The Intensity of Heavily Ionizing Penetrating Particles at 4300-Meter Altitude*

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The counting efficiency of a low pressure Geiger-Mueller counter for the ionizing cosmic radiation penetrating various thicknesses of lead has been measured at 4300-meter altitude. From existing data it is possible to determine the counting efficiency expected for mesotrons alone at that altitude. For thicknesses greater than 100 g/cm² electrons are excluded, and the observed excess in counting efficiency over that expected for mesotrons can be interpreted as due to heavily ionizing particles. On the assumption that these are protons the data indicate, for example, that 17 ± 5 percent of the radiation penetrating 167 g/cm² Pb consists of protons.

THE ionizing component of cosmic radiation in the atmosphere is thought to be comprised of electrons, mesotrons (of various kinds), protons, and heavier nuclear particles. The intensity variations of the different sub-components with atmospheric depth furnish the most useful information from which conclusions may be drawn regarding the collision processes involved in their genesis and absorption.

The principal means[1] which have been employed heretofore to distinguish these various particles are the cloud chamber and the proportional counter. Geiger counter experiments have also been performed in which some particular property of the radiation concerned (e.g., radioactive decay, absorption properties, shower production) is utilized to distinguish the component. In addition it was suggested some years ago[2] that the use of low pressure Geiger counters might afford a means of discriminating between components without sacrificing the simplicity and ruggedness of ordinary coincidence circuits.

We have recently given further attention to this suggestion with the aim of using such methods to resolve the components at various altitudes. The basis of the method lies in the dependence of the efficiency of a low pressure counter upon the primary ionization of a particle which traverses its sensitive volume. The dependence which is well known is as follows:

$$\epsilon=1-e^{-l_a JP}. \tag{1}$$

Here l_a is the average path length through the sensitive volume, J the primary ionization (N.T.P.), P the counter pressure in atmospheres. Such a counter could count 10^8-ev mesotrons ($J=$min.) with 20 percent efficiency but 10^8-ev protons ($J=4\times$min.) with about 60 percent efficiency. The relation given above depends only upon the assumptions that the initiation of a single ion pair within the counter will cause a discharge and that a negligible number of secondaries emerge from the counter wall upon the passage of a particle. Both points have received indirect experimental confirmation;[2,3] the latter is investigated more thoroughly in the appendix.

The low pressure counter method has been successfully applied to the determination of the primary ionization in various gases by sea-level cosmic radiation[2,3] and to the variation of primary ionization with the energy of fast electrons.[4] This paper reports the results of an experiment carried out at 4300-meters altitude (summit of Mt. Evans, Colorado) in which was observed the variation of the counting efficiency of a low pressure counter with the penetrating power of the ionizing radiation at that altitude. The results indicate the presence of heavily ionizing penetrating particles. If it is assumed that they are protons, a rough determination of their intensity relative to the total ionizing component penetrating various thicknesses of lead

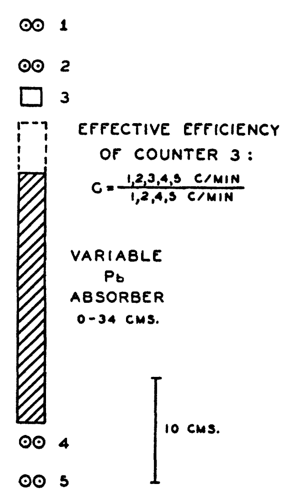

FIG. 1. Counter telescope.

* This investigation was assisted by the joint program of the ONR and the AEC. Also supported in part by the National Geographic Society.
[1] See, for example, W. F. G. Swann, Reports on Progress in Physics 10, 1 (1946).
[2] W. E. Danforth and W. E. Ramsey, Phys. Rev. 49, 854 (1936).
[3] M. G. E. Cosyns, Bull. Acad. Belg. 5, 498 (1937).
[4] F. L. Hereford, Phys. Rev. 74, 579 (1948).

is possible. With the exception of the part played by σ-mesons,[5] this component is probably responsible for the observed nuclear reactions in cosmic-ray phenomena.

