Letters to the Editor

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Coincident Bursts in Small Ionization Chambers*

C. G. MONTGOMERY AND D. D. MONTGOMERY
*Sloane Physics Laboratory, Yale University, New Haven, Connecticut***
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EVIDENCE has already been reported[1] to show that many bursts of ionization occurring in a large unshielded ionization chamber result from nuclear disintegrations in the chamber walls rather than from showers of electrons from the atmosphere. Similar conclusions have also been drawn by other experimenters.[2] These observations have been extended to smaller chambers by measuring bursts occurring simultaneously in two spherical chambers 3 inches in diameter and coincidences between bursts and the discharges of a counter set that detected electrons in air showers. The chambers were similar to those of R. T. Young[3] who made observations of bursts under lead shielding. A pressure of 200 lbs./in.^{2} of argon was used and the bursts were recorded photographically by means of an electrometer tube, a galvanometer, and a motor-driven camera. The counter discharges flashed a lamp which also made a record on the sensitive paper. The two chambers were separated 9 cm center to center and three counter trays were placed at various distances in the same horizontal plane. The ionization produced in the chamber corresponded to that from about 10 or more electrons traversing the chamber, or to an energy loss in the chamber of about 1.5 Mev. The counter trays each had an area of 350 cm^2 and hence their efficiency for the detection of an air shower in the range of densities involved was very nearly 100 percent.

Observations were made both at sea level and at an elevation of 3500 meters at Climax, Colorado. The size-frequency distribution of bursts is approximately a power-law distribution with an exponent which increases slightly with increasing size of burst. For the integral distribution the exponent is about 3. It was found that no coincidences between the chambers or between chambers and counters occurred that could not be explained by the finite resolving time of the apparatus. This resolving time was determined by the galvanometer period and by the speed of the photographic recorder and was found to be 1.5 sec. The fraction f of the bursts which were coincident fortuitously is equal to $2NT$, where N is the rate of occurrence of the bursts and T the resolving time. The observed values of N were between 100 per hour and 10 per hour and hence f was between 8.6 percent and less than 1 percent, depending on the sizes of bursts considered. Similarly the observed triple coincidences could also be accidental. We must conclude therefore that air showers do not contribute appreciably as a cause of the bursts in these chambers.

Since the chamber walls were thin ($\frac{1}{32}$-inch steel) and care was taken to remove all heavy material from the neighborhood of the chambers, cascade showers produced locally also cannot be the cause of the bursts observed. Heavily ionizing particles must therefore be invoked. By a lucky accident, one of the spheres was contaminated with a source of alpha-particles and observations made at low pressure showed about 10 times as many bursts in one chamber as in the other. At the high pressure[3] used, however, the numbers of bursts in the two chambers were the same. The alpha-particle ionization was suppressed by recombination in the weak field near the chamber walls and did not contribute. The bursts must therefore be explained by heavily ionizing particles of long enough range to bring them within the high field region of the chamber. Protons, fast alpha-particles, and slow mesons produced by nuclear disintegrations in the chamber walls remain as possible sources of the ionization bursts. From the known range-energy relation, a single slow meson would have to have an energy between 4.1 and 1.5 Mev to produce a burst. It seems unlikely that many such particles are present.

* A preliminary account of these observations was presented at the Cambridge meeting of the New England Section of the American Physical Society, May, 1948.

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[2] Bridge, Hazen, Rossi, and Williams, Phys. Rev. **74**, 1083 (1948); H. Carmichael, Phys. Rev. **74**, 1667 (1948).
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Microwave Absorption Spectra of Paramagnetic Salts*

YU TING, ROY C. WEIDLER, AND DUDLEY WILLIAMS
Ohio State University, Columbus, Ohio
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WE have recently observed magnetic resonance absorption in the following paramagnetic salts: chromium fluoride, CrF_3, chromium bromide, $CrBr_3$, chromium sulfite, $Cr_2(SO_3)_3$, manganese tartrate, $MnC_4H_4O_6$, manganese carbonate, $MnCO_3$, manganese acetate, $Mn(C_2H_3O_2)_2$, and ferric ammonium sulfate $FeNH_4(SO_4)_2\cdot 12H_2O$. With incident radiation of wave-length 3.3 cm, intense absorption was noted when the external magnetic field was in the range 3000–4000 gauss; the observed absorption peaks were in the vicinity of 3500 gauss. In the measurements completed thus far, powdered samples of the salts were mounted in a resonant cavity. Further work on single crystals of the above salts is in progress.

