In the non-relativistic limit, $\beta \ll 1$,

$$K_n = [1/(2n+1)]\beta^2 \sin^2(\vartheta/2),$$
$$L = [(4/3)(\log 2 - 1) - \tfrac{1}{5}]\beta^2 \sin^2(\vartheta/2), \quad (5)$$

and

$$\delta = (8\alpha/3\pi)\beta^2 \sin^2(\vartheta/2)[\log(mc^2/2\Delta E) + (19/30)], \quad (6)$$

which increases linearly with the kinetic energy of the particle. For a slowly moving particle, it is an elementary matter to include the additional scattering produced by the real emission of quanta with energies in the interval from ΔE to W. One thereby obtains the following fractional decrease in the differential cross section for scattering through an angle ϑ, irrespective of the final energy:

$$\delta = (8\alpha/3\pi)\beta^2 \sin^2(\vartheta/2)[\log(mc^2/8W) + (19/30) + (\pi - \vartheta)\tan(\vartheta/2) + [\cos\vartheta/\cos^2(\vartheta/2)]\log\csc(\vartheta/2)]. \quad (7)$$

We may remark, parenthetically, that in the same non-relativistic approximation, the radiative correction to the energy of a particle moving in an external field with potential energy $V(r)$ is[5]

$$\begin{aligned}\delta E &= (\alpha/3\pi)[\log(mc^2/2\Delta w) + (31/120)](\hbar/mc)^2\langle\nabla^2 V\rangle \\ &\quad + (\alpha/2\pi)(\hbar/2mc)\langle -i\beta\boldsymbol{\alpha}\cdot\nabla V\rangle \\ &= (\alpha/3\pi)(\hbar/mc)^2[(\log(mc^2/2\Delta W) + (19/30))\langle\nabla^2 V\rangle \\ &\quad + \tfrac{3}{4}\langle\boldsymbol{\sigma}\cdot\mathbf{L}(1/r)(dV/dr)\rangle],\end{aligned} \quad (8)$$

where $\mathbf{L}$ is the orbital angular momentum operator in in units of $\hbar$, and ΔW is an average excitation energy of the system.[6] Applied to the relative displacement of the $2^2S_{\frac{1}{2}}$ and $2^2P_{\frac{1}{2}}$ levels of hydrogen, this formula yields 1051 mc/sec., to be compared with the experimental value[7] of 1062 ± 5 mc/sec.

The extreme relativistic limit of (1) is

$$\delta = (4\alpha/\pi)[(\log(E/\Delta E) - (13/12)) \times (\log(2E/mc^2)\sin(\vartheta/2) - \tfrac{1}{2}) + (17/72) + \phi(\vartheta)], \quad (9)$$

where

$$\phi(\vartheta) = \tfrac{1}{2}\sin(\vartheta/2)\int_{\cos(\vartheta/2)}^{1}\left[\frac{\log\frac{1}{2}(1+x)}{1-x} - \frac{\log\frac{1}{2}(1-x)}{1+x}\right] \times \frac{dx}{(x^2-\cos^2(\vartheta/2))^{\frac{1}{2}}}. \quad (10)$$

The integral can be performed analytically for $\vartheta = \pi$, $\phi(\pi) = \pi^2/24$, but must be evaluated numerically for other angles. An approximation in excess, which has the correct asymptotic form at small angles, is provided by

$$\phi(\vartheta) \sim \frac{1-\cos(\vartheta/2)}{[2\cos(\vartheta/2)(1+\cos(\vartheta/2))]^{\frac{1}{2}}} \times \left[\log\frac{1}{2(1-\cos(\vartheta/2))} + \frac{1-\cos(\vartheta/2)}{2} + 1\right]. \quad (11)$$

This approximation is reasonably accurate even at $\vartheta = \frac{1}{2}\pi$, where the value yielded by (11) exceeds by only 8.6 percent the following result of a numerical calculation, $\phi(\pi/2) = 0.292$.

