\beta)/2(1+\alpha\beta)-m_0, \tag{53}$$

(3) $w=M_z/M_0$

$$A=0,\quad B=m_0-\alpha\beta/(1+\alpha\beta), \tag{54}$$
$$C=(\beta-\alpha)m_0/2+\alpha\left[1-\frac{\beta(\alpha+\beta)}{2(1+\alpha\beta)}\right].$$

(b) If $\alpha=\beta$ (i.e., $T_1=T_2$) we again find as a root of $\Delta(x)=0$, $x=-\beta$. Thence

$$a=\beta, \tag{55a}$$

and from (48)

$$b=\beta, \tag{55b}$$
$$s^2=1+\delta^2. \tag{55c}$$

We find for the coefficients A, B, and C in this case (taking $u_0=v_0=0$).

(a) $w=u/M_0$

$$\left.\begin{aligned}A&=-\delta(1-m_0)/(1+\delta^2),\\ B&=\frac{\delta}{1+\delta^2}\left(\frac{\beta^2}{1+\beta^2+\delta^2}-m_0\right),\\ C&=-\delta\beta/(1+\beta^2+\delta^2),\end{aligned}\right\} \tag{56}$$

(b) $w=v/M_0$

$$\left.\begin{aligned}A&=0,\\ B&=\beta/(1+\beta^2+\delta^2),\\ C&=-m_0+\beta^2/(1+\beta^2+\delta^2),\end{aligned}\right\} \tag{57}$$

(c) $w=M_z/M_0$

$$\left.\begin{aligned}A&=(\beta^2+\delta^2)/(1+\beta^2+\delta^2),\\ B&=(m_0-\beta^2/(1+\beta^2+\delta^2))/(1+\delta^2),\\ C&=\beta/(1+\beta^2+\delta^2).\end{aligned}\right\} \tag{58}$$

(c) A third special case of interest occurs when the radio-frequency magnetic field H_1 is sufficiently large so that $\alpha\ll 1$ and $\beta\ll 1$. Since $\alpha<\beta$ the second of these conditions implies the first. It is possible to evaluate the time factors a, b, and s under the less restrictive condition $\beta-\alpha\ll 1$.

A real root of $\Delta(x)=0$ can be extracted in this case as follows: Putting

$$z=x+\beta=\beta-a.$$

We find that $\Delta(x)=0$ reduced to

$$z=(\beta-\alpha)[1-1/(1+\delta^2+z^2)].$$

It is evident that $z<(\beta-\alpha)$ and so if $\beta-\alpha\ll 1$, $z\ll 1$ and by iteration we find

$$z=(\beta-\alpha)\frac{\delta^2}{1+\delta^2}\left\{1+\frac{\delta^2(\beta-\alpha)^2}{(1+\delta^2)^3}+\frac{\delta^4(2-\delta^2)(\beta-\alpha)^4}{(1+\delta^2)^6}+O[(\beta-\alpha)^6]\right\}.$$

For most purposes it suffices to include only the leading term.

This gives

$$a=(\beta+\alpha\delta^2)/(1+\delta^2)+O[(\beta-\alpha)^3], \quad (59a)$$
$$b=\beta-\tfrac{1}{2}(\beta-\alpha)/(1+\delta^2)+O[(\beta-\alpha)^3], \quad (59b)$$
$$s^2=1+\delta^2+O[(\beta-\alpha)^2]. \quad (59c)$$

For the coefficients A, B, and C we find in this case (again taking $u_0=v_0=0$).

(1) $w=u/M_0$

$$\left.\begin{aligned} A&=\frac{\delta}{1+\delta^2}\left(m_0-\frac{1+\delta^2}{\delta^2+\beta/\alpha}\right)+O(\beta^2),\\ B&=-\frac{m_0\delta}{1+\delta^2}+O(\beta^2),\\ C&=\frac{\delta}{1+\delta^2}\left[\frac{m_0(\beta-\alpha)(\frac{1}{2}-\delta^2)}{1+\delta^2}-\alpha\right]+O(\beta^3),\end{aligned}\right\} \quad (60)$$

(2) $w=v/M_0$

$$\left.\begin{aligned} A&=-\frac{\beta-\alpha}{(1+\delta^2)^2}\cdot\delta^2\left(m_0-\frac{1+\delta^2}{\delta^2+\beta/\alpha}\right)+O(\beta^3),\\ B&=\frac{1}{1+\delta^2}\left[\frac{(\beta-\alpha)\delta^2 m_0}{1+\delta^2}+\alpha\right]+O(\beta^3),\\ C&=-m_0+O(\beta^2).\end{aligned}\right\} \quad (61)$$

(3) $w=M_z/M_0$

$$\left.\begin{aligned} A&=\frac{\delta^2}{1+\delta^2}\left(m_0-\frac{1+\delta^2}{\delta^2+\beta/\alpha}\right)+O(\beta^2),\\ B&=m_0/(1+\delta^2)+O(\beta^2),\\ C&=\alpha+m_0(\beta-\alpha)(\tfrac{1}{2}+2\delta^2)/(1+\delta^2)+O(\beta^3).\end{aligned}\right\} \quad (62)$$