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The Variational Method for the Continuous Wave Function of an Electron in the Field of a Neutral Atom

SU-SHU HUANG
Yerkes Observatory, University of Chicago, Williams Bay, Wisconsin
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SINCE L. Hulthén first formulated the variational method for the treatment of the continuous spectrum[1] in the quantum theory, several papers have appeared[2] in which his method has been applied to problems of nuclear scattering. In this note we shall generalize Hulthén's principle for the case of electron scattering by a neutral atom.

Considering first the simplest case of a free electron in the field of a neutral hydrogen atom, we have the following asymptotic form for the wave function

$$\psi_\infty=(2l+1/4\pi)^{\frac{1}{2}}\{(l/\gamma_2)^{-\gamma_1}P_l(\cos\theta_2)\sin(k\gamma_2-l\pi/2+\eta)\pm(e/\gamma_1)^{-\gamma_2}P_l(\cos\theta_1)\sin(k\gamma_1-l\pi/2+\eta)\}. \quad (1)$$

(We have adopted the Bohr radius and Rh as the unit of length and of energy, respectively. The other symbols have their standard meanings.)

Considering

$$\mathfrak{L}=\int\int_{\infty}\psi(L-E)\psi d\tau_1 d\tau_2, \quad (2)$$

where

$$L=-\nabla_1^2-\nabla_2^2-(2/\gamma_1)-(2/\gamma_2)+(2/\gamma_{12}), \quad (3)$$

we have by Green's theorem

$$\int\psi\nabla_1^2\delta\psi d\tau_1=\int\delta\psi\nabla_1^2\psi d\tau_1 +\oint_{\infty}[\psi(\partial\delta\psi/\partial\gamma_1)-\delta\psi(\partial\psi/\partial\gamma_1)]dS_1. \quad (4)$$

Now substituting the asymptotic expansion of the wave function (1) in the surface integral of (4) (evaluated at $\gamma_1\rightarrow\infty$), we can show that

$$\int\int_{\infty}\psi\nabla_1^2\delta\psi d\tau_1 d\tau_2=\int\int_{\infty}\delta\psi\nabla_1^2\psi d\tau_1 d\tau_2-\pi k\delta\eta. \quad (5)$$

The corresponding integral with the subscript 2 can be similarly transformed. Hence

$$\delta\mathfrak{L}=2\int\int\delta\psi(L-E)\psi d\tau_1 d\tau_2+2\pi k\delta\eta. \quad (6)$$

If ψ satisfies the wave equation, then we have

$$\delta\mathfrak{L}=2\pi k\delta\eta, \quad (7)$$

which provides the generalization of Hulthén's principle for the present case.

From this derivation it is evident that the same result would follow, also, for the case of a free electron in the field of a *neutral* atom with more than one electron. In case the atom is not neutral the asymptotic form of ψ will be different from (1); but the necessary modifications can be easily made.

In practical applications, some form for ψ, involving certain constants, can be assumed and the constants be then determined by the condition $\mathfrak{L}=0$ and $\delta\mathfrak{L}=0$. This procedure will be analogous to what Hulthén has adopted in his paper.

In spite of the general nature of the above method, the mathematical formula for actually working out the variational integral are very complicated. It would therefore seem that a somewhat less general but a mathematically simpler formulation of the principle is more suitable. This is the generalization of Tamm's formulation for the case considered by Hulthén.

For the case of S scattering by a neutral hydrogen atom we can choose the three coordinates γ_1, γ_2, and γ_{12} (which is the distance between the two electrons) to describe the wave function, since for this case, we have spherical symmetry. The wave equation can be written in terms of these coordinates and with the substitution

$$\psi=(e^{-\gamma_1}/\gamma_2)[\sin k\gamma_2+u(\gamma_1,\gamma_2,\gamma_{12})\cos k\gamma_2], \quad (8)$$

it can be shown that the resulting differential equation for $u(\gamma_1,\gamma_2,\gamma_{12})$ is the Euler equation of the following variational integral:

$$\delta\mathfrak{F}=\delta\int_0^{\infty}d\gamma_2\int_0^{\infty}d\gamma_1\int_{|\gamma_1-\gamma_2|}^{\gamma_1+\gamma_2}d\gamma_{12} \times F(u_1,u_2,u_{12},u;\gamma_1,\gamma_2,\gamma_{12})=0, \quad (9)$$