The asymptotic formula (9) is quite accurate for even moderate energies. Thus, with $\vartheta = \frac{1}{2}\pi$, $\Delta E = 10$ kev, and $W = 3.1$ Mev, which corresponds to $(E/mc^2)\sin(\vartheta/2) = 5$, the value of δ computed from (9) differs from the correct value, $\delta = 8.6\ 10^{-2}$, by only a few tenths of a percent. It is evident from this numerical result that radiative corrections to scattering cross sections can be quite appreciable. For the particular conditions chosen, ΔE can be materially increased (but still subject to $\Delta E \ll W$), without seriously impairing δ. Thus, with $\Delta E = 40$ kev, $\delta = 6.3\ 10^{-2}$, while $\Delta E = 80$ kev yields $\delta = 5.1\ 10^{-2}$. As to the energy dependence of δ, we remark that with a given accuracy in the determination of the energy, say $\Delta E/E = 0.04/3.6 = 1.1\ 10^{-2}$, an increase in the total energy by a factor of four produces an addition of $4.4\ 10^{-2}$ to δ. Thus, for a kinetic energy of 14 Mev, $\delta = 11\ 10^{-2}$.

The variation of δ with angle, at moderate energies, cannot be studied with the asymptotic formula (9) alone, for at small angles the condition $(p/mc)^2\sin^2(\vartheta/2) \gg 1$, which underlies this formula, will not be maintained. It is evident from (1) that δ is proportional to $\sin^2(\vartheta/2)$ at angles such that $(p/mc)^2\sin^2\frac{1}{2}\vartheta \ll 1$. However, for $W = 3.1$ Mev, $\Delta E = 40$ kev, and $\vartheta = \pi/4$, which corresponds to $(p/mc)\sin(\vartheta/2) = 2.7$, the correct value of δ, $4.2\ 10^{-2}$, exceeds that deduced from (9) by only 2 percent. For the same choice of W and ΔE, the value of δ associated with $\vartheta = 3\pi/4$ is $\delta = 7.2\ 10^{-2}$.

The verification of these predictions would provide valuable conformation for the relativistic aspects of the radiative corrections to the electromagnetic properties of the electron.

[1] W. E. Lamb and R. C. Retherford, Phys. Rev. **72**, 241 (1947).
[2] J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947); D. E. Nagle, R. S. Julian, and J. R. Zacharias, Phys. Rev. **72**, 971 (1947); P. Kusch and H. M. Foley, Phys. Rev. **72**, 1256 (1947); **73**, 412 (1948).
[3] J. Schwinger, Phys. Rev. **74**, 1939 (1948); **75**, 651 (1949).
[4] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).
[5] This result agrees with that obtained by an earlier method [J. Schwinger, Phys. Rev. **73**, 416 (1948)], and announced at the January 1948 meeting of the American Physical Society. However, in the previous method the contribution of the additional magnetic moment to the energy in an electric field had to be artificially corrected in order to obtain a Lorentz invariant result. This difficulty is attributable to the incorrect transformation properties of the electron self-energy obtained from the conventional Hamiltonian treatment, and is completely removed in the covariant formulation now employed. Independent calculations by J. B. French and V. F. Weisskopf [Phys. Rev. **75**, 338 (1949)], as well as N. M. Kroll and W. E. Lamb, Jr. [Phys. Rev. **75**, 388 (1949)], are also in agreement with Eq. (8).
[6] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
[7] R. C. Retherford and W. E. Lamb, Jr., Bull. Am. Phys. Soc. **24**, No. 1, (1949).

Measurements of Behavior and Mobility of Polyatomic Ions

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A TECHNIQUE capable of tracing continuously in time the motion of a body of ions in an electric field is desirable for the measurement of mobility and could indicate changes of mobility occurring due to changes in the nature of the moving charges (due to transfer of charge in ion-molecule collision, dissociation to smaller, or association to larger ionic masses, etc.).

The technique used by H. G. Stever[1] in obtaining a "recovery curve" for Geiger-Mueller tube discharges has been extended to give information concerning the actual progress of the sheath $r = r(t)$ by simultaneously performing three experiments:

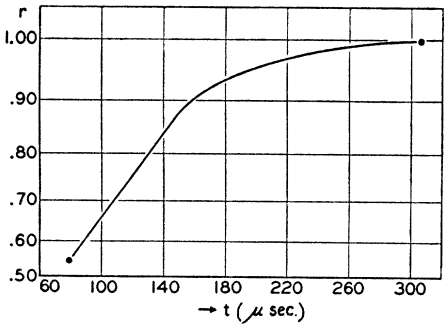

FIG. 1. Plot of r^2 *vs.* the dead-time t, showing linear relation up to $r=0.9$.

1. The "recovery curve," i.e., pulse-size of subsequent discharge pulses S as a function of time elapsed since the previous discharge pulse t, at an applied potential V_0.
2. The pulse-size S *versus* applied potential V for ordinary space-charge free operation.
3. The space-charge density q developed per pulse at $V=V_0$.