EXPERIMENTAL METHOD

The counter telescope employed in the present experiment (Fig. 1) consisted of four trays of argon-butane counters (1, 2, 4, 5) and a single low pressure hydrogen counter (3) constructed of square cross-section brass tubing. The total counter wall thickness in the telescope was 6.1 gm/cm^2. Counter 3 was filled with pure hydrogen to a pressure of 4.0 cm Hg, the hydrogen being admitted through a palladium leak after the usual cleaning and baking procedures were completed. In the coincidence circuits this counter was quenched externally by a circuit of the Neher-Harper type. The square cross-section cathode was used in order that existing calculations[6] could be applied directly to the determination of the average path length through the counter.

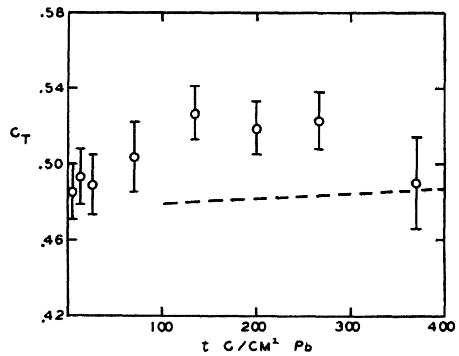

FIG. 2. The observed ratio of fivefold to fourfold coincidence rates for the radiation penetrating various thicknesses of lead. The dotted line shows the expected ratio for mesotrons alone at 4300-meters altitude.

The actual experiment consisted only of determining the ratio $G_T(t)$,

$$G_T(t)=\frac{(1,\,2,\,3,\,4,\,5)c/\text{min.}}{(1,\,2,\,4,\,5)c/\text{min.}}. \tag{2}$$

This clearly gives the effective efficiency of counter 3 for all particles of sufficient energy to penetrate the thickness t g/cm^2 Pb or *of sufficient energy to create secondaries in the lead which penetrate trays 4 and 5.* The data which were obtained are shown in Fig. 2 with the statistical errors indicated by the vertical lines through the experimental points.

The following factors are of interest concerning the possibility of instrumental errors. Fivefold and fourfold coincidences were recorded simultaneously. With proper normalization the fourfold rates yield an absorption curve in lead which is in agreement with that obtained by Hall[7] at the same altitude. Data were taken with the thicknesses of lead chosen in random order (e.g., 34 cm, 2 cm, 18 cm, etc.). Hence a progressive instrumental change could not account for the shape of the experimental curve in Fig. 2. The data, however, are in error due to various incidental but real effects, and these have been considered and dealt with as follows:

Accidental coincidences.—The resolving time of the circuits employed was approximately 8×10^{-6} sec. rendering both the fourfold and fivefold accidental rates completely negligible with respect to the genuine coincidence rates.

Side showers.—The contribution to the counting rates as a result of side showers was determined by placing tray 1 out of line, and inserting enough lead to prevent real (2, 4, 5) coincidences due to electrons belonging to cascade showers which might also discharge tray 1. Fivefold and fourfold rates were taken and found to be less than 1.5 percent of the genuine coincidence rates. Their ratio showed that counter 3 had an *over-all efficiency* of 25 ± 5 percent for side showers. On the other hand, the observed values of the counting efficiency for real events (Fig. 2) were in the neighborhood of 50 percent. It is then clear that side shower effects tend to lower the experimental points of Fig. 2; hence the choice to neglect them is a conservative one.

Secondaries from walls accompanying mesotrons.—A more important process is the production in the counter walls above counter 3 of secondary electrons which penetrate into the sensitive volume and discharge the counter coincidentally with the passage of a mesotron. Though the number of such events is small their effect must be considered in interpretation of the results. The probability that a secondary electron will be produced by a mesotron traversing a small thickness of material ($t\ll$radiation length) is easily computed from the differential collision probability for spin 0 mesotrons given by Bhabba[8] (see Appendix). It is further possible to estimate for a given mesotron energy the probability that such an electron will penetrate the remaining thickness of the material and emerge in this case into the sensitive volume of the counter. The results of this calculation are given in Fig. 3 where the solid line shows ν, the probability in percent that a mesotron of momentum p in traversing the counter walls will produce a secondary which penetrates into the sensitive volume of the counter.