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Atomic Beam Magnetic Resonance Experiments with Radioactive Elements Na^{22}, K^{40}, Cs^{135}, and Cs^{137}*

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Modifications of the atomic beam magnetic resonance method for the determination of nuclear spins and moments by observation of the hyperfine structure of atomic ground states are described which make it possible to work with very small quantities in samples with low concentrations. These modifications include the analysis of the beam by means of a mass spectrometer while performing the resonance experiment. A new source of beams of atomic alkali metals is described. The results are shown in the following tabulation:

	Spin	h.f.s. $\Delta\nu$ mgc/sec.	Nuclear magnetic moment nuclear magnetons
Na^{22}	3	1220.64 ±0.04	1.746 ±0.003
K^{40}	4	1285.73 ±0.05	−1.290 ±0.005
Cs^{135}	7/2	9724 ±8	2.724 ±0.010
Cs^{137}	7/2	10,126 ±7	2.837 ±0.010

ATTEMPTS to compare the various forms of the Fermi theory of beta-ray disintegration with experiments usually meet with the difficulty that those characteristics of the decay which make it possible to observe the nuclear angular momentum directly are just those characteristics which make the observation of the energetics of the decay more difficult and less precise. In particular, the long half-lives of C^{14}, K^{40}, and Rb^{87} permit the use of large amounts of material in observation of nuclear spin,[1,2] but the determination of the shapes [3,4] of their beta-ray spectra, especially in the low energy regions, is less reliable than could be obtained with more active materials because of the effect of scattering in the source. For tritium, on the other hand, one can work with extremely active sources because the radiation (beta-rays of less than 18 kev) is all absorbed by the containing vessel, but only recently has the spectrum shape been observed.[5] It is therefore desirable to develop a method for the observation of nuclear moments which can be applied to more energetic substances with shorter half-lives, since it is for such materials that decay schemes and spectra can be well studied. Preliminary calculations and experience with K^{40} indicated that the molecular beam magnetic resonance method might well be applicable to radio isotopes of the alkali metals because of the excellent existing method of detection. The techniques described in the present paper, devoted to Na^{22}, K^{40}, Cs^{135}, and Cs^{137}, can be extended to other nuclear species with some additional development. From the point of view of radioactive decay, Na^{22} has been most carefully studied by Good, Peaslee, and Deutsch[6] and Cs^{137} is being extensively examined by a number of workers including Townsend, Cleland, and Hughes[7] and Mitchell and Peacock.[8]

* Preliminary reports of some of these results are given by Luther Davis, Jr., Phys. Rev. **74**, 1193 (1948), Phys. Rev. **76**, 435 (1949), and D. E. Nagle, Phys. Rev. **76**, 847 (1949).

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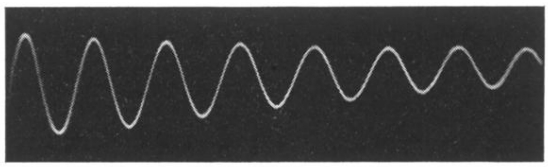

FIG. 4. Proton resonance in glycerine (exact resonance). Sweep time = 0.01 sec., $H_1 = 0.17$ gauss, $\nu_0 = 9.0$ mc/sec.

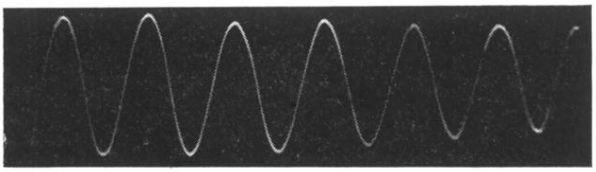

FIG. 5. Proton resonance in distilled water (exact resonance). Sweep time 0.01 sec., $H_1=0.15$ gauss, $\nu_0=9.0$ mc/sec.

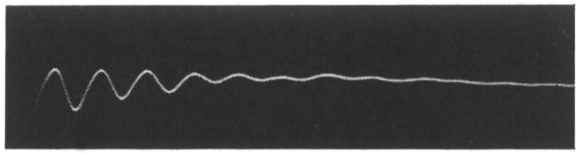

FIG. 6. Proton resonance in glycerine. Conditions same as in Fig. 4 except that H_0 is displaced off resonance by 0.21 gauss.

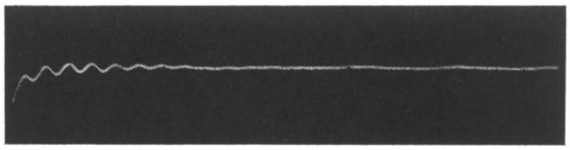

FIG. 7. Proton resonance in glycerine. Conditions same as in Fig. 4 except that H_0 is displaced off resonance by 0.51 gauss.