where

$$F=e^{-2\gamma_1}\cos^2 k\gamma_2\bigg[\frac{\gamma_1\gamma_{12}}{\gamma_2}(u_1^2+u_2^2+2u_{12}^2) +\frac{\gamma_1^2-\gamma_2^2+\gamma_{12}^2}{\gamma_2}u_1u_{12}+\frac{\gamma_1(\gamma_2^2-\gamma_1^2+\gamma_{12}^2)}{\gamma_2^2}u_2u_{12} -\frac{2\gamma_1\gamma_{12}}{\gamma_2}\left(\frac{1}{\gamma_2}-\frac{1}{\gamma_{12}}\right)(2u\tan k\gamma_2+u^2)\bigg] \quad (10)$$

and

$$u_1=\partial u/\partial\gamma_1,\quad u_2=\partial u/\partial\gamma_2,\quad u_{12}=\partial u/\partial\gamma_{12}. \quad (11)$$

The foregoing two formulations of the variational principle can be shown to be equivalent: For, with the form of the wave function expressed in (8) $\delta\mathfrak{L}$ becomes $(\frac{1}{2})\delta\mathfrak{F}$ provided $u(\gamma_1,\gamma_2,\gamma_{12})$ satisfies the equation governing it.

It is of interest to note that the integral reduces to that given by Tamm in the special case $u=u(\gamma_2)$ only, and $-2(1/\gamma_2-1/\gamma_{12})$ can be approximated by an expression which involves γ_2 only (as for example by a potential $V(\gamma_2)$ of the Hartree field of the hydrogen atom). For, in this special case, the integration with respect to γ_{12} and γ_1 can be effected directly and the final integral

$$\mathfrak{F}=\tfrac{1}{2}\int_0^{\infty}d\gamma_2[u_2^2\cos^2 k\gamma_2+Vu^2\cos^2 k\gamma_2 +2Vu\cos k\gamma_2\sin k\gamma_2] \quad (12)$$

is identical with what Tamm gives.

Further work on this problem is in progress.

My sincere thanks are due Professor S. Chandrasekhar who suggested this problem to me and also made invaluable discussions.

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Čerenkov Radiation Effect in Meson Theory

W. W. Wada
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan
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IN the present note the idea of Čerenkov radiation is applied to the equations of a scalar meson in the hope of accounting for some instances of multiple meson production in a phenomenological way. The mesonic Čerenkov radiation is expected to occur if the source of the mesonic field, *viz.*, the nucleon, moves with velocity greater than the phase velocity of the propagation of the meson field itself.

The relativistic energy-momentum relationship gives $\omega/\kappa=c(1+\kappa_0^2/\kappa^2)^{\frac{1}{2}}$ as the phase velocity of the meson wave in vacuum, where ω and κ are the frequency and the wave number of the phase wave respectively, κ_0 is the reciprocal Compton wave-length of the meson, and c the velocity of light. This phase velocity is always greater than c. Therefore, in order to have conditions necessary for the mesonic Čerenkov radiation, phenomenological field equations endowed with constants analogous to the ϵ and μ of Maxwellian theory will have to be used.

In the simplest case, that of a scalar meson field, at most three such nuclear material constants may occur; but when the Gordon-Klein equation is formed by elimination, only two constants n_1 and n_2 can appear:

$$\Delta\phi-(n_1^2/c^2)(\partial^2\phi/\partial t^2)-(\kappa_0^2/n_2^2)\phi=0.$$

The fact that the Gordon-Klein equation has three terms indicates that, no matter what kind of meson field is considered, there would occur at most two "indices of refraction," one corresponding to the electromagnetic case, *viz.*, n_1, and the other to be associated with the Compton wave-length, *viz.*, n_2.

Assuming a plane wave solution to this equation one obtains

$$\omega/\kappa=(c/n_1)(1+\kappa_0^2/(n_2^2\kappa^2))^{\frac{1}{2}}$$

as the phase velocity of meson wave in nuclear matter. It is clear from this expression that for $n_1>1$ and for large values of κ, the phase velocity of the meson wave may become less than c and hence less than the velocity of the incident nucleon.

Considerations similar to those of Cox,[1] who obtained the expression for the electromagnetic Čerenkov radiation in which