Also shown is the mesotron momentum spectrum

[5] Lattes, Muirhead, Occhialini, Powell, Nature **160**, 486 (1947).
[6] E. E. Witmer and M. A. Pomerantz, J. Frank. Inst. **246**, 293 (1948).
[7] D. B. Hall, Phys. Rev. **66**, 321 (1944).
[8] H. J. Bhabba, Proc. Roy. Soc. **164**, 257 (1938).

deduced from Hall's absorption curve in lead taken at 4300-meters altitude.[7] From these two distributions can be obtained the average probability of a secondary emerging from the wall per mesotron passage for all mesotrons with momenta greater than p, viz.,

$$\text{av. } \nu(p)=\int_p^\infty \nu(p)I(p)dp\Big/\int_p^\infty I(p)dp. \qquad (3)$$

If we now convert from momentum to range in lead (g/cm²), we can plot $f(t)$, the fraction of mesotrons capable of penetrating t gm/cm² Pb which are accompanied by a secondary electron emerging from the counter wall. It is this curve (Fig. 4) which will be of use below in correcting the data.

DISCUSSION OF RESULTS

In analysis of the results it was felt that for thicknesses of lead less than 100 g/cm², the presence of electrons in addition to mesotrons and heavier particles complicated matters to the extent that unambiguous interpretation was impossible. As a consequence it was decided to consider only those experimental points taken for greater thicknesses which amounts to an analysis of the hard component only. We shall assume then that the total vertical intensity $T(t)$ penetrating t g/cm² ($t>100$) consists of a mesotron component $M(t)$ plus an N-component, $N(t)$, consisting of protons and heavier particles. We shall adopt *coincidences per minute* as our unit of vertical intensity and denote by $G_T(t)$, $G_M(t)$, and $G_N(t)$ the ratio of fivefold to fourfold coincidence rates for that part of the indicated components penetrating t g/cm² Pb. The fraction of mesotrons penetrating t g/cm² which are accompanied by secondary electrons from the counter wall has been denoted by $f(t)$. The counter 3 will have a 100 percent counting efficiency for this fraction. We may then write for a given thickness t g/cm²,

$$T=M+N=(1,2,4,5)c/\text{min}.$$
$$G_TT=G_MM(1-f)+fM+G_NN=(1,2,3,4,5)c/\text{min}.$$

The ratio, N/T, of the N-component to the total intensity, T, penetrating t g/cm² is then,

$$N/T=\frac{G_T-G_M(1-f)-f}{G_N-G_M(1-f)-f}. \qquad (4)$$

Our experimental data furnish values of $G_T(t)$, and the fraction $f(t)$ which is small has been estimated above. It remains then to determine G_M and G_N, the expected efficiency of counter 3 for the mesotron and N-components alone.

The value of G_M for various lead thicknesses is easily determined. The operation of the telescope at sea level with 66 g/cm² Pb yields the efficiency of counter 3 for mesotrons with the sea-level momentum distribution which is well known. The observed value, corrected in this case for side showers, was

$$G_T'(66)=G_M'=.485\pm.012 \quad \text{(Swarthmore)}.$$

It is assumed that there are a negligible number of protons at sea level and relatively few mesotrons of momentum less than 10^8 ev/c both of which facts are born out by experiment. With the knowledge of the mesotron momentum distributions at sea level[9] and at 4300 meters[7] and the ratio $J_M/J_{\min}$[4] (ratio of the primary ionization by sea-level mesotrons to the minimum ionization, viz., 1.2) one can determine with the help of expression (1) values of $G_M(t)$ at 4300-meters. The above value of G_M at sea-level yields $J_M=5.8$; hence, $J_{\min}=4.85$. The variation of J with momentum