These three measurements together with the relationship

$$V=V_0-2q\log(b/r),$$

which defines the effect of the ionic sheath of space-charge q on the field condition near the wire by comparing it to the effect of an equivalent potential V in absence of space-charge is used to derive the progress of the sheath $r=r(t)$. In the case of the cylindrical geometry at hand, the mobility k can then be shown to be

$$k=\tfrac{1}{2}(C/V_0)(d/dt)(r^2),$$

so that the slope of a r^2 *versus* t plot will reveal the effective mobility at each point.

For a tube in which $r_{max}=b=1.0$ cm, plots of r^2 *versus* t obtained from mixtures of 5.0 cm Hg of argon and 2.5 cm Hg of polyatomic component P, for the six normal hydrocarbons C_nH_{2n+2} from ethane (C_2H_8) to heptane (C_7H_{16}), straight lines were obtained between the dead-time radius (at which the measurement becomes possible) to about $r=0.9$. Thereafter, a "tail" is observed for all experiments which is believed to be caused mainly by electrical end-effect and by eccentricity. Figure 1 is an example.

Figure 2 indicates relative mobility figures derived from the respective straight-line slopes.

Mass spectrometer research together with calculations of fields in the G-M tube would indicate that in the discharge-avalanche process large concentrations of fragment ions are to be expected having masses considerably smaller than those of the parent molecules. The rather straightforward trend of mobility with parent mass, however, seems to indicate that the bulk of ionic masses consists of ionized parent molecules.

The observations made concern ions over a path of some 10^4 collisions but are also *begun* only after elapse of some 10^4 collisions since their creation (because of the "dead-time" phenomenon). Charge transfer from initial "fragment" ions to ultimate parent ions may have occurred, therefore.

It can be shown that a transfer mechanism to the complete parent such as $C_3H_7^+ + C_6H_{14} \rightarrow C_3H_7 + C_6H_{14}^+$ requires additional energy, about 2.6 ev in this case, while the type of transfer exemplified by

$$C_3H_7^+ + C_6H_{14} \rightarrow C_3H_8 + C_6H_{13}^+$$

can proceed "exothermally" as is probably true generally for the latter type of transfer, namely:

$$(C_fH_{2f+1})^+ + (C_pH_{2p+2}) \rightarrow (C_fH_{2f+2}) + (C_pH_{2p+1})^+,$$

as long as $f<p$ which is true by definition of fragment and parent.

However, there is a region in the G-M discharge in which kinetic energies of the order of 2.6 ev can be obtained[2] so that the first type of exchange may also proceed.[3]

Incidentally, the measurement on ethyl alcohol demonstrates that a relationship of mobility with molecular mass is applicable only to a homologous series of molecules.

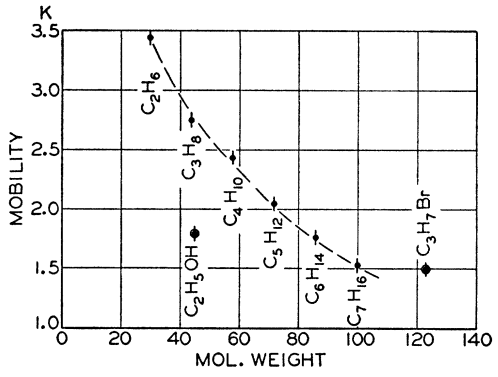

FIG. 2. Mobilities of various molecular ions as a function of molecular weight.

The transfer of ionic charge from one type of parent ion such as ethane to parent molecules of lower characteristic ionization potentials by admixing a controlled impurity of the latter molecules has also been demonstrated by the same technique of measurement.

While the experimental work is as yet incomplete, it seemed worth pointing out that the technique employed may well aid in studying a number of aspects of polyatomic ion behavior in a perhaps unique manner.

[1] H. G. Stever, Phys. Rev. **61**, 38 (1942).
[2] Serge A. Korff, Phys. Rev. **72**, 477 (1947).
[3] A. O. Allen pointed out to us that the experiment on C_3H_7Br would be indicative of the first-mentioned type of transfer reaction.

Scintillation Counting with Chrysene

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TO extend the range of response of scintillation counters to low energy beta-particles, various organic phosphors have been investigated. Chrysene has proved to be at least as efficient as anthracene for crystals of comparable size. However, with respect to ease of crystallization of relatively large size crystals, chrysene is superior to anthracene. Slow cooling of a heated solution of toluene and