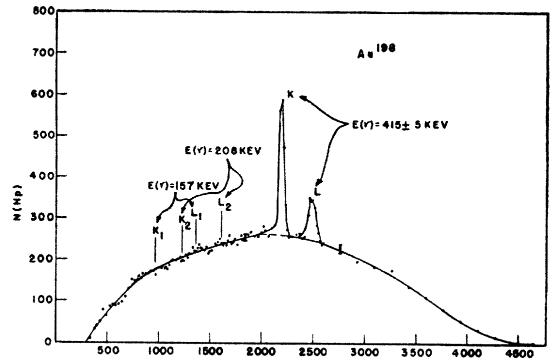

FIG. 1. The β-ray spectrum of Au^{198}.

pile. The final source was ~3 mg/cm^2, mounted on 0.5-mg/cm^2 Nylon. The counter window, as before, was 0.3-mg/cm^2 Nylon, while the resolution $\Delta H\rho/H\rho$ was 2 percent.

The momentum plot of the resulting spectrum is given in Fig. 1. The K and L lines of the high energy γ give 415±5 kev, close to the precision value of DuMond *et al.*[2] From the areas under the peaks and curve we get the following conversion coefficients, correction having been made for the window absorption at low energies:

$E(\gamma)$	K	$L+M$	$K+L+M$	$K/(L+M)$
415 kev	3.50%	1.30%	4.80%	2.69

The spin change of the γ can be estimated from the K/L ratio, assuming the M contribution to be a small part of the L peak. Extrapolating the theoretical curves of Hebb and Nelson[8] to high Z, for electric multipole radiation, the spin change is found to be $\Delta l=3$. The value of 4.80 percent for the total conversion coefficient agrees well with that of 4.70±0.24 percent determined by Wiedenbeck and Chu[3] using an entirely different method. As to the 0.157- and 0.208-Mev γ-rays, there is no sign of a conversion line at either the K or L positions. From the statistical errors in this region of the spectrum, it is possible to estimate an upper limit for the presence of the low energy γ's, after making some reasonable assumptions as to their conversion properties. If both γ's were electric dipole, then conversion coefficients of ~5 percent would be expected from Hulme's[9] theoretical data. This leads to an upper limit of ~4 percent for the presence of the γ-rays. Similarly, for electric quadrapole radiation, using the theoretical curve of Taylor and Mott,[10] this upper limit is decreased to ~1 percent. It is clear that γ's present in 15 percent ratio to the β's, would be easily detected even with conversion coefficients as low as 1 percent.

The only reasonable conclusion seems to be that the low energy γ's are associated with an impurity. Mitchell, in a communication to DuMond,[2] has suggested that this impurity is mercury. An analysis of our samples by F. Tompkins' group shows <0.01 percent Hg and Pt.

Jnanananda[11] found a conversion line at 58.4 kev, $H\rho=837$ gauss cm, which was interpreted as an Auger electron due to the 70.3 kev Hg $K\alpha$ x-ray converting in the L_{III} shell of Au^{198}. We do not find this line in our curve, even though the window is thin enough to let most of the continuous β's through at the line energy. If we use our value of 3.50 percent for the K conversion coefficient of the 411-kev γ-rays, and assume a value of ~0.9 for the fluorescence yield of the Hg K level, the expected yield of the Auger line is ~0.35 percent of the continuous β-spectrum. This value is about the limit of our sensitivity, indicating why the line was not found.

The previous value for the Au^{198} half-life,[5] 2.66±0.01 days, has been raised slightly, after recalculating some of the background corrections. Our final value is 2.69±0.02.

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Coherent Scattering of Radiation and Negative Energy States

OTTO HALPERN
University of Southern California, Los Angeles, California
AND
HARVEY HALL
U. S. Navy Department, Washington, D. C.
January 7, 1949

FOLLOWING Dirac,[1] it has been generally[2] accepted that in the absence of real pairs, the theoretical expressions for the scattering cross section are identical, whether negative energy states are assumed to be free or occupied. On the basis of this reasoning Waller[2] calculates the scattering cross section of neutral atoms assuming the negative energy states to be unoccupied, and then claims that his results apply equally to the case of occupied negative energy states (hole theory proper). Heitler[2] reports in detail the proof originally given by Dirac that the matrix elements in both versions of the theory are identical; for every matrix element of a transition leading from a state of positive energy through an intermediate occupied state of negative energy to a final state of positive energy, there exists an *identical* matrix element of a transition starting from the (previously intermediate) occupied state of negative energy and leading to the final state of positive energy, while the second part of the transition leads from the initial state of positive energy to the vacated state of negative energy.

This theorem is obviously true for phenomena like the Comptom effect, where the final state differs from the initial state of positive energy. But it is equally obvious, though so far overlooked, that the theorem cannot apply, e.g. to coherent scattering processes in which the initial and final state are the same. In this latter case, the transition from the negative energy state to the final state is as impossible as the transition from the initial state of positive energy to the not yet vacated state of negative energy. Somewhat similar remarks can be made concerning incoherent scattering processes leading to final bound states, but the discussion of them would not add anything new to the principal question.

It is therefore certain that no matrix elements exist in the hole theory proper which are *identical* with the matrix elements calculated by Waller that are proportional to Z, and give the main contribution to the coherent scattering of x-rays by atoms. One has, therefore, to turn to a closer study of those matrix elements which are characteristic for the hole theory, and have no direct analog in the other version of the theory. These new matrix elements correspond to transitions from a state of negative energy to an intermediate state of positive energy and back to the vacated state of negative energy. The intermediate states of positive energy can be either bound or free states; they, as well as the states of negative energy, have eigenfunctions which are determined by the combined potential of the nucleus and the atomic electrons.

If the quantum of incident radiation is large compared to the ionization energy of the atom but small compared to mc^2, then the Waller terms which give the main contribution to x-ray scattering are replaced by two groups of terms of the following character. The first group is of the same form as the Waller term with the characteristic difference that the form factor of the atom is not that of the real scattering atom but that of a "substitute atom." The substitute atom is defined as an assembly of electrons which occupy (as far as these solutions exist) states with the same quantum numbers as the electrons in the scattering atom. But the eigenfunctions of these electrons are the solutions of a *one-electron problem* in the field of the nucleus modified by the potential produced by all the electrons in the *scattering* atom.[3] The form factor of these terms is, therefore, in principle, different from the Waller terms.

In addition there exist terms of the type

$$\Sigma[(\psi_-{}^*(\boldsymbol{\alpha}\cdot\mathbf{A}_0)\psi_+)(\psi_+{}^*(\boldsymbol{\alpha}\cdot\mathbf{A})\psi_-)/E]$$

where the notation is customary, and the sum is to be extended over *all positive and negative* energy states. Here again the eigenfunctions of the positive and negative energy states are defined by the "substitute potential" which is, as before, the sum of the nuclear and electronic potential of the scattering atom. This additional sum over positive and negative energy states requires special treatment to avoid divergences.

The results obtained indicate that not only the proof concerning the equivalence of the two versions of the theory is erroneous, but also that the two theories lead to different predictions concerning experimental results; it is of course to be hoped that after elimination of the pseudodivergences the hole theory proper will be confirmed by observation.

We shall in the near future present the calculations leading to the results quoted, as well as extensions to the case of large quanta and bare nuclei.

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