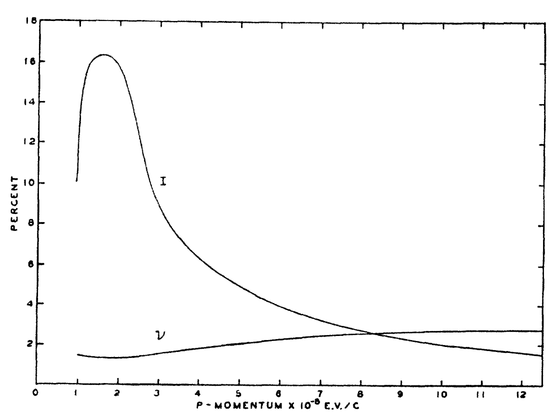

FIG. 3. The curve I represents the differential momentum spectrum for mesotrons at 4300-meters as given by Hall. ν represents the probability that a mesotron of momentum p will produce at least one secondary electron in the counter walls above counter 3 which will penetrate into the sensitive volume of this counter (see Appendix).

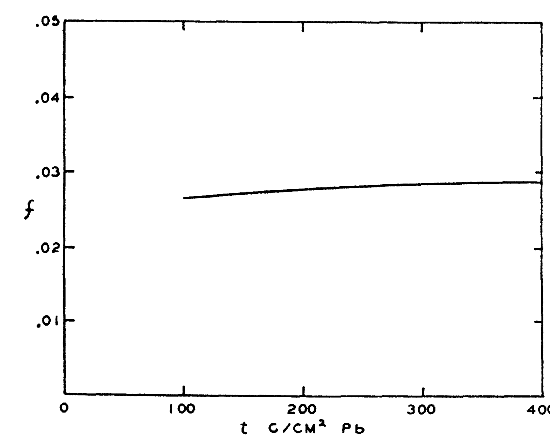

FIG. 4. The fraction of mesotrons f, penetrating t gm/cm² accompanied by secondary electrons reaching the sensitive volume of counter 3.

[9] H. Jones, Rev. Mod. Phys. 11, 235 (1939).

has been given by Bethe[10] and has been experimentally confirmed.[4] Using Bethe's curve we may then compute the effective efficiency $G_M(p)$ of counter 3 for all mesotrons of momenta greater than p on the basis of the momentum distribution $I(p)$ at 4300 meters given by Hall.

$$G_M(p)=\int_p^\infty \epsilon(p)I(p)dp \Big/ \int_p^\infty I(p)dp, \tag{5}$$

where

$$\epsilon(p)=1-e^{-l_a J(p)P}.$$

It should be realized that this seemingly complicated process is, in principal, very simple, and is not dependent upon arbitrary assumptions. The value of the minimum primary ionization used in the graphical integrations of expression (5) was determined from the sea-level data taken with the same apparatus with which the data of Fig. 2 were obtained. If in the expression (5) we now convert from momentum to range in g/cm² Pb we have $G_M(t)$ as shown in Fig. 5. The fact that the expected values of $G_M(t)$ at 4300-meters are generally less than the observed value $G_M'(66)$ at sea level is in accordance with the greater intensity of mesotrons with the minimum ionization ($p\sim 10^8$ ev/c) at 4300 meters.

The values of N/T are relatively insensitive to the choice of $G_N(t)$. Undoubtedly, some particles heavier than protons exist at 4300 meters but they are presumably few in number. Consequently, for a first approximation we shall assume that the N-component consists of protons alone and use the values of $J_P(p)$ given by Bethe[10] and an assumed p^{-2} differential intensity distribution to determine $G_N(t)$. This curve is also shown in Fig. 5.

Now with all quantities on the right side of Eq. (4) determined for various values of lead thickness

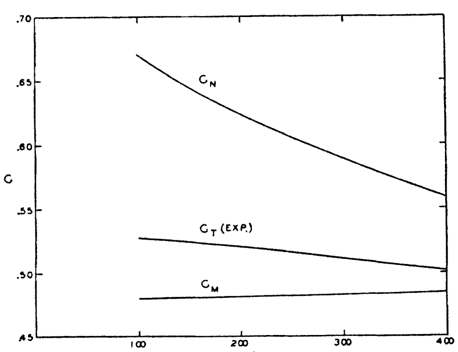

FIG. 5. The curve G_N represents the expected counting efficiency for protons penetrating various thicknesses of lead assuming a p^{-2} differential momentum distribution. G_T is a smooth curve through the experimental points of Fig. 2. G_M represents the expected counting efficiency for mesotrons penetrating various thicknesses of lead at 4300-meters altitude.

[10] H. A. Bethe, *Handbuch der Physik* (1933), 24, p. 522.

we can compute N/T, the fraction of the total ionizing component penetrating t g/cm² which consists of protons (Fig. 6). The relative probable errors of the points defining the N/T curve are about 30 percent.† It is seen, for example, that the fraction of protons in the hard component ($t>167$ gm/cm²) is approximately 17 ± 5 percent. Though this value is somewhat higher than the estimates of proton intensity from previous cloud chamber work at 4300 meters,[11] it seems to be in fair agreement with the results of Anderson *et al.*[12] who deduced that approximately 33 percent of the particles observed by them in a cloud chamber at 30,000 ft. (9100 meters) with momenta greater than 4×10^8 ev/c were protons. The unpublished Pike's Peak data of Anderson and Neddermeyer is also quoted in reference 12 as indicating that the change in the characteristics of the penetrating component is much smaller between 4300 meters and 9100 meters than between sea level and 4300 meters. This fact is, of course, supported by the above data. The absolute intensity of protons, N, based on the experimental data is also shown in Fig. 6.

In arriving at this absorption curve, $N(t)$, we have assumed a p^{-2} differential distribution of protons to exist in order to determine $G_N(t)$. Actually these two distributions very nearly coincide in the region of momenta considered. However, the finally deduced curve depends most heavily upon the experimental data (i.e., the difference between $G_T(t)$ and $G_M(t)$). The choice of a $p^{-2.5}$ distribution, for example, does not appreciably alter the values of N/T or the shape of the absorption curve $N(t)$.

The point should be made that if the computation were made from the experimental points of Fig. 2 rather than the smooth curve through these points, the values of N/T and hence of N might depart from the curves of Fig. 6. Indeed for the point at 370 gm/cm² N/T is negative with a sufficiently large probable error to render this particular value meaningless. The values of N/T at smaller thicknesses, however, differ from zero by two to three times the probable error.

ACKNOWLEDGMENTS

The assistance of Mr. Alden Stevenson in the construction of the apparatus was a substantial contribution toward the success of the experiment. Thanks are also due Mr. M. A. Pomerantz for frequent suggestions, and to Drs. W. F. G. Swann and L. Eisenbud for their part in helpful discus-

† For thicknesses greater than 300 g/cm² the error is considerably larger (see Fig. 2).

[11] W. M. Powell, Phys. Rev. 69, 385 (1946).

[12] Adams, Anderson, Lloyd, Rau, and Saxena, Rev. Mod. Phys. 20, 334 (1948).

sions. The work at Mt. Evans was made possible through the financial support of the National Geographic Society and the facilities of the Inter-University High Altitude Laboratory. All counters used were constructed by Mr. Austin Nester.

APPENDIX

We wish to determine the probability that a mesotron of energy E will produce a secondary electron in the counter walls above counter 3 which will penetrate into the sensitive volume of this counter.

The probability $\chi(E, Q)dQ$ for a mesotron of mass μ, charge ± 1, spin 0, and energy E, traversing a thickness dx (g/cm^2), to transfer an amount of energy between Q and $Q+dQ$ to a free electron as given by Bhabba[8] is,

$$\chi(E, Q)dQ = \frac{2Cm}{\beta^2}\frac{dQ}{Q^2}\left(1-\beta^2\frac{Q}{Q_M}\right). \tag{1}$$

Here m is the electronic mass, Q_M the maximum transferable energy, and C a constant which in the notation and units of Rossi and Greisen[13] is

$$C = 0.15(Z/A).$$

Denoting by p the momentum of the incident mesotron, if $p \ll \mu^2/m \sim 2\times 10^{10}$(e.v./$c$), it can be shown that,

$$Q_M = 2m(p/\mu c) = 2m(\beta^2/(1-\beta^2)). \tag{2}$$

If further Q is small compared with both E and Q_M, it can be shown that the collision probability is independent of spin justifying the use of the spin 0 collision probability of Eq. (1). These inequalities are satisfied in the problem to be considered.

The probability of a collision occurring in the interval x to $x+dx$ in which a mesotron of energy E produces a secondary electron of energy between Q and $Q+dQ$ *which penetrates the remaining wall thickness* is

$$\chi(E, Q)P(Q, x)dQdx, \tag{3}$$

where $P(Q, x)$ is the probability that an electron of energy Q will penetrate x g/cm^2, viz.,

$$P(Q, x) = \begin{cases} 1, \text{ for } x \leqq R(Q)\cos\theta(E, Q) \\ 0, \text{ for } x > R(Q)\cos\theta(E, Q). \end{cases} \tag{4}$$

In this expression $R(Q)$ and $\theta(E, Q)$ are, respectively, the range and angle of emission of an electron of energy Q produced by a mesotron of energy E (Fig. 7). Hence the probability of a collision occurring in a thickness x g/cm^2 in which a mesotron of energy E produces a secondary electron of energy greater than η which penetrates the remaining wall thickness is

$$\nu_\eta(E) = \int_\eta^{Q_M} dQ \int_0^x \chi(E, Q)P(Q, x)dx.$$

In view of Eq. (4) this may be written,

$$\nu_\eta(E) = \int_\eta^{Q_M} \chi(E, Q)R(Q)\cos\theta(E, Q)dQ.$$

The computation of this integral can be greatly simplified if one removes the function $\cos\theta(E, Q)$ from under the integral sign and gives it a mean value $\langle\cos\theta(E)\rangle_{\text{Av}}$. The exact computation of a few cases justifies this step; the error incurred is negligible for our purposes. One can then approximate $\langle\cos\theta(E)\rangle_{\text{Av}}$ as follows. From the theory of elastic collisions,

$$\langle\cos\theta(E)\rangle_{\text{Av}} = \left[\frac{(p/\mu c)^2+1}{(p/\mu c)^2+Q_M/\bar{Q}}\right]^{\frac{1}{2}}, \tag{5}$$

where

$$\bar{Q} = \frac{\int_\eta^{Q_M} Q\chi(E, Q)dQ}{\int_\eta^{Q_M}\chi(E, Q)dQ} = \eta \log\frac{Q_M}{\eta}.$$

Hence the expression for $\nu_\eta(E)$ can be written,

$$\nu_\eta(E) = \langle\cos\theta(E)\rangle_{\text{Av}} \int_\eta^{Q_M} \chi(E, Q)R(Q)dQ. \tag{6}$$

In our case, however, for counter-wall thickness x_0 (2 g/cm^2 in our case) there is a maximum value

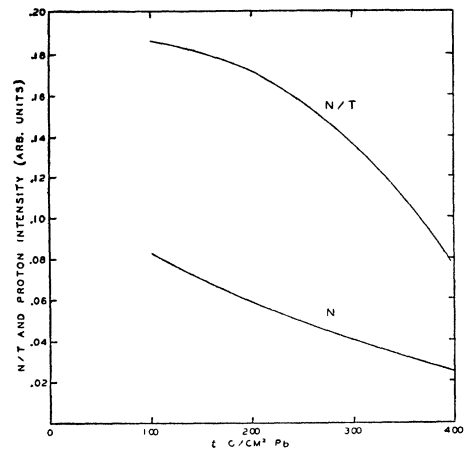

FIG. 6. The curve N/T shows the fraction of the total ionizing component penetrating various thicknesses of lead which consists of protons as derived from the data. The curve N shows the corresponding proton absorption curve in arbitrary intensity units.

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[13] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

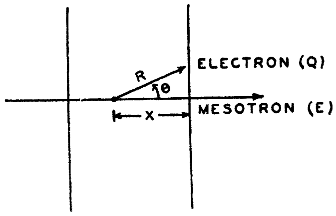

FIG. 7.

which $R(Q)$ can have, viz., $R(Q_0) \sim x_0$. Beyond this point $R(Q)$ is constant and equal to x_0. Thus Eq. (6) becomes the sum of two integrals,

$$\nu_\eta(E) = \langle \cos\theta(E) \rangle_{\text{Av}} \times \left\{ \int_\eta^{Q_0} \chi(E, Q) R(Q) dQ + x_0 \int_{Q_0}^{Q_M} \chi(E, Q) dQ \right\} \cdot \quad (7)$$

Now for energies less than Q_0 (3 Mev in our case) the range energy curve for electrons can be closely approximated by,[14]

$$R(Q) = 0.4 Q^{1.5} \qquad (Q \text{ in Mev}).$$

In view of this relation and that of Eq. (1) one obtains by performing the integrations in Eq. (7) and setting $x_0 = 2$ g/cm² for our particular case,

$$\nu_\eta(E) = \frac{2Cm}{\beta^2} \langle \cos\theta(E) \rangle_{\text{Av}} \left[k \left(\sqrt{Q_0} - \sqrt{\eta} - \frac{\beta^2 Q_0^{\frac{3}{2}}}{3Q_M} \right) + \frac{1}{Q_0} - \frac{1}{Q_M} - \frac{\beta^2}{Q_M} \log \frac{Q_M}{Q_0} \right], \quad (8)$$

where $\langle \cos\theta(E) \rangle_{\text{Av}}$ is given by Eq. (5), Q_M by Eq. (2), and $k = 8 \times 10^{-10}$ g/cm². Secondary electrons of energy less than 10^4 ev will not contribute appreciably to the values of $\nu_\eta(E)$. Hence, if one sets $\eta = 10^4$ ev, takes the value of the constant C appropriate for brass, and converts from energy to momentum of the incident mesotron, $\nu(p)$ can be plotted from Eq. (8). This curve is shown in Fig. 3 of the text for mesotron momenta between 10^8 and 1.2×10^9 ev/c. It should be noted that through the choice of the constant C appropriate to brass the number of secondaries produced in the glass counter envelopes is overestimated. Furthermore, scattering losses reduce the number reaching the sensitive volume of the counter. The computed values of $\nu(p)$ are consequently high which makes the estimated proton intensity (Fig. 6) conservatively low.

[14] L. E. Glendenin, Nucleonics 2, 12 (1948).

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Reflection and Refraction of Plane Shear Waves in Viscoelastic Media

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The shear elasticity and viscosity of liquids have been measured at ultrasonic frequencies by utilizing plane shear waves in an elastic solid and measuring the reflection loss and phase shift caused by reflection at a plane interface of the solid and a liquid. The first measurements of this type involved normal incidence. In a recent modification of the method, oblique incidence results in an enhanced effect. This paper derives the theoretical relations between the constants of the two media, the complex reflection coefficient and the angle of incidence. The theory describes some of the general properties of reflected and refracted shear waves in isotropic viscoelastic media.

INTRODUCTION

THIS paper is concerned with some theoretical relations involved in a method for measuring the shear wave parameters of viscous liquids at ultrasonic frequencies. The experimental details are described in a companion paper by W. P. Mason *et al.*, in this issue. The method is based on the measurement of a reflection coefficient for plane shear waves in an elastic solid, reflected obliquely from a plane interface of the solid and the liquid which is being investigated. A pulsing technique is used to separate the reflections of different orders at the detecting crystal, and the primary shear waves are suitably polarized so that reflection does not produce compressional waves in addition to the reflected shear waves. The complex reflection coefficient is measured by comparing the amplitudes and phases of waves reflected from the solid-liquid interface and from the same surface when the liquid is absent. Compared with normal incidence methods which had been developed previously, the oblique incidence method possesses some experimental advantages, resulting in greater accuracy of measurement, but oblique incidence involves more complicated relationships between the reflection coefficient and the constants of the two media. Solutions for these relations are derived in this paper. The theory describes some of the general properties of plane shear waves in viscoelastic media.

The elements of the system, omitting the